Journal of Zhejiang University-SCIENCE C (Computers & Electronics)
ISSN 1869-1951 (Print); ISSN 1869-196X (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

# Stochastic computer network with multiple terminals under total accuracy rate*

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Received July 11, 2012; Revision accepted Sept. 29, 2012; Crosschecked Nov. 12, 2012

**Abstract:** From the viewpoint of service level agreements, data transmission accuracy is one of the critical performances for assessing Internet by service providers and enterprise customers. The stochastic computer network (SCN), in which each edge has several capacities and the accuracy rate, has multiple terminals. This paper is aimed mainly to evaluate the system reliability for an SCN, where system reliability is the probability that the demand can be fulfilled under the total accuracy rate. A minimal capacity vector allows the system to transmit demand to each terminal under the total accuracy rate. This study proposes an efficient algorithm to find all minimal capacity vectors by minimal paths. The system reliability can then be computed in terms of all minimal capacity vectors by the recursive sum of disjoint products (RSDP) algorithm.

**Key words:** Multiple terminals, Accuracy rate, Service level agreements (SLAs), System reliability, Minimal path
**doi:**10.1631/jzus.C1200220 **Document code:** A **CLC number:** TN914

## 1 Introduction

Computer network is one of the major medium for transmitting data/information in most enterprises. Usually, the Internet service provider (ISP) should transmit the specific demand to different customers. As the stability of computer networks strongly influences the quality of data transmissions from a source to many terminals/customers, especially for accuracy traffic measurement and monitoring, the system reliability of the computer network is always of concern for information technology departments. Many enterprises regard system reliability evaluation or improvement as crucial for network management, traffic engineering, and security tasks. In general, a computer network is usually modeled as a network topology with nodes and edges, in which each edge represents a transmission line and each node represents a transmission device such as a hub, router, or switch. In fact, a transmission line is combined with several physical lines such as twisted pairs, coaxial cables, or fiber cables. Each physical line provides a capacity and may fail; this implies that a transmission line has several states where state $c$ means that $c$ physical lines are operational. Hence, the capacity of each arc has several values. In other words, the computer network should be multistate due to the various capacities of each transmission line. Such a network is a typical stochastic flow network (Aven, 1985; Xue, 1985; Jane *et al*., 1993; Lin *et al*., 1995; Cheng, 1998; Yeh, 1998; 2004; 2005; Ramirez-Marquez and Rocco, 2009; Ramirez-Marquez *et al*., 2009; Lin, 2001; 2002; 2009; 2010; Lin and Yeh, 2011), and is called a stochastic computer network (SCN) herein. The related works about system reliability and accuracy rate for computer networks are introduced as follows.

### 1.1 Related works in assessing system reliability

Traditionally, system reliability is the probability that demand can be successfully transmitted from the single source to single terminal through the network. Several studies have evaluated the system

* Project (No. NSC 99-2221-E-011-066-MY3) supported in part by the National Science Council, Taiwan

reliability in terms of minimal cuts (Aven, 1985; Jane *et al.*, 1993; Lin, 2002; 2010; Lin and Yeh, 2011) or minimal paths (Aven, 1985; Xue, 1985; Lin *et al.*, 1995; Lin, 2001; 2009). A minimal path is an order sequence of edges from the source to the terminal with no cycle being used to decide the flow assignments in a network. For the perfect node case with only a capacity constraint, several researchers (Xue, 1985; Lin *et al.*, 1995; Yeh, 1998) have presented algorithms to generate all minimal capacity vectors for demand to evaluate the system reliability.

### 1.2 Related works involving accuracy rate

From the viewpoint of network operations, management, and engineering, service level agreements (SLAs) are an important part of the networking industry (Wang *et al.*, 2011). SLAs are used in contracts between ISPs and their customers. An SLA can be measured by many criteria: for instance, availability, delay, loss, and out-of-order packets. A basic index is the accuracy rate, which is often used to measure the performance of enterprise networks. The file is said to be transmitted correctly only if the file received at the terminal is identical to the original file. In fact, data transfer is done through packet transmission. Thus, the accuracy rate is the ratio of correct packets to all packets. The ISP should and promise to furnish with a specific accuracy rate and fulfill the requirements from all customers according to their contracts of SLAs. Therefore, from the perspective of quality of service (QoS) (Huang *et al.*, 2009; Sausen *et al.*, 2010; Zibanezhad *et al.*, 2011), maintaining a high network accuracy rate is essential for enterprises to survive in a competitive environment. Many researchers have discussed the issues related to measuring local area network (LAN) traffic (Chlamtac, 1980; Amer, 1982; Jain and Routhier, 1986), and previous studies have considered flow accuracy in traffic classification. Such flows are called elephant flows. Since high packet-rate flows have a great impact on network performance, identifying them promptly is important in network management and traffic engineering (Mori *et al.*, 2007). A conventional method for estimating the accuracy rate of large or elephant flows is the use of packet sampling. However, packet sampling is the main challenge in network or flow measurements. Statistical sampling of network traffic has been used to measure traffic on NSFNET (Claffy *et al.*, 1993). Jedwab *et al.* (1992) proposed an algorithm to limit the flow packet count estimation error of the largest flows using a statistical traffic model. Feldmann *et al.* (2001) presented a model for traffic demands to support traffic engineering and performance debugging of large ISP networks. Choi *et al.* (2003) used packet sampling to accurately estimate large flows under dynamic traffic conditions.

### 1.3 The addressed problem

In the previous references mentioned above, the system reliability is evaluated for the network with one source to one sink. However, multiple terminals and accuracy rate in a computer network are quite important criteria, but to the best of our knowledge there has been no literature that includes both criteria for assessing system reliability. From the perspective of SLAs, providers need to fulfill each customer's required demand with the accuracy rate. In the SCN, the provider can be regarded as a source, and customers are regarded as multiple terminals. The purpose of this paper is to evaluate the system reliability, i.e., whether the SCN satisfies each customer's requirement, including demand and accuracy rate. The system reliability for multiple terminals is defined as the probability that the demand at each terminal can be successfully satisfied, and all received flows should meet the total accuracy rate. An efficient algorithm is proposed to generate all minimal capacity vectors fulfilling all customers' requirements. The system reliability can be computed easily in terms of such vectors. This value of system reliability allows the network supervisor to comprehend the ability of the SCN and how it can be used to improve the SCN to enhance the system reliability.

## 2 Stochastic-flow network model

Let $G=(E, N, M)$ be a stochastic-flow network where $E=\{e_i|1\le i\le n\}$ is the set of edges, $N$ is the set of nodes, and $M=\{M_1, M_2, \ldots, M_n\}$ with $M_i$ being the maximum capacity of edge $e_i$. The network $G$ further satisfies the following assumptions:

1. The capacity of each edge is stochastic with a given probability distribution.

2. The capacities of different edges are statistically independent.

3. Flow in the network must satisfy the flow-conservation law (Ford and Fulkerson, 1962). This means no flow is lost during transmission.

4. Each node is perfectly reliable.

### 2.1 System reliability evaluation

Suppose there are totally $m$ minimal paths from the source to all terminals: $\mathrm{mp}_1$, $\mathrm{mp}_2$, ..., $\mathrm{mp}_m$. The flow model can be described in terms of capacity vector $\boldsymbol{X}=(x_1, x_2, \ldots, x_n)$ and flow vector $\boldsymbol{F}=(f_1, f_2, \ldots, f_m)$, where $x_i$ denotes the current capacity of edge $e_i$ and $f_j$ denotes the current flow on $\mathrm{mp}_j$.

The total accuracy rate is the ratio of correct data to all received data. The system reliability $R_{\boldsymbol{D},A}$ is the probability that the network can fulfill the demand $\boldsymbol{D}=(d_1, d_2, \ldots, d_r)$ and total accuracy rate $A$, where $d_u$ is the demand on terminal $t_u$. That is, $R_{\boldsymbol{D},A}\equiv\Pr\{\boldsymbol{X}|\boldsymbol{X}$ meets $(\boldsymbol{D}, A)\}$. For convenience, let $\Omega\equiv\{\boldsymbol{X}|\boldsymbol{X}$ meets $(\boldsymbol{D}, A)\}$. Then, the system reliability $R_{\boldsymbol{D},A}$ is

$$R_{\boldsymbol{D},A}=\Pr\{\Omega\}=\sum_{\boldsymbol{X}\in\Omega}\Pr\{\boldsymbol{X}\},$$

where $\Pr\{\boldsymbol{X}\}=\Pr\{x_1\}\Pr\{x_2\}\ldots\Pr\{x_n\}$ by assumption 2.

### 2.2 Relationship between X and F

Any flow vector $\boldsymbol{F}$ that is a feasible flow-pattern under $M$ satisfies

$$\sum_{j=1}^{m}\{f_j \mid e_i\in\mathrm{mp}_j\}\le M_i,\ \ i=1, 2, \ldots, n, \tag{1}$$

where $\sum_{j=1}^{m}\{f_j \mid e_i\in\mathrm{mp}_j\}$ is the amount of flow through $e_i$. Constraint (1) means that the total flow on $e_i$ (i.e., the loading of $e_i$) cannot exceed the maximum capacity of $e_i$. For convenience, the set of $\boldsymbol{F}$'s that are feasible under $M$ is denoted by $F_M$. Similarly, $\boldsymbol{F}$ is feasible under $\boldsymbol{X}$ if it satisfies

$$\sum_{j=1}^{m}\{f_j \mid e_i\in\mathrm{mp}_j\}\le x_i,\ \ i=1, 2, \ldots, n. \tag{2}$$

Thus, $F_{\boldsymbol{X}}=\{\boldsymbol{F}|\boldsymbol{F}$ is feasible under $\boldsymbol{X}\}$. Furthermore, the flow $\boldsymbol{F}$ is said to meet the demand $\boldsymbol{D}=(d_1, d_2, \ldots, d_r)$ and total accuracy rate $A$ if it satisfies

$$\sum_{j=1}^{m}\{f_j \mid \mathrm{mp}_j\in Q_u\}\ge d_u,\ \ u=1, 2, \ldots, r, \tag{3}$$

and

$$\sum_{j=1}^{m}\Big(f_j\prod(a_i \mid e_i\in\mathrm{mp}_j)\Big)\Big/\sum_{j=1}^{m}f_j\ge A, \tag{4}$$

where $Q_u$ is the set of minimal paths from the source to the terminal $t_u$. The value $a_i$ is the accuracy rate of edge $e_i$, $i=1, 2, \ldots, n$. Constraint (3) implies that the each received flow at terminal $t_u$ is larger than or equal to the demand $d_u$. In constraint (4), the value $\prod(a_i \mid e_i\in\mathrm{mp}_j)$ is the accuracy rate of minimal path $j$, and $f_j\prod(a_i \mid e_i\in\mathrm{mp}_j)$ is the value of accuracy flow through $\mathrm{mp}_j$. Thus, $\sum_{j=1}^{m}\big(f_j\prod(a_i \mid e_i\in\mathrm{mp}_j)\big)$ is the total value of accuracy flow, and the value of $\sum_{j=1}^{m}\big(f_j\prod(a_i \mid e_i\in\mathrm{mp}_j)\big)\big/\sum_{j=1}^{m}f_j$ is the total accuracy rate of the SCN.

For terminals $(t_1, t_2, \ldots, t_r)$, a capacity vector $\boldsymbol{X}$ is defined to meet the demand $\boldsymbol{D}=(d_1, d_2, \ldots, d_r)$ and total accuracy rate $A$ if there exists a flow vector $\boldsymbol{F}\in F_{\boldsymbol{X}}$ which meets the demand $\boldsymbol{D}$ and total accuracy rate $A$. We thus have the following definition:

**Definition 1** The capacity vector $\boldsymbol{X}$ meets $(\boldsymbol{D}, A)$ if and only if there exists an $\boldsymbol{F}\in F_{\boldsymbol{X}}$ meeting $(\boldsymbol{D}, A)$.

### 2.3 Generation of (D, A)-MPs

Although $R_{\boldsymbol{D},A}=\sum_{\boldsymbol{X}\in\Omega}\Pr\{\boldsymbol{X}\}$, it is an inefficient method to find all such $\boldsymbol{X}$ and then sum their probabilities for obtaining $R_{\boldsymbol{D},A}$. Let $\Omega_{\min}\equiv\{\boldsymbol{X}|\boldsymbol{X}$ is minimal w.r.t. $\le$ in $\Omega\}$ be the set of minimal vectors in $\Omega$. Each $\boldsymbol{X}\in\Omega_{\min}$ is called a $(\boldsymbol{D}, A)$-MP in this paper.

**Definition 2** $\boldsymbol{X}$ is a $(\boldsymbol{D}, A)$-MP if and only if (1) $\boldsymbol{X}\in\Omega$ and (2) $\boldsymbol{Y}\notin\Omega$ for any capacity vector $\boldsymbol{Y}$ with $\boldsymbol{Y}<\boldsymbol{X}$ (where $\boldsymbol{Y}\le\boldsymbol{X}$ if and only if $y_i\le x_i$ for $i=1, 2, \ldots, n$ and $\boldsymbol{Y}<\boldsymbol{X}$ if and only if $\boldsymbol{Y}\le\boldsymbol{X}$ and $y_i<x_i$ for at least one $i$). Then the system reliability can be rewritten as

$$R_{\boldsymbol{D},K}=\Pr\{\Omega\}=\Pr\{\boldsymbol{Y}|\boldsymbol{Y}\ge\boldsymbol{X}\text{ for a }(\boldsymbol{D}, A)\text{-MP }\boldsymbol{X}\}. \tag{5}$$

A critical property for any $\boldsymbol{X}$ meeting $(\boldsymbol{D}, A)$ is stated in the following lemma:

**Lemma 1** $\boldsymbol{X}\in\Omega$ if and only if there exists an $\boldsymbol{F}\in F_{\boldsymbol{X}}$ which satisfies

$$\sum_{j=1}^{m}\{f_j \mid \text{mp}_j \in Q_u\}=d_u,\ u=1, 2, \ldots, r. \quad (6)$$

**Proof** Suppose $\boldsymbol{X}\in\Omega$; i.e., $\boldsymbol{X}$ meets ($\boldsymbol{D}$, $A$) and there exists an $\boldsymbol{F}\in F_{\boldsymbol{X}}$ such that $\sum(f_j \mid \text{mp}_j \in Q_u) \geq d_u$, $u$=1, 2, …, $r$. Without loss of generality, for $f_j$ and $d_u$, assume $\text{mp}_1\in Q_1$, $f_1>0$, and $\sum(f_j \mid \text{mp}_j \in Q_1)=d_1+1$. Let $\boldsymbol{F}'=(f_1', f_2', \ldots, f_m')=(f_1-1, f_2', \ldots, f_m')$. Then $\boldsymbol{F}'<\boldsymbol{F}$ ($\boldsymbol{F}'\in F_{\boldsymbol{X}}$) and $\sum(f_j' \mid \text{mp}_j \in Q_1)=\sum(f_j \mid \text{mp}_j \in Q_1)-1=d_1$. This implies that we can find an $\boldsymbol{F}$ with $\sum(f_j \mid \text{mp}_j \in Q_u)=d_u$ for $u$=1, 2, …, $r$. Conversely, if there exists an $\boldsymbol{F}\in F_{\boldsymbol{X}}$ which satisfies Eq. (6), then $\boldsymbol{X}\in\Omega$ by Definition 1.

For each flow vector $\boldsymbol{F}$ satisfying both constraints (4) and (6), the corresponding capacity vector $\boldsymbol{Z}_{\boldsymbol{F}}=(x_1, x_2, \ldots, x_n)$ is generated via

$$x_i=\sum(f_j \mid e_i \in \text{mp}_j),\ i=1, 2, \ldots, n. \quad (7)$$

Since any $\boldsymbol{F}$ satisfying constraints (4) and (6) meets ($\boldsymbol{D}$, $A$), $\boldsymbol{Z}_{\boldsymbol{F}}$ also meets ($\boldsymbol{D}$, $A$), i.e., $\boldsymbol{Z}_{\boldsymbol{F}}\in\Omega$ (by Lemma 1). An important property of ($\boldsymbol{D}$, $A$)-MP is subsequently stated as follows:

**Theorem 1** If $\boldsymbol{X}$ is a ($\boldsymbol{D}$, $A$)-MP, then there exists at least one $\boldsymbol{F}\in F_M$ such that $\boldsymbol{X}=\boldsymbol{Z}_{\boldsymbol{F}}$.

**Proof** If $\boldsymbol{X}=(x_1, x_2, \ldots, x_n)$ is a ($\boldsymbol{D}$, $A$)-MP, then there exists an $\boldsymbol{F}$ such that $\boldsymbol{F}\in F_{\boldsymbol{X}}$. Let $\boldsymbol{Z}_{\boldsymbol{F}}=(y_1, y_2, \ldots, y_n)$ be generated from $\boldsymbol{F}$. Constraint (2) implies that $\boldsymbol{Z}_{\boldsymbol{F}}\leq\boldsymbol{X}$. Hence, $x_i\geq y_i$ for each $i$. Without loss of generality, for $x_i$, let $x_1>y_1$. Set $\boldsymbol{Y}=(x_1-1, x_2, \ldots, x_n)$ with $\boldsymbol{Y}<\boldsymbol{X}$. Then $\boldsymbol{Y}\geq\boldsymbol{Z}_{\boldsymbol{F}}$ and $\boldsymbol{F}\in F_{\boldsymbol{Y}}$. This implies that $\boldsymbol{Y}$ satisfies ($\boldsymbol{D}$, $A$). It contradicts that $\boldsymbol{X}$ is a ($\boldsymbol{D}$, $A$)-MP. We conclude that $x_i=y_i$ for each $i$, i.e., $\boldsymbol{X}=\boldsymbol{Z}_{\boldsymbol{F}}$.

Let $\Psi\equiv\{\boldsymbol{Z}_{\boldsymbol{F}}|\boldsymbol{F}$ satisfies constraints (4) and (6)$\}$ and $\Psi_{\min}\equiv\{\boldsymbol{X}|\boldsymbol{X}$ is minimal w.r.t. $\leq$ in $\Psi\}$. By Theorem 1, $\Psi$ contains all ($\boldsymbol{D}$, $A$)-MPs, i.e., $\Psi\supseteq\Omega_{\min}$. The following theorem further proves that $\Psi_{\min}$ is the set of ($\boldsymbol{D}$, $A$)-MPs (i.e., $\Psi_{\min}=\Omega_{\min}$):

**Theorem 2** $\{(\boldsymbol{D}, A)\text{-MPs}\}=\Psi_{\min}$.

**Proof** Suppose $\boldsymbol{X}\in\Psi_{\min}$ (note that $\boldsymbol{X}\in\Omega$) but it is not a ($\boldsymbol{D}$, $A$)-MP. Then there exists a ($\boldsymbol{D}$, $A$)-MP $\boldsymbol{Y}$ such that $\boldsymbol{Y}<\boldsymbol{X}$, which implies that there exists an $\boldsymbol{F}\in F_M$ such that $\boldsymbol{Y}=\boldsymbol{Z}_{\boldsymbol{F}}$ and $\boldsymbol{Y}\in\Psi$. This contradicts $\boldsymbol{X}\in\Psi_{\min}$. Hence, $\boldsymbol{X}$ is a ($\boldsymbol{D}$, $A$)-MP. Conversely, let $\boldsymbol{X}$ be a ($\boldsymbol{D}$, $A$)-MP (note that $\boldsymbol{X}\in\Psi$) but $\boldsymbol{X}\notin\Psi_{\min}$; i.e., there exists a $\boldsymbol{Y}\in\Psi$ such that $\boldsymbol{Y}<\boldsymbol{X}$. Hence, $\boldsymbol{Y}\in\Omega$, which contradicts the supposition that $\boldsymbol{X}$ is a minimal vector in $\Omega$. Hence, $\boldsymbol{X}\in\Psi_{\min}$.

## 3 Solution procedure

### 3.1 Algorithm to generate all (*D*, *A*)-MPs

Similar to the approaches in the literature (Xue, 1985; Lin *et al*., 1995; Yeh, 1998; 2004; 2005; Lin, 2001; 2009; Zuo *et al*., 2007; Lin and Yeh, 2011), suppose all minimal paths have been pre-computed. Given demand $\boldsymbol{D}=(d_1, d_2, \ldots, d_r)$ at terminal $(t_1, t_2, \ldots, t_r)$ respectively and total accuracy rate $A$, all ($\boldsymbol{D}$, $A$)-MPs can be derived by the following steps.

Algorithm to generate all ($\boldsymbol{D}$, $A$)-MPs
/* derive all ($\boldsymbol{D}$, $A$)-MPs */
**Input:** $G$, $d_u$, and $A$.
**Output:** All ($\boldsymbol{D}$, $A$)-MPs.

Step 1: Find all feasible flow vectors $(f_1, f_2, \ldots, f_m)$ satisfying the following constraints:

1. Demand constraint:

$$\sum_{j=1}^{m}\{f_j \mid \text{mp}_j \in Q_u\}=d_u,\ u=1, 2, \ldots, r. \quad (8)$$

2. Capacity constraint for arc:

$$\sum_{j=1}^{m}\{f_j \mid e_i \in \text{mp}_j\}\leq M_i,\ i=1, 2, \ldots, n. \quad (9)$$

3. Total accuracy rate constraint:

$$\sum_{j=1}^{m}\left(f_j\prod(a_i \mid e_i \in \text{mp}_j)\right)\Big/\sum_{u=1}^{r}d_u \geq A,\ u=1, 2, \ldots, r. \quad (10)$$

Step 2 (Obtaining $\Psi$): Transform each feasible flow $(f_1, f_2, \ldots, f_m)$ to the corresponding $\boldsymbol{X}=(x_1, x_2, \ldots, x_n)$ via

$$x_i=\sum(f_j \mid e_i \in \text{mp}_j),\ i=1, 2, \ldots, n. \quad (11)$$

Step 3 (Generating $\Psi_{\min}$): Suppose the result of step 2 is $\Psi=\{\boldsymbol{X}^1, \boldsymbol{X}^2, \ldots, \boldsymbol{X}^h\}$. Check each $\boldsymbol{X}$ in $\Psi$

whether it is a (**D**, *A*)-MP or not by the following procedures:

```
Initialization: I=∅ /* I is the stack which stores the index of
each non-minimal one after checking */
1  for i=1 to h and i∉I
2    for j=i+1 to h and j∉I
3      if X^i>X^j then
4        I=I∪{i}  /* X^i is not a (D, A)-MP and go to line 9 */
5      else if X^i≤X^j then
6        I=I∪{j} /* X^j is not a (D, A)-MP and go to line 7 */
7      end if
8      j=j+1
9    end for
10  i=i+1
11  X^i is a (D, A)-MP
12 end for
```

In step 1, we find all feasible **F** fulfilling demand constraint (8), capacity constraint (9), and total accuracy rate constraint (10). Hence, each feasible solution **F** is transformed to **X** according to Eq. (11) in step 2. Each such **X** is a candidate of (**D**, *A*)-MP. Finally in step 3, the comparison rule (lines 1–12) is used to delete the non-minimal **X**. The remaining **X**'s are (**D**, *A*)-MPs.

### 3.2 RSDP algorithm

Suppose $\boldsymbol{X}^1, \boldsymbol{X}^2, \ldots, \boldsymbol{X}^q$ are all (**D**, *A*)-MPs. To calculate $R_{\boldsymbol{D},A}$, set $B_i=\{\boldsymbol{X}|\boldsymbol{X}\geq\boldsymbol{X}^i\}$, $i$=1, 2, …, $q$. Then,

$$R_{\boldsymbol{D},A}=\Pr\{\bigcup_{i=1}^{q}B_i\}. \quad (12)$$

The value $\Pr\{\bigcup_{i=1}^{q}B_i\}$ can be computed by applying the RSDP algorithm (Hudson and Kapur, 1985; Yarlagadda and Hershey, 1991; Zuo *et al*., 2007; Lin, 2010; Lin and Yeh, 2011), the inclusion-exclusion rule (Lin, 2001; 2002; 2009; 2010), the disjoint-event method (Hudson and Kapur, 1985; Yarlagadda and Hershey, 1991), or state-space decomposition (Amer, 1982; Aven, 1985; Alexopoulos, 1995; Lin *et al*., 1995). Note that $\Pr\{\boldsymbol{X}=\boldsymbol{Y}\}=\prod_{i=1}^{n}\Pr\{x_i=y_i\}$ and $\Pr\{\boldsymbol{X}\geq\boldsymbol{Y}\}=\prod_{i=1}^{n}\Pr\{x_i\geq y_i\}$.

Since all (**D**, *A*)-MPs have been found in Section 3.1, the RSDP algorithm is applied to derive the system reliability. The system reliability is the union of events where the capacity vector is greater than or equal to at least one of (**D**, *A*)-MPs. The RSDP algorithm is a recursive algorithm combined by the sum-of-disjoint-product principle (Zuo *et al*., 2007). In this algorithm, a maximum operator, '⊕', is defined as

$$\boldsymbol{X}^{1,2}=\boldsymbol{X}^1\oplus\boldsymbol{X}^2\equiv\max(x_i^1, x_i^2),\ i=1, 2, \ldots, n. \quad (13)$$

For example, suppose $\boldsymbol{X}^1$=(1, 0, 1, 1, 0, 0, 1, 1) and $\boldsymbol{X}^2$=(0, 0, 2, 0, 0, 0, 2, 2). By Eq. (13), $\boldsymbol{X}^{1,2}=\boldsymbol{X}^1\oplus\boldsymbol{X}^2$ =(max(1, 0), max(0, 0), max(1, 2), max(1, 0), max(0, 0), max(0, 0), max(1, 2), max(1, 2))=(1, 0, 2, 1, 0, 0, 2, 2). The following pseudo-code procedure summarizes the RSDP algorithm.

RSDP algorithm

```
/* Compute system reliability R_{D,A}=Pr{∪_{i=1}^{q} B_i} */
  function SR=RSDP(X^1, X^2, …, X^q)
        /* Input q (D, A)-MPs */
  for i=1:q
    if i==1
       R_{D,A}=Pr(X≥X^i)
    else
       Temp R_{D,A}_1=Pr(X≥X^i)  /* Temporary R_{D,A} */
    if i==2
       Temp R_{D,A}_2=Pr(X≥max(X^1, X^i))
          /* max(X^1, X^i)=X^1⊕X^i */
    else
       for j=1:i–1
          X^j=max(X^j, X^i)  /* max(X^j, X^i)=X^j⊕X^i */
       end for
       q=q–1
       Temp R_{D,A}_2=RSDP(X^1, X^2, …, X^q)
          /* Execute recursive procedure */
    end if
  end for
  R_{D,A}=R_{D,A}+Temp R_{D,A}_1–Temp R_{D,A}_2
        /* Return the system reliability */
```

## 4 Numerical examples

### 4.1 A benchmark network

The benchmark network shown in Fig. 1 is used to illustrate the proposed method. Table 1 lists the edge data including capacity, probability, and accuracy rate. In this example, it is known that $n$=9. We list all minimal paths for two terminals, $t_1$ and $t_2$, in the following order:

For $t_1$: mp$_1$={$a_1$, $a_2$}, mp$_2$={$a_1$, $a_4$, $a_5$}, mp$_3$={$a_7$, $a_5$}.

For $t_2$: $\text{mp}_4=\{a_1, a_2, a_3\}$, $\text{mp}_5=\{a_1, a_2, a_6, a_9\}$, $\text{mp}_6=\{a_1, a_4, a_5, a_3\}$, $\text{mp}_7=\{a_1, a_4, a_5, a_6, a_9\}$, $\text{mp}_8=\{a_1, a_4, a_8, a_9\}$, $\text{mp}_9=\{a_7, a_5, a_3\}$, $\text{mp}_{10}=\{a_7, a_5, a_6, a_9\}$, $\text{mp}_{11}=\{a_7, a_8, a_9\}$.

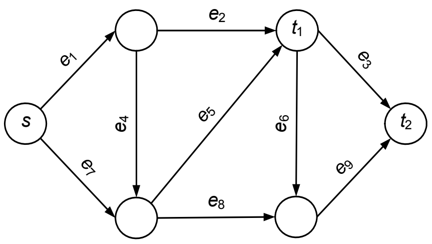


**Fig. 1 A benchmark network**

**Table 1 Data of edges in Fig. 1**

| Edge | Accuracy rate | Probability | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 1 Mbps | 2 Mbps | 3 Mbps | 4 Mbps |
| $e_1$ | 0.98 | 0.05 | 0.05 | 0.1 | 0.25 | 0.55 |
| $e_2$ | 0.97 | 0.05 | 0.05 | 0.2 | 0.7 | 0 |
| $e_3$ | 0.99 | 0.1 | 0.1 | 0.1 | 0.1 | 0.6 |
| $e_4$ | 0.99 | 0.05 | 0.2 | 0.75 | 0 | 0 |
| $e_5$ | 0.94 | 0.1 | 0.1 | 0.1 | 0.7 | 0 |
| $e_6$ | 0.96 | 0.1 | 0.1 | 0.8 | 0 | 0 |
| $e_7$ | 0.98 | 0.05 | 0.05 | 0.2 | 0.7 | 0 |
| $e_8$ | 0.98 | 0.1 | 0.1 | 0.1 | 0.7 | 0 |
| $e_9$ | 0.99 | 0.05 | 0.05 | 0.15 | 0.2 | 0.55 |

For the case $\boldsymbol{D}=(d_1, d_2)=(2, 3)$ and $A=0.935$, the system reliability $R_{\boldsymbol{D},A}$ can be calculated using the following steps.

Step 1: Find all feasible flows $\boldsymbol{F}=(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11})$ with the following constraints:

$$f_1+f_2+f_3=2,\ f_4+f_5+f_6+f_7+f_8+f_9+f_{10}+f_{11}=3. \tag{14}$$

$$\begin{aligned}&f_1+f_2+f_4+f_5+f_6+f_7+f_8\le4,\\ &f_1+f_4+f_5\le3,\ f_4+f_6+f_9\le4,\\ &f_2+f_6+f_7+f_8\le2,\ f_2+f_3+f_6+f_7+f_9+f_{10}\le3,\\ &f_5+f_7+f_{10}\le2,\ f_3+f_9+f_{10}+f_{11}\le3,\\ &f_8+f_{11}\le3,\ f_5+f_7+f_8+f_{10}+f_{11}\le4.\end{aligned} \tag{15}$$

$$\begin{aligned}&0.9506f_1+0.912f_2+0.9212f_3+0.95\times0.9411f_4\\ &+0.9035f_5+0.9029f_6+0.8668f_7+0.9413f_8\\ &+0.912f_9+0.8755f_{10}+0.9508f_{11}\\ &\ge5\times0.935=4.675.\end{aligned} \tag{16}$$

Table 2 shows the 40 feasible flow vectors.

Step 2: Transform each solution $\boldsymbol{F}$ into $\boldsymbol{X}=(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)$ via

$$\begin{aligned}&x_1=f_1+f_2+f_4+f_5+f_6+f_7+f_8,\ x_2=f_1+f_4+f_5,\\ &x_3=f_4+f_6+f_9,\ x_4=f_2+f_6+f_7+f_8,\\ &x_5=f_2+f_3+f_6+f_7+f_9+f_{10},\ x_6=f_5+f_7+f_{10},\\ &x_7=f_3+f_9+f_{10}+f_{11},\ x_8=f_8+f_{11},\ x_9=f_5+f_7+f_8+f_{10}+f_{11}.\end{aligned} \tag{17}$$

Table 2 lists the results after transformation.

Step 3: Remove those non-minimal ones in $\Psi$ to obtain $\Psi_{\min}$.

$I=\varnothing$
$i=1$
$j=2$
$\boldsymbol{X}^2\ge\boldsymbol{X}^1$, and $\boldsymbol{X}^1\ge\boldsymbol{X}^2$ or $\boldsymbol{X}^1=\boldsymbol{X}^2$
$j=3$
$\boldsymbol{X}^3\ge\boldsymbol{X}^1$, and $\boldsymbol{X}^1\ge\boldsymbol{X}^3$ or $\boldsymbol{X}^1=\boldsymbol{X}^3$
$j=4$
$\boldsymbol{X}^4=(2, 0, 0, 2, 2, 0, 3, 3, 2)>\boldsymbol{X}^1=(2, 0, 0, 2, 2, 0, 3, 3, 1)$.
Thus, $\boldsymbol{X}^4$ is not a $(\boldsymbol{D}, A)$-MP and $I=4$.
$j=5$
…
END

Since 40 feasible $\boldsymbol{X}$ need to be checked whether they are $(\boldsymbol{D}, A)$-MP or not, step 3 will be executed at most $40^2$ times. We show only three repeats to explain the process of removing non-minimal $\boldsymbol{X}$. There are totally 18 $(\boldsymbol{D}, A)$-MPs after the comparison step. The final result is summarized in Table 2. Subsequently, the system reliability $R_{\boldsymbol{D},A}=\Pr\{\bigcup_{i=1}^{18} B_i\}$ is 0.6465 by applying the RSDP algorithm.

## 4.2 A large-scale case study at Taiwan

We employ a practical computer network, the TANet with 30 edges (Fig. 2), to demonstrate the utility of the proposed approach for assessing larger stochastic computer networks. The TANet is the backbone network, connecting all educational and academic organizations in Taiwan. Consider the case where NTU is the source and NSYSU and NTTU are two terminals. In this example, each edge is composed of several Optical Carrier 18 (OC-18) physical lines and each physical line provides two possible capacities, 1 Gbps (Gb/s) and 0 bps (bit/s). Since the lines are provided by different suppliers, the capacity of each edge follows a distinct probability distribution. For instance, $e_i$ is an edge combined by $v$ independent physical lines where each line can provide 1 Gbps

**Table 2 Results of each step for Example 1**

| Step 1: ($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}$) | Step 2: ($x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$) | Step 3: ($\boldsymbol{D}$, $A$)-MP? | Remark |
|---|---|---|---|
| (0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 1) | (2, 0, 0, 2, 2, 0, 3, 3, 1) | Yes | |
| (0, 0, 2, 1, 0, 0, 0, 1, 0, 0, 1) | (2, 1, 1, 1, 2, 0, 3, 2, 1) | Yes | |
| (0, 0, 2, 2, 0, 0, 0, 0, 0, 0, 1) | (2, 2, 2, 0, 2, 0, 3, 1, 1) | Yes | |
| (0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 2) | (2, 0, 0, 2, 2, 0, 3, 3, 2) | | $\boldsymbol{X}^4>\boldsymbol{X}^1$ |
| (0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 2) | (2, 1, 1, 1, 2, 0, 3, 2, 2) | | $\boldsymbol{X}^5>\boldsymbol{X}^2$ |
| (0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 3) | (2, 0, 0, 2, 2, 0, 3, 3, 3) | | $\boldsymbol{X}^6>\boldsymbol{X}^1$ |
| (1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 2) | (2, 1, 0, 1, 1, 0, 3, 3, 2) | Yes | |
| (1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1) | (2, 1, 1, 1, 2, 0, 3, 2, 1) | | $\boldsymbol{X}^8>\boldsymbol{X}^1$ |
| (1, 0, 1, 0, 0, 0, 0, 2, 0, 0, 1) | (3, 1, 0, 2, 1, 0, 2, 3, 1) | Yes | |
| (1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 2) | (2, 1, 1, 1, 2, 0, 3, 2, 2) | | $\boldsymbol{X}^{10}>\boldsymbol{X}^2$ |
| (1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 2) | (2, 2, 0, 0, 1, 1, 3, 2, 3) | Yes | |
| (1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 2) | (2, 2, 1, 0, 1, 0, 3, 2, 2) | Yes | |
| (1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1) | (2, 2, 2, 0, 2, 0, 3, 1, 1) | | $\boldsymbol{X}^{13}\geq\boldsymbol{X}^3$ |
| (1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1) | (3, 2, 1, 1, 1, 0, 2, 2, 1) | Yes | |
| (1, 0, 1, 1, 0, 0, 0, 2, 0, 0, 0) | (4, 2, 1, 2, 1, 0, 1, 2, 0) | Yes | |
| (1, 0, 1, 2, 0, 0, 0, 0, 0, 0, 1) | (3, 3, 2, 0, 1, 0, 2, 1, 1) | Yes | |
| (1, 0, 1, 2, 0, 0, 0, 1, 0, 0, 0) | (4, 3, 2, 1, 1, 0, 1, 1, 0) | Yes | |
| (1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 3) | (2, 1, 0, 1, 1, 0, 3, 3, 3) | | $\boldsymbol{X}^{18}>\boldsymbol{X}^7$ |
| (1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 2) | (2, 1, 1, 1, 2, 0, 3, 2, 2) | | $\boldsymbol{X}^{19}>\boldsymbol{X}^2$ |
| (1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 2) | (3, 1, 0, 2, 1, 0, 2, 3, 2) | | $\boldsymbol{X}^{20}>\boldsymbol{X}^9$ |
| (1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 2) | (3, 2, 1, 1, 1, 0, 2, 2, 2) | | $\boldsymbol{X}^{21}>\boldsymbol{X}^9$ |
| (1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1) | (4, 2, 1, 2, 1, 0, 1, 2, 1) | | $\boldsymbol{X}^{22}>\boldsymbol{X}^{15}$ |
| (1, 1, 0, 2, 0, 0, 0, 0, 0, 0, 1) | (4, 3, 2, 1, 1, 0, 1, 1, 1) | | $\boldsymbol{X}^{23}>\boldsymbol{X}^{17}$ |
| (2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3) | (2, 2, 0, 0, 0, 0, 3, 3, 3) | Yes | |
| (2, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2) | (2, 2, 0, 0, 1, 1, 3, 2, 3) | | $\boldsymbol{X}^{25}\geq\boldsymbol{X}^{11}$ |
| (2, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2) | (2, 2, 1, 0, 1, 0, 3, 2, 2) | | $\boldsymbol{X}^{26}\geq\boldsymbol{X}^{12}$ |
| (2, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1) | (2, 2, 2, 0, 2, 0, 3, 1, 1) | | $\boldsymbol{X}^{27}\geq\boldsymbol{X}^3$ |
| (2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2) | (3, 2, 0, 1, 0, 0, 2, 3, 2) | Yes | |
| (2, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1) | (3, 2, 1, 1, 1, 0, 2, 2, 1) | | $\boldsymbol{X}^{29}\geq\boldsymbol{X}^{14}$ |
| (2, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1) | (4, 2, 0, 2, 0, 0, 1, 3, 1) | Yes | |
| (2, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0) | (4, 2, 1, 2, 1, 0, 1, 2, 0) | | $\boldsymbol{X}^{31}\geq\boldsymbol{X}^{15}$ |
| (2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2) | (3, 2, 1, 1, 1, 0, 2, 2, 2) | | $\boldsymbol{X}^{32}>\boldsymbol{X}^9$ |
| (2, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1) | (4, 2, 1, 2, 1, 0, 1, 2, 1) | | $\boldsymbol{X}^{33}>\boldsymbol{X}^{15}$ |
| (2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2) | (3, 3, 0, 0, 0, 1, 2, 2, 3) | Yes | |
| (2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1) | (4, 3, 0, 1, 0, 1, 1, 2, 2) | Yes | |
| (2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2) | (3, 3, 1, 0, 0, 0, 2, 2, 2) | Yes | |
| (2, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1) | (3, 3, 2, 0, 1, 0, 2, 1, 1) | | $\boldsymbol{X}^{37}\geq\boldsymbol{X}^{16}$ |
| (2, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1) | (4, 3, 1, 1, 0, 0, 1, 2, 1) | Yes | |
| (2, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0) | (4, 3, 2, 1, 1, 0, 1, 1, 0) | | $\boldsymbol{X}^{39}\geq\boldsymbol{X}^{17}$ |
| (2, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1) | (4, 3, 2, 1, 1, 0, 1, 1, 1) | | $\boldsymbol{X}^{40}>\boldsymbol{X}^{17}$ |

with probability $p$. Thus, the probability that $e_i$ provides $w$ Gbps is $C_w^v p^v (1-p)^{v-w}$. Both the capacity and accuracy rate of each edge are given in Table 3. For the case where the TANet has to preserve a minimal service level that delivers to NSYSU at least 5 Gbps of data and NTTU at least 3 Gbps of data, the system reliability for $\boldsymbol{D}$=(5, 3) and $A$=0.86 can be evaluated using the proposed solution procedure. Fifty-two solutions are generated from steps 1 and 2, and only 10 ($\boldsymbol{D}$, $A$)-MPs are obtained from step 3. The system reliability $R_{\boldsymbol{D},A}$ calculated by the RSDP algorithm is 0.6081.

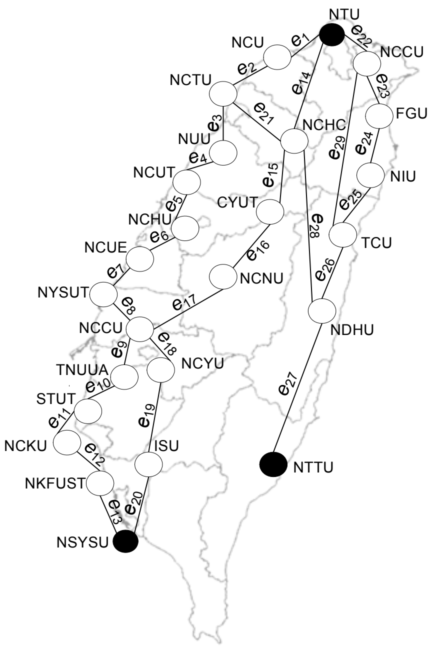


**Fig. 2 Taiwan Academic Network (TANet)**

## 5 Computational complexity analysis for generating all (*D*, *A*)-MPs

The storage and computational time complexity of the proposed algorithm are analyzed.

### 5.1 Storage complexity

For each demand at terminal $t_u$, the number of feasible solutions satisfying $\sum_{j=1}^{m}\{f_j \mid \text{mp}_j \in Q_u\} = d_u$ is bounded by $\binom{|Q_u|+d_u-1}{d_u}$, where $|Q_u|$ is the number of minimal paths in $Q_u$. Hence, the total number of feasible solutions of constraints (8)–(10) is bounded by $\prod_{u=1}^{r}\binom{|Q_u|+d_u-1}{d_u}$. It requires at most $\prod_{u=1}^{r}\binom{|Q_u|+d_u-1}{d_u}$ storage space to execute the

**Table 3 Data of edges in Fig. 2**

| Edge | Accuracy rate | Probability | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0 | 1 Gbps | 2 Gbps | 3 Gbps | 4 Gbps | 5 Gbps |
| $e_1$ | 0.99 | 0.000064 | 0.004608 | 0.110592 | 0.884736 | 0.000000 | 0.000000 |
| $e_2$ | 0.99 | 0.000343 | 0.013671 | 0.181629 | 0.804357 | 0.000000 | 0.000000 |
| $e_3$ | 0.98 | 0.000100 | 0.003600 | 0.048600 | 0.291600 | 0.656100 | 0.000000 |
| $e_4$ | 0.95 | 0.000041 | 0.001884 | 0.032502 | 0.249180 | 0.716393 | 0.000000 |
| $e_5$ | 0.96 | 0.000216 | 0.010152 | 0.159048 | 0.830584 | 0.000000 | 0.000000 |
| $e_6$ | 0.99 | 0.000125 | 0.007125 | 0.135375 | 0.857375 | 0.000000 | 0.000000 |
| $e_7$ | 0.99 | 0.001000 | 0.027000 | 0.243000 | 0.729000 | 0.000000 | 0.000000 |
| $e_8$ | 0.97 | 0.000008 | 0.001176 | 0.057624 | 0.941192 | 0.000000 | 0.000000 |
| $e_9$ | 0.97 | 0.000064 | 0.004608 | 0.110592 | 0.884736 | 0.000000 | 0.000000 |
| $e_{10}$ | 0.98 | 0.000027 | 0.002619 | 0.084681 | 0.912673 | 0.000000 | 0.000000 |
| $e_{11}$ | 0.98 | 0.000064 | 0.004608 | 0.110592 | 0.884736 | 0.000000 | 0.000000 |
| $e_{12}$ | 0.99 | 0.000343 | 0.013671 | 0.181629 | 0.804357 | 0.000000 | 0.000000 |
| $e_{13}$ | 0.99 | 0.000125 | 0.007125 | 0.135375 | 0.857375 | 0.000000 | 0.000000 |
| $e_{14}$ | 0.99 | 0.000010 | 0.000450 | 0.008100 | 0.072900 | 0.328050 | 0.590490 |
| $e_{15}$ | 0.98 | 0.000125 | 0.007125 | 0.135375 | 0.857375 | 0.000000 | 0.000000 |
| $e_{16}$ | 0.99 | 0.000024 | 0.001276 | 0.025428 | 0.225220 | 0.748052 | 0.000000 |
| $e_{17}$ | 0.99 | 0.000008 | 0.001176 | 0.057624 | 0.941192 | 0.000000 | 0.000000 |
| $e_{18}$ | 0.98 | 0.000024 | 0.001276 | 0.025428 | 0.225220 | 0.748052 | 0.000000 |
| $e_{19}$ | 0.99 | 0.000003 | 0.000188 | 0.004334 | 0.049836 | 0.286557 | 0.659082 |
| $e_{20}$ | 0.97 | 0.000003 | 0.000188 | 0.004334 | 0.049836 | 0.286557 | 0.659082 |
| $e_{21}$ | 0.97 | 0.000003 | 0.000188 | 0.004334 | 0.049836 | 0.286557 | 0.659082 |
| $e_{22}$ | 0.98 | 0.008100 | 0.163800 | 0.828100 | 0.000000 | 0.000000 | 0.000000 |
| $e_{23}$ | 0.98 | 0.002500 | 0.095000 | 0.902500 | 0.000000 | 0.000000 | 0.000000 |
| $e_{24}$ | 0.97 | 0.001600 | 0.076800 | 0.921600 | 0.000000 | 0.000000 | 0.000000 |
| $e_{25}$ | 0.95 | 0.003600 | 0.112800 | 0.883600 | 0.000000 | 0.000000 | 0.000000 |
| $e_{26}$ | 0.95 | 0.000125 | 0.007125 | 0.135375 | 0.857375 | 0.000000 | 0.000000 |
| $e_{27}$ | 0.95 | 0.000024 | 0.001276 | 0.025428 | 0.225220 | 0.748052 | 0.000000 |
| $e_{28}$ | 0.99 | 0.003600 | 0.112800 | 0.883600 | 0.000000 | 0.000000 | 0.000000 |
| $e_{29}$ | 0.99 | 0.000006 | 0.000475 | 0.013538 | 0.171475 | 0.814506 | 0.000000 |

proposed algorithm. Let $L=\prod_{u=1}^{r}\binom{|Q_u|+d_u-1}{d_u}$ be the maximum number of feasible flow solutions.

### 5.2 Computational time complexity

For step 1: In the worst case, each feasible solution of Eq. (8) takes $O(m)$ time to test whether it satisfies $\sum_{j=1}^{m}\{f_j \mid e_i \in \mathrm{mp}_j\} \le M_i$ (constraint (9)) for each edge, and $O(mn)$ time for all edges. Hence, it needs at most $O(mn)$ time to check whether it satisfies the total accuracy rate (constraint (10)). Thus, we need at most $O(mnL)$ time to generate all feasible solutions of constraints (8)–(10).

For step 2: Virtually, $\sum_{j=1}^{m}\{f_j \mid e_i \in \mathrm{mp}_j\}$ has been computed in constraint (9). So, we do not need any more time to execute step 2.

For step 3: It further needs to take at most $O(nL)$ time to test each solution from step 3 whether it is minimal, and at most $O(nL^2)$ time to check all solutions from step 3 in the worst case.

In sum, the computational time complexity of the proposed algorithm in the worst case is $O(mnL)$+$O(nL^2)$=$O(nL^2)$ (Note that $m=\sum_{u=1}^{r}|Q_u|$ is less than $L$ in the first summand).

## 6 Conclusions

For a stochastic flow network with multiple terminals, this paper proposes an algorithm based on minimal paths to describe $\boldsymbol{X}$ and $\boldsymbol{F}$ for deriving all ($\boldsymbol{D}$, $A$)-MPs. The system reliability $R_{\boldsymbol{D},K}$, the probability that the network fulfills both demand $\boldsymbol{D}$ and total accuracy rate $A$, can be computed in terms of all ($\boldsymbol{D}$, $A$)-MPs by the RSDP. The proposed algorithm needs $O(nL^2)$ time in the worst case.

It seems that applying repeatedly the algorithm proposed by Xue (1985) or Lin *et al.* (1995) is a straight way to calculate the system reliability for multiple terminals. Their approaches can evaluate $R_u$=Pr{$\boldsymbol{X}$|there exists an $\boldsymbol{F}\in F_{\boldsymbol{X}}$ which meets ($d_u$, $A$)} for each terminal $t_u$. Then let $R'_{\boldsymbol{D},A}\equiv\prod_{u=1}^{r}R_u$ be an approximate $R_{\boldsymbol{D},A}$. However, such a method is not reasonable by comparing the values of $R'_{\boldsymbol{D},A}$ with $R_{\boldsymbol{D},A}$. For the benchmark example, $R_{\boldsymbol{D},A}=R_1R_2$=0.8891×0.8520=0.7575 overestimates the true system reliability 0.6465.

Future research can extend the SCN to the overall-terminal problem. Transmission time is also an important issue for computer network systems. Hence, we can consider the time effects on each edge. In addition, considering the transmission cost and time together, it is also worth studying the system reliability under budget, time, and accuracy rate constraints.

## References


Alexopoulos, C., 1995. A note on state-space decomposition methods for analyzing stochastic flow networks. *IEEE Trans. Rel.*, **44**(2):354-357. [doi:10.1109/24.387394]

Amer, P.D., 1982. A measurement center for the NBS local area computer network. *IEEE Trans. Comput.*, **C-31**(8): 723-729. [doi:10.1109/TC.1982.1676078]

Aven, T., 1985. Reliability evaluation of multistate systems with multistate components. *IEEE Trans. Rel.*, **R-34**(5): 473-479. [doi:10.1109/TR.1985.5222235]

Cheng, S.T., 1998. Topological optimization of a reliable communication network. *IEEE Trans. Rel.*, **47**(3):225-233. [doi:10.1109/24.740489]

Chlamtac, I., 1980. Issues in Design and Measurement of Local Area Networking. 6th Int. Computer Measurement Group Conf., p.32-34.

Choi, B.Y., Park, J., Zhang, Z.L., 2003. Adaptive Random Sampling for Traffic Load Measurement. IEEE Int. Conf. on Communications, p.1552-1556. [doi:10.1109/ICC.2003.1203863]

Claffy, K.C., Polyzos, G.C., Braun, H.W., 1993. Application of Sampling Methodologies to Network Traffic Characterization. Proc. ACM SIGCOMM, p.194-203. [doi:10.1145/166237.166256]

Feldmann, A., Greenberg, A., Reingold, N., Lund, C., Rexford, J., True, F., 2001. Deriving traffic demands for operational IP networks: methodology and experience. *IEEE/ACM Trans. Network.*, **9**(3):265-279. [doi:10.1109/90.929850]

Ford, L.R., Fulkerson, D.R., 1962. Flow in Networks. Princeton University Press, USA.

Huang, F.M., Lan, C.W., Yang, J.H., 2009. An optimal QoS-based Web service selection scheme. *Inf. Sci.*, **179**(19):3309-3322. [doi:10.1016/j.ins.2009.05.018]

Hudson, J.C., Kapur, K.C., 1985. Reliability bounds for multistate systems with multistate components. *Oper. Res.*, **33**(1):153-160. [doi:10.1287/opre.33.1.153]

Jain, R., Routhier, S.A., 1986. Packet trains: measurements and a new model for computer network traffic. *IEEE J. Sel. Areas Commun.*, **4**(6):986-995. [doi:10.1109/JSAC.1986.1146410]

Jane, C.C., Lin, J.S., Yuan, J., 1993. On reliability evaluation of a limited-flow network in terms of minimal cutsets. *IEEE Trans. Rel.*, **42**(3):354-361. [doi:10.1109/24.257

817]

Jedwab, J., Phaal, P., Pinna, B., 1992. Traffic Estimation for the Largest Sources, on a Network, Using Packet Sampling with Limited Storage. Technical Report HPL-92-35, HP Labs Technical Report.

Lin, J.S., Jane, C.C., Yuan, J., 1995. On reliability evaluation of a capacitated-flow network in terms of minimal pathsets. *Networks*, **25**(3):131-138. [doi:10.1002/net.3230250306]

Lin, Y.K., 2001. A simple algorithm for reliability evaluation of a stochastic-flow network with node failure. *Comput. Oper. Res.*, **28**(13):1277-1285. [doi:10.1016/S0305-0548(00)00039-3]

Lin, Y.K., 2002. Using minimal cuts to evaluate the system reliability of a stochastic flow network with failures at nodes and arcs. *Rel. Eng. Syst. Safety*, **75**(1):41-46. [doi:10.1016/S0951-8320(01)00110-7]

Lin, Y.K., 2009. Optimal routing policy of a stochastic-flow network. *Comput. Ind. Eng.*, **56**(4):1414-1418. [doi:10.1016/j.cie.2008.09.002]

Lin, Y.K., 2010. A stochastic model to study the system capacity for supply chains in terms of minimal cuts. *Int. J. Prod. Econ.*, **124**(1):181-187. [doi:10.1016/j.ijpe.2009.10.022]

Lin, Y.K., Yeh, C.T., 2011. Reliability optimization of component assignment problem for a multistate network in terms of minimal cuts. *J. Ind. Manag. Optim.*, **7**(1):211-227. [doi:10.3934/jimo.2011.7.211]

Mori, T., Takine, T., Pan, J., Kawahara, R., Uchida, M., Goto, S., 2007. Identifying heavy-hitter flows from sampled flow statistics. *IEICE Trans. Commun.*, **E90-B**(11):3061-3072. [doi:10.1093/ietcom/e90-b.11.3061]

Ramirez-Marquez, J.E., Rocco, C.M., 2009. Stochastic network interdiction optimization via capacitated network reliability modeling and probabilistic solution discovery. *Rel. Eng. Syst. Safety*, **94**(5):913-921. [doi:10.1016/j.ress.2008.10.006]

Ramirez-Marquez, J.E., Rocco, S.C.M., Levitin, G., 2009. Optimal protection of general source-sink networks via evolutionary techniques. *Rel. Eng. Syst. Safety*, **94**(10):1676-1684. [doi:10.1016/j.ress.2009.05.002]

Sausen, P.S., Spohn, M.A., Perkusich, A., 2010. Broadcast routing in wireless sensor networks with dynamic power management and multi-coverage backbones. *Inf. Sci.*, **180**(5):653-663. [doi:10.1016/j.ins.2009.11.016]

Wang, J.Z., Varman, P., Xie, C.S., 2011. Optimizing storage performance in public cloud platforms. *J. Zhejiang Univ.-Sci. C (Comput. & Electron.)*, **12**(12):951-964. [doi:10.1631/jzus.C1100097]

Xue, J., 1985. On multistate system analysis. *IEEE Trans. Rel.*, **R-34**(4):329-337. [doi:10.1109/TR.1985.5222178]

Yarlagadda, R., Hershey, J., 1991. Fast algorithm for computing the reliability of communication network. *Int. J. Electron.*, **70**(3):549-564. [doi:10.1080/00207219108921308]

Yeh, W.C., 1998. A revised layered-network algorithm to search for all d-minpaths of a limited-flow acyclic network. *IEEE Trans. Rel.*, **47**(4):436-442. [doi:10.1109/24.756087]

Yeh, W.C., 2004. Multistate network reliability evaluation under the maintenance cost constraints. *Int. J. Prod. Econ.*, **88**(1):73-83. [doi:10.1016/S0925-5273(03)00180-4]

Yeh, W.C., 2005. A new approach to evaluating reliability of multistate networks under the cost constraint. *Omega*, **33**(3):203-209. [doi:10.1016/j.omega.2004.04.005]

Zibanezhad, B., Zamanifar, K., Sadjady, R.S., Rastegari, Y., 2011. Applying gravitational search algorithm in the QoS-based Web service selection problem. *J. Zhejiang Univ.-Sci. (Comput. & Electron.)*, **12**(9):730-742. [doi:10.1631/jzus.C1000305]

Zuo, M.J., Tian, Z., Huang, H.Z., 2007. An efficient method for reliability evaluation of multistate networks given all minimal path vectors. *IIE Trans.*, **39**(8):811-817. [doi:10.1080/07408170601013653]