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Ray targeting for optimizing smooth freeform surfaces for LED non-rotational illumination*

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Abstract: We propose an effective optimization method for generating smooth freeform surfaces for light-emitting diode (LED) non-rotational illumination based on ray targeting. This method begins with a starting design and goes through two optimization steps. An initial estimate is determined using a partial differential equation (PDE) method and a variable separation mapping. In the first optimization step the merit function is developed with ray targeting to ensure the shape of the illumination pattern. The purpose of the second optimization is to further improve the optical performance by constructing the merit function with uniformity and efficiency. Smooth freeform reflective and refractive surfaces, which can produce a uniform rectangular illumination without rotational symmetry, are designed using this method. The results show that uniform rectangular illumination is achieved and that smooth freeform surfaces are obtained. With ray targeting, the design efficiency can be significantly enhanced, and excellent optical performance can be achieved.

Key words: Illumination design, Freeform surface, Non-imaging optics, Computation methods
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1 Introduction

Freeform surfaces differ from each other through high degrees of design freedom that can be used to generate compact designs with better performance and also produce special illuminations. With the advancements in design and manufacturing, freeform surfaces are being used extensively in non-imaging optics, such as road lighting (Luo *et al.*, 2010), rectangular illumination in projectors (Ding *et al.*, 2008), automotive lighting (Cvetkovic *et al.*, 2006), and off-axis illumination in the optical lithography system (Wu *et al.*, 2011). Freeform surface design is an inverse problem, and a key issue in non-imaging optics. To solve this problem, Oliker (2000) proposed a method of supporting paraboloids (SP method) to generate smooth freeform reflectors, and Michaelis *et al.* (2011) generalized this method to design freeform lenses. With the SP method, the illumination area should be discretized into thousands of discrete target points, and a quadratic surface patch (a paraboloid, a superellipse, or a Cartesian oval) is created for each discrete target point with an iterative algorithm. For example, 20 000 Cartesian ovals had to be determined in Michaelis *et al.* (2011) for constructing a freeform refractive surface. It is hard to imagine that designing a smooth surface with this method would be convenient. Freeform surfaces can also be designed using the partial differential equation (PDE) method (Ries and Winston, 1994; Oliker, 2007; Rubinstein and Wolansky, 2007; Ding *et al.*, 2008; Fournier *et al.*, 2010; Wu *et al.*, 2011). With a PDE method, a set of first-order PDEs (or a second-order PDE) through which the freeform surface is governed, can be obtained by applying the Snell law and the conservation law of energy. The PDE method is more efficient than

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the SP method in solving this inverse design problem, and can be used to tackle complex illumination tasks. In general, an integrable mapping is required for generating a smooth freeform surface with the PDE method, because the continuity of the freeform surface is strongly determined by integrability of the mapping. Unfortunately, it is very difficult to obtain such an integrable mapping (Rubinstein and Wolansky, 2007; Fournier *et al*., 2010). With variable separation mappings, however, which are usually easier to obtain, the freeform surfaces have to be constructed with step discontinuities to obtain a non-rotational illumination (Ding *et al*., 2008; Fournier *et al*., 2010; Wu *et al*., 2011). Undoubtedly, these discontinuities pose a great obstacle for manufacturing and providing a practical application of the freeform surface. Recently, some optimization methods have been proposed for generating smooth freeform surfaces, such as the Luo method (Luo *et al*., 2010) and Wang method (Wang *et al*., 2010). Although the results generated using these methods are desirable to a certain extent, these methods will not work when the actual irradiances at all discrete points on the illumination area are approximately equal, and the stray light cannot be controlled very well. To the authors' knowledge, there is still an urgent need for a practical and effective method that can be used to design the smooth freeform reflective and refractive surfaces.

In this paper, we propose an effective optimization method based on ray targeting to generate smooth optical freeform surfaces for light-emitting diode (LED) non-rotational illumination. This method goes through three main design stages: an initial design step and two optimization steps. The initial design is aimed to find a beginning design for the optimization process through a PED method and a variable separation mapping. The first optimization is then used to ensure the shape of the illumination pattern by constructing the merit function with a ray targeting strategy. The second optimization is used to further improve the optical performance by constructing the merit function with irradiance uniformity and energy efficiency. Through this optimization design, non-rotational illumination can be achieved and smooth freeform surfaces obtained. Two design examples are given to verify this design method.

2 Optimization method

The flowchart for this design method is as depicted in Fig. 1. With this optimization method, first a starting design should be given. Then, the freeform surface will be optimized during the first optimization step to approach the target. If the results obtained from this step do not meet the requirements, the freeform surface will be further optimized during the second optimization step. Details of this design method are presented in the following.

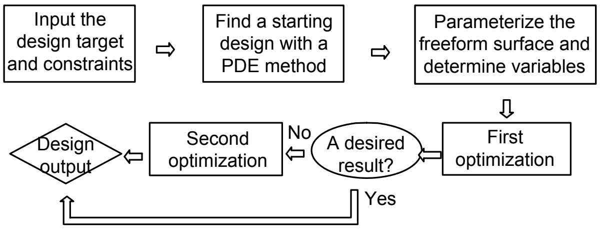

Fig. 1 Flowchart of the optimization design method

2.1 Optimization algorithm and model parameterization

As Monte Carlo ray tracing is required in the design of freeform surface illuminations, local optimization algorithms always appear to be more efficient than global optimization algorithms. In an illumination system, the relationship between target illumination and the parameters of the system is usually highly nonlinear, and cannot be obtained in an analytic form. The downhill simplex method (Koshel, 2005), the core of which is a derivative-free optimization algorithm, is used in this optimization method, and the mathematical design strategy is employed here to find an initial estimate.

Model parameterization is an important aspect of setting up this optimization. It can be divided into two parts: representing the initial estimate and determining the variables. The non-rational bi-cubic B-spline surface, which has the properties of both stability and flexibility, is used to represent the smooth freeform surface. The length of the position vector (in the spherical coordinate system) or the z-coordinate (in the cylindrical coordinate system or Cartesian coordinate system) of each data point on the optical surface can be chosen as a variable. Then a smooth B-spline surface is constructed by global surface interpolation with these discrete data points during each of the optimization iterations.

2.2 Two optimization phases and construction of the merit function

For a freeform illumination without rotational symmetry, a variable separation mapping usually violates the integrability condition, and the actual illumination will deviate from the desired one. In our method, an optimization process is employed to improve the actual illumination produced by the initial estimate. The optimization process is composed of two optimization steps, each playing a different role with a different merit function. The merit function characterizes the optical performance of an illumination system and determines the search direction, and thus should be constructed carefully for each optimization step.

As variable separation mapping does not satisfy the integrability condition, the shape of the illumination pattern may deviate significantly from the desired one. The first optimization aims to first ensure the shape of the illumination pattern. An ideal light source is used here. Based on ray targeting, the merit function (MF) in this step is defined as

$$\mathrm{MF}=\frac{1}{N}\sum_{i=1}^{N}\sqrt{(x_i-x_{\mathrm{t}i})^2+(y_i-y_{\mathrm{t}i})^2+(z_i-z_{\mathrm{t}i})^2},\quad (1)$$

where N is the number of the variables, (x_i, y_i, z_i) is the actual position of the ith ray on the target plane, and $(x_{\mathrm{t}i}, y_{\mathrm{t}i}, z_{\mathrm{t}i})$ is the corresponding target position defined by the mapping. The difference between the actual and target positions can be minimized in this optimization step. Only several hundred rays are needed, and using the result of this step as the initial estimate of the second optimization can significantly improve the search efficiency and generate a better result. This will be discussed in detail in Section 3.

In the second optimization step, an LED light source is used and the merit function is constructed with irradiance uniformity and energy efficiency to find the best compromise to improve both uniformity and efficiency. Irradiance uniformity is represented by the relative standard deviation (RSD) of irradiance. A smaller RSD value means a higher uniformity (Wu *et al.*, 2012). Transmission efficiency (TE) is the ratio of energy received by the target illumination area to energy of the output light of the source. Then, the merit function of the second optimization is defined by

$$\mathrm{MF}=w_1\cdot\mathrm{RSD}+w_2\cdot(1-\mathrm{TE}),\quad (2)$$

where w_1 and w_2 are weights. w_1 should be larger than w_2 when irradiance uniformity is of more concern. The number of rays should be chosen by taking statistical noise and the time spent into account. Use of an actual extended source usually represents good practicability of a method in designing freeform illumination. Two freeform illumination tasks without rotational symmetry will be given in Section 3 to verify this practical design method.

3 Design examples and results

Smooth freeform reflective and refractive surfaces, which can produce uniform illumination on a target plane, have been designed using this optimization method.

3.1 Designing a smooth freeform reflective surface

Schematic illustration of the freeform reflector design is as shown in Fig. 2, and the design parameters are as listed in Table 1. We use the PDE method presented in Ding *et al.* (2008) to obtain the starting design. A set of first-order PDEs can be obtained:

$$\begin{cases}\rho_\theta=f_1(\theta,\varphi,\rho),\\ \rho_\varphi=f_2(\theta,\varphi,\rho),\end{cases}\quad (3)$$

where $(\theta, \varphi, \rho(\theta, \varphi))$ are the spherical coordinates of point P, and ρ_θ and ρ_φ are the first-order partial derivatives of ρ with respect to θ and φ, respectively. This equation is numerically solved using the fourth-order Runge-Kutta formula. Then, a set of discrete data points are obtained, and the freeform surface is represented by a non-rational bi-cubic B-spline surface, which is defined by (Piegl and Tiller, 1997)

$$S(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)N_{j,q}(v)P_{i,j},\ 0\le u\le 1,\ 0\le v\le 1,\quad (4)$$

where $P_{i,j}$ are the control points, $N_{i,p}(u)$ and $N_{j,q}(v)$ are the pth- and qth-degree B-spline basis functions defined on its knot vectors respectively, and $p=q=3$. Since this PDE method uses a variable separation mapping, the actual irradiance distribution obtained from the initial estimate deviates significantly from the desired one (Fig. 3a). Obviously, the design requirements are not met. Then, the optimization process should be employed to improve the optical performance.

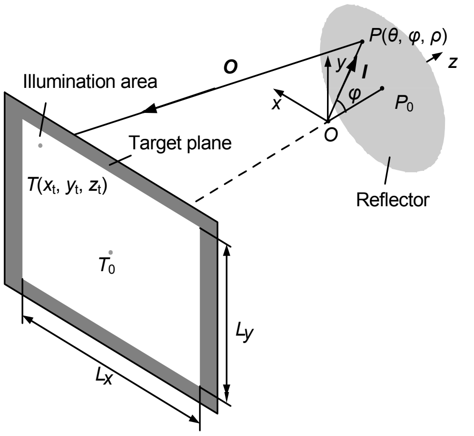

Fig. 2 Schematic illustration of the smooth freeform reflector

Table 1 Design parameters and the stopping criteria for the second optimization

Parameter	Value
z-coordinate of vertex P_0, H	6 mm
z-coordinate of point T_0, Z_0	−250 mm
Length of the illumination area, L_x	200 mm
Width of the illumination area, L_y	150 mm
Maximum emission angle, $\varphi_{\max}$	π/4
Termination tolerance on the variable, TolX	1e-6 mm
Maximum number of iterations, MaxIter	600

Considering the symmetry of the freeform reflector, we could only focus on designing the part located in the first quadrant. More variables do not mean a better result for a local optimization algorithm. Taking both the success ratio and the time spent during optimization into account, 12 variables are used. Then, another 12 mirror points are obtained, and the freeform surface in the first quadrant is constructed based on interpolation theory and Boolean operation (Fig. 4). As mentioned above, the first optimization is used to ensure the shape of illumination pattern. Only 121 rays are traced. After 300 iterations, the ray

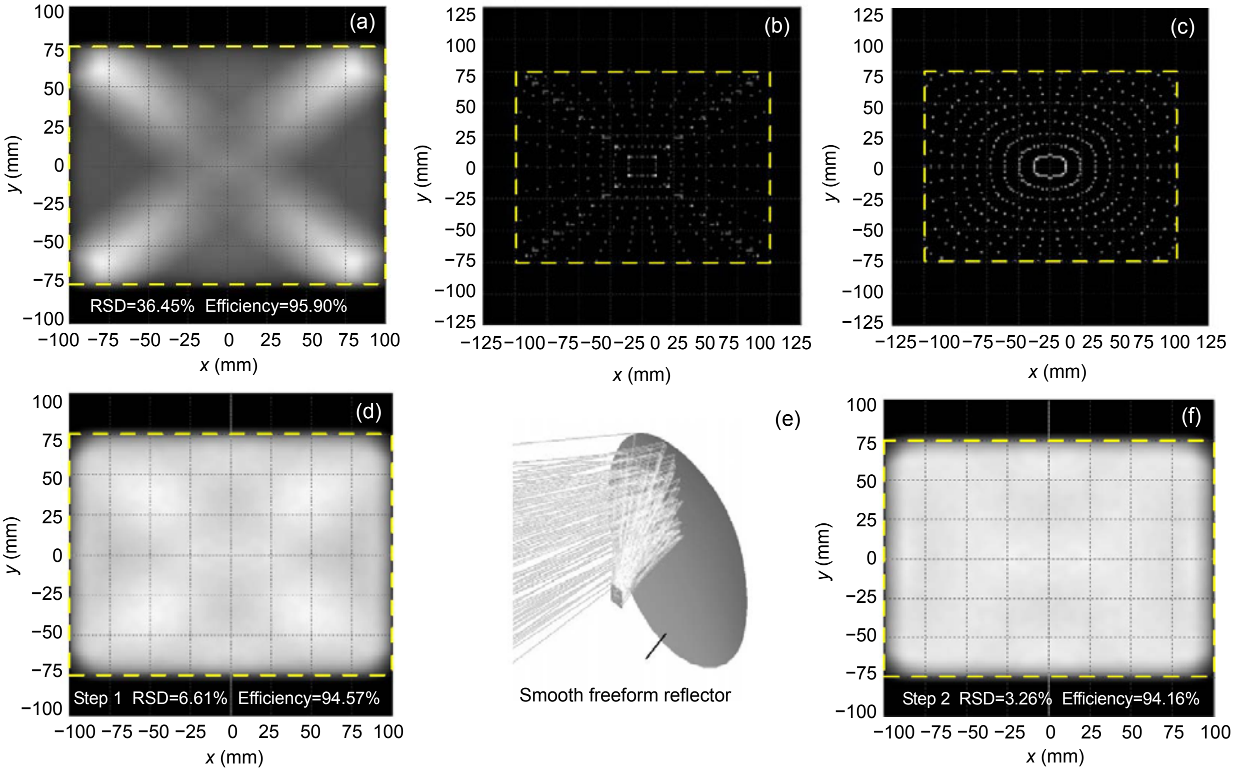

Fig. 3 Design of the freefm reflector

(a) The irradiance distribution; (b) The ray intercept plot on the illumination plane produced by the starting design; (c) The ray intercept plot; (d) The irradiance distribution obtained from the first optimization; (e) The optimized reflector; (f) The irradiance distribution obtained from the second optimization. w_1=1 and w_2=0.05

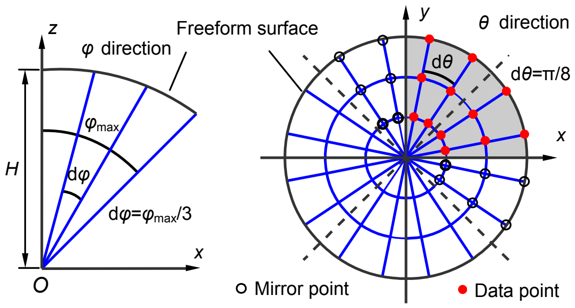

Fig. 4 Schematic illustration of determining the twelve variables

Length of the position vector of each data point is chosen as a variable

intercept plot and the irradiance distribution are as shown in Figs. 3c and 3d, respectively. RSD declines significantly from 36.45% to 6.61%. A better optical performance is obtained from the first optimization.

If the results obtained from the first optimization do not meet the design requirement, the second optimization can be employed to further improve the optical performance. The initial estimate of the second optimization is the result obtained from the first optimization. A 1 mm×1 mm LED Lambertian emitter is employed, and 400000 rays are traced during this optimization process. The stopping criterion TolX is satisfied after 265 iterations, and it takes about 12 h to finish the second optimization using a computer with a 3.06 GHz Pentium Dual-Core CPU. Then, two million rays are traced to reduce statistical noise. The optimized reflector and the irradiance distribution are as shown in Figs. 3e and 3f, respectively. This optimization step produces a 0.41% efficiency drop, and an approximately 3.35% change in uniformity. The optical performance is further improved.

To further discuss the influence of the first optimization, we employ only the second optimization during the whole optimization process. The optimization stops after 449 iterations, and the total computation time (TCT) is about 16 h. Fig. 5 shows the optimized irradiance distribution and the relationship between the merit function convergence and the number of iterations.

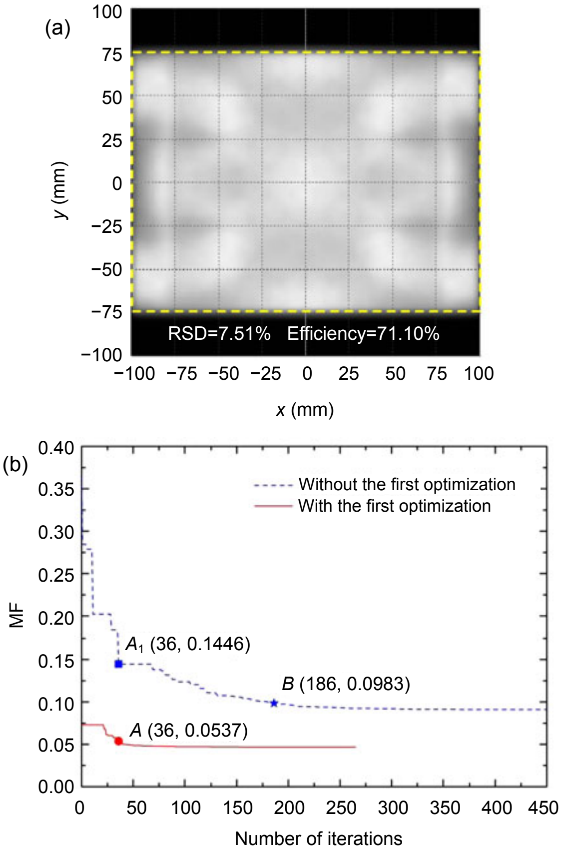

Fig. 5 Irradiance distribution on the target illumination area obtained from the optimization process without the first optimization (a) and the merit function (MF) vs. the number of iterations (b)

The RSD is 7.51% and the efficiency is 71.10% for the case in which the first optimization is not used. In Fig. 3d, an optical performance with RSD=6.61% and efficiency=94.57% is obtained from the first optimization. Obviously, a better result is obtained from the first optimization even without the second optimization. As shown in Fig. 5b, the TCT is about 101 min at point A, and an approximately 0.71% change in MF is produced after A. The TCT is about 217 min at point B, and an approximately 0.73% change in MF is produced after B (Table 2). Besides, with the same number of iterations, the result obtained from the design with the first optimization is better than that without the first optimization. For example, MF at point A_1 is 0.1446, which is greatly inferior to MF at point A. With the first optimization, the optimization process converges faster, the search efficiency of the second optimization is significantly enhanced, and a better optical performance is obtained.

3.2 Designing a smooth freeform refractive surface

Similarly, a smooth freeform refractive surface is designed to create a uniform rectangular illumination, and this optimization method is also used to improve the optical performance. Fig. 6 shows the optimized optical performance. The influence of the first optimization is also analyzed. Fig. 7b and Table 3 clearly show that the search efficiency of the second optimization is significantly enhanced when the first optimization is used, and a better optical performance is obtained.

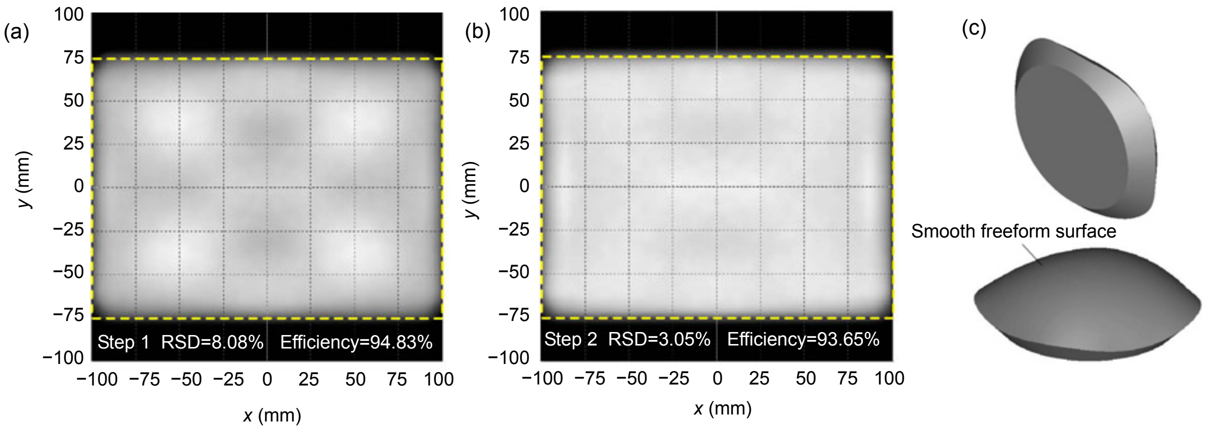

Fig. 6 Design of the freeform lens

(a) The irradiance distribution obtained from the first optimization; (b) The irradiance distribution obtained from the second optimization; (c) The optimized smooth freeform refractive surface. w_1=1 and w_2=0.05

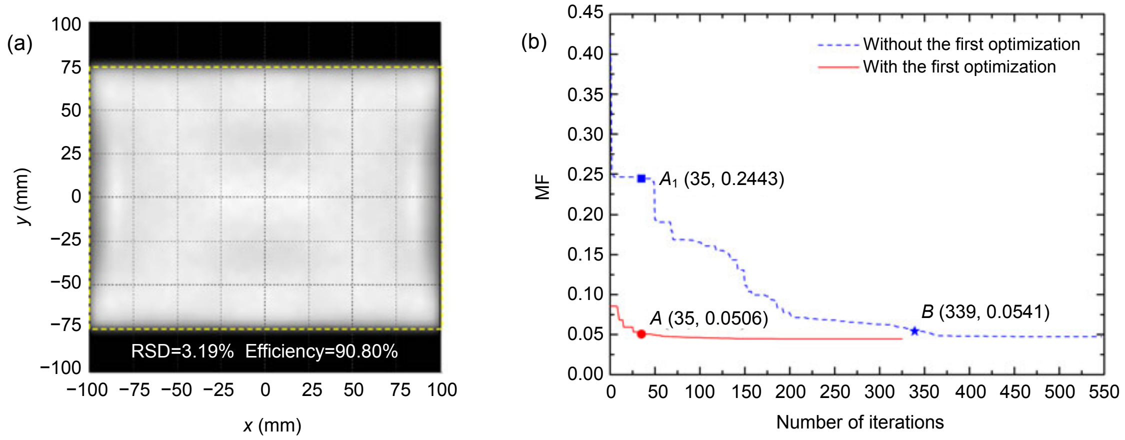

Fig. 7 Irradiance distribution obtained from the optimization process without the first optimization (a) and the merit function (MF) vs. the number of iterations (b)

The total computation time is about 80 min at point A, and an approximately 0.62% change in MF is produced after A. The total computation time is about 568 min at point B, and an approximately 0.65% change in MF is produced after B

Table 2 Comparison of the results in case of with or without the first optimization in the design of a smooth freeform reflective surface

Parameter	Value	
	With	Without
Relative standard deviation of irradiance	3.26%	7.51%
Transmission efficiency	94.16%	71.10%
Number of iterations	265	449
Total computation time (h)	12	16

Table 3 Comparison of the results in case of with or without the first optimization in the design of a smooth freeform refractive surface

Parameter	Value	
	With	Without
Relative standard deviation of irradiance	3.05%	3.19%
Transmission efficiency	93.65%	90.80%
Number of iterations	325	547
Total computation time (h)	13	18

4 Conclusions

Designing smooth freeform surfaces for producing non-rotational illumination is a challenge in freeform illumination design. In this paper, we demonstrate an effective and practical optimization method for solving this problem based on ray targeting. This optimization method begins with a starting design and goes through two optimization steps. The starting design is used to find an initial estimate for the optimization process with a PDE method. The first optimization is aimed to ensure the shape of the illumination pattern, and the second optimization is aimed to further improve the optical performance. The ray targeting technology used in the first optimization significantly enhances the design efficiency of this optimization method and ensures excellent optical performance. With this optimization design, non-rotational illumination can be achieved and smooth freeform surfaces obtained. In the future, we will generalize this method to tackle complex illumination tasks.

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