



## Fuzzy cost-profit tradeoff model for locating a vehicle inspection station considering regional constraints<sup>\*</sup>

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**Abstract:** Facility location allocation (FLA) is one of the important issues in the logistics and transportation fields. In practice, since customer demands, allocations, and even locations of customers and facilities are usually changing, the FLA problem features uncertainty. To account for this uncertainty, some researchers have addressed the fuzzy profit and cost issues of FLA. However, a decision-maker needs to reach a specific profit, minimizing the cost to target customers. To handle this issue it is essential to propose an effective fuzzy cost-profit tradeoff approach of FLA. Moreover, some regional constraints can greatly influence FLA. By taking a vehicle inspection station as a typical automotive service enterprise example, and combined with the credibility measure of fuzzy set theory, this work presents new fuzzy cost-profit tradeoff FLA models with regional constraints. A hybrid algorithm integrating fuzzy simulation and genetic algorithms (GA) is proposed to solve the proposed models. Some numerical examples are given to illustrate the proposed models and the effectiveness of the proposed algorithm.

**Key words:** Cost-profit tradeoff, Credibility theory, Fuzzy simulation, Fuzzy programming, Genetic algorithm  
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### 1 Introduction

Since facility location allocation (FLA) problems were initialized, there have been many advances in their solution methods, variants, and applications e.g., emergency service systems, telecommunications, minerals prospection (Escavy and Herrero, 2013), layout optimization of factories, public services, and automotive service firms. The early work mainly focused on an uncapacitated FLA problem. For example, Vasko *et al.* (2003) proposed a single source

capacitated FLA problem. It is considered as one of the most important problems in this field with their focus on locating facilities that have capacity constraints. Meanwhile, many different models have been formed in the field of FLA (Min *et al.*, 1997; Badri, 1999), i.e., median models (Rajagopalan *et al.*, 2008; Farahani *et al.*, 2012), center models (Church and ReVelle, 1974; Nickel and Puetro, 2005; Farahani and Hekmatfar, 2009), covering models (Daskin, 1995), hub location modes (O'Kelly, 1987), and hierarchical location models (Drezner and Hamacher 2002; Klose and Drex1, 2005; Sahin and Sural, 2007). One of the main objectives of models is to minimize the weighted distance between demand points and candidate locations. They are capable of cost or time of response minimization or profit maximization (Arabani and Farahani, 2012). Their model parameters are deterministic. A large number of solution approaches for different models have been proposed,

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e.g., exact and approximate methods, simulated annealing, genetic algorithms, and tabu search (Kuenne and Soland, 1972; Gong *et al.*, 1995; Murray and Church, 1996; Ernst and Krichnamoorthy, 1999; Arostegui *et al.*, 2006; Lu and Bostel, 2007; Farahani *et al.*, 2010; Wang *et al.*, 2011).

In practice, many parameters of FLA, such as demands of clients and cost of operating facilities, may be uncertain. To address these issues, researchers have addressed some uncertain FLA problems. For example, Logendran and Terrell (1988) solved an uncapacitated FLA problem with price-sensitive stochastic demands. Zhou and Liu (2003) established stochastic models for capacitated FLA problems to minimize the transportation distance. Taaffe *et al.* (2010) presented supply capacity acquisition and allocation with uncertain customer demands. Their goal is to determine stock levels and customer assignments for each facility in order to minimize expected procurement, holding, and shortage costs across all facilities. Locating an automotive service enterprise is one of the important applications of FLA problems. To achieve sustainable development of automotive service industry and ensure smooth automotive service activities, e.g., an automotive inspection station, parking, and 4S (sale, sparepart, service, and survey) car shop, some researchers have discussed some uncertainty related issues. For example, by considering the uncertainty of the number of inspection vehicles, Qiang *et al.* (2013) established a stochastic location model for a vehicle inspection station to achieve minimum transportation cost. A hybrid algorithm integrating the genetic algorithm (GA) and neural networks (NN) was proposed to solve this vehicle inspection station model.

Probability theory can be regarded as a tool for describing objective uncertainty. However, to obtain the probability distribution of an uncertain factor, we need many samples to apply the statistics inference approach. In fact, a decision-maker lacks data of related factors. Instead, expert opinion is used to provide estimations. In this case, credibility theory, as a branch of fuzzy set theory, can better deal with this ambiguous information. It is a fuzzy measure with self-duality and many application perspectives, e.g., industrial process planning (Tian *et al.*, 2011). In addition, some researchers have addressed the fuzzy location problems. Wen and Iwamura (2008) dis-

cussed the fuzzy FLA problem with fuzzy demands under the Hurwicz criterion to minimize the transportation cost. Wang *et al.* (2009) analyzed the two-stage fuzzy facility location problems to minimize the transportation cost. Mousavi and Niaki (2013) built a capacitated FLA model with stochastic location and fuzzy demand to minimize the transportation cost of customers. Zhou and Liu (2007) established fuzzy models for capacitated FLA problems with fuzzy demands to minimize the transportation distance/cost. Wang and Watada (2012) addressed the value-at-risk (VaR) based facility location problem in a fuzzy random environment. They discussed the recourse-based facility location problems in a hybrid uncertain environment to minimize the transportation cost (Wang and Watada, 2010).

Based on the above overview, we note that fuzzy FLA research focuses mainly on the analysis of fuzzy transportation cost or distance of customers. However, in an actual FLA process, a decision-maker must minimize the total transportation cost of customers, ensuring the specific profit that investors need to achieve. Thus, it is necessary to establish the cost-profit tradeoff model for fuzzy FLA problems. Moreover, in the actual location process, due to government policy and environmental considerations, one cannot build in places like marshes, lakes, tourist areas, parks, residential areas, and other special areas. These regions should be excluded from the location process. Based on the above reasons, by taking a vehicle inspection station as a typical automotive service enterprise and an example, we establish the fuzzy cost-profit tradeoff models of its location with regional constraints using credibility theory.

## 2 Problem statements

To easily establish our models, we first give the concept of fuzzy variable, assumptions, and parameters.

### 2.1 Fuzzy variable

Fuzzy set theory has been well developed and applied in a wide variety of practical problems. In the fuzzy world, there are three important types of measures: possibility, necessity, and credibility (Liu, 2004).

Let  $\xi$  be a fuzzy variable with membership function  $\mu$ , and let  $u$  and  $r$  be real numbers. The possibility, necessity, and credibility of a fuzzy event  $\{\xi \leq r\}$  are defined respectively by

$$\text{Pos}\{\xi \leq r\} = \sup_{t \leq r} \mu(t), \quad (1)$$

$$\text{Nec}\{\xi \leq r\} = 1 - \text{Pos}\{\xi > r\} = 1 - \sup_{t > r} \mu(t), \quad (2)$$

$$\text{Cr}\{\xi \leq r\} = \frac{1}{2}(\text{Pos}\{\xi \leq r\} + \text{Nec}\{\xi \leq r\}). \quad (3)$$

After giving the concept of credibility, the expected value of a fuzzy variable  $t$  can be defined as follows:

$$E(\xi) = \int_0^{+\infty} \text{Cr}\{\xi \geq r\} dr - \int_{-\infty}^0 \text{Cr}\{\xi \leq r\} dr. \quad (4)$$

## 2.2 Assumptions

In this study we make the following four assumptions:

**Assumption 1** The distribution condition of vehicle inspection customer demand is ignored and the center of the vehicle inspection demand region is treated as the coordinates of vehicle inspection demand; namely, the vehicles are at the center of each of the demand regions.

**Assumption 2** The cost per kilometer from the inspection demand customer to vehicle inspection station is constant; that is, the relationship between the transportation cost and transportation distance is viewed as linear.

**Assumption 3** The inspection capability of a vehicle inspection station is ignored; namely, inspection capability is great enough to meet the requirements of users.

**Assumption 4** The transportation costs are the same in vehicle inspection demand regions; that is, road conditions in different areas are assumed to be the same.

## 2.3 Parameters

$i$ —The index of vehicle inspection demand region,  $i=1, 2, \dots, m$ .

$j$ —The index of the vehicle inspection station that needs to be established,  $j=1, 2, \dots, n$ .

$(l_i, z_i)$ —The coordinates of the location of the  $i$ th vehicle inspection demand region.

$(x_j, y_j)$ —The coordinates of the location of the  $j$ th vehicle inspection station. Note that it is a decision variable in this work.

$c_{ij}$ —Cost per kilometer transportation from vehicle inspection demand region  $i$  to inspection station  $j$  (in CNY).

$\xi_{ij}$ —The number of vehicles from vehicle inspection region  $i$  to inspection station  $j$ . It is a fuzzy variable.

$b_j$ —The obtained benefit of inspecting one vehicle at vehicle inspection station  $j$  (in CNY).

$e_j$ —The fixed cost to establish the vehicle inspection station  $j$  (in CNY).

## 3 Fuzzy cost-profit tradeoff models for locating a vehicle inspection station

Based on the presented concept and assumptions, we build two fuzzy cost-profit tradeoff models with regional constraints for a vehicle inspection station, i.e., the fuzzy expected cost-profit tradeoff model with regional constraints and the fuzzy chance-constrained cost-profit tradeoff model with regional constraints.

### 3.1 Fuzzy expected cost-profit tradeoff model with regional constraints

In the actual process of locating a vehicle inspection station, a decision-maker wants to seek the minimum expected total transportation cost of vehicle inspection customers while obtaining the assuredly specific expected profit of investors. Furthermore, regional constraints should be excluded from consideration when a vehicle inspection is located. To deal with this issue, regional constraints should be introduced to the established model. Thus, we establish a fuzzy expected cost-profit tradeoff model with regional constraints for locating a vehicle inspection station as follows:

$$\min E(C) \quad (5)$$

subject to

$$\begin{cases} E\left[\sum \sum b_j \xi_{ij} - \sum e_j\right] \geq B^0, \\ h(x, y) \leq 0, \quad g(x, y) \geq 0, \\ x \in (x_1, x_u), \quad y \in (y_1, y_u), \end{cases} \quad (6)$$

where  $C$  is the total transportation cost of vehicle

inspection customers,  $C = \sum_i \sum_j \xi_{ij} c_{ij} d_{ij}$ .  $\xi_{ij}$  is the

number of vehicles from vehicle inspection region  $i$  to inspection station  $j$ ,  $c_{ij}$  is the cost per kilometer transportation from vehicle inspection demand area  $i$  to inspection station  $j$ , and  $d_{ij}$  is the distance between vehicle inspection demand region  $i$  and inspection station  $j$ ,  $d_{ij} = \sqrt{(x_j - l_i)^2 + (y_j - z_i)^2}$ .  $\sum_i \sum_j b_j \xi_{ij}$

$-\sum_j e_j \triangleq B$  is the obtained total profit of investors of

building vehicle inspection station, and  $B^0$  is a given expected profit of investors.  $x_l$  and  $x_u$  are the lower and upper bounds along the  $x$  axis, respectively, and  $y_l$  and  $y_u$  are the lower and upper bounds along the  $y$  axis, respectively.  $x_l, x_u, y_l,$  and  $y_u$  can be determined by the coordinates of vehicle inspection demand regions.  $h(x, y) \leq 0$  and  $g(x, y) \geq 0$  are regional constraints.

### 3.2 Fuzzy chance-constrained cost-profit tradeoff model with regional constraints

In some cases, a decision-maker wants to seek the minimum total transportation cost of vehicle inspection customers for obtaining an assuredly specific profit for investors with at least some given confidence levels. Combined with the chance-constrained programming concept, the following fuzzy chance-constrained cost-profit tradeoff model with regional constraints is established to locate a vehicle inspection station:

$$\min \bar{C} \tag{7}$$

subject to

$$\begin{cases} \text{Cr}\left\{\sum_i \sum_j \xi_{ij} c_{ij} d_{ij} \leq \bar{C}\right\} \geq \alpha, \\ \text{Cr}\left\{\left(\sum_i \sum_j b_j \xi_{ij} - \sum_j e_j\right) \geq \bar{B}\right\} \geq \beta, \\ h(x, y) \leq 0, \quad g(x, y) \geq 0, \\ x \in (x_l, x_u), \quad y \in (y_l, y_u), \end{cases} \tag{8}$$

where  $\bar{B}$  is a given profit of investors,  $\alpha$  and  $\beta$  are predetermined confidence levels, and

$$\begin{aligned} &\text{Cr}\left\{\sum_i \sum_j \xi_{ij} c_{ij} d_{ij} \leq \bar{C}\right\} \geq \alpha \text{ and} \\ &\text{Cr}\left\{\left(\sum_i \sum_j \xi_{ij} b_j - \sum_j e_j\right) \leq \bar{B}\right\} \geq \beta \end{aligned}$$

are credibility constraints.

## 4 Solution algorithm

Fuzzy simulation is an effective means to assess and calculate fuzzy functions. It has been used to effectively solve many fuzzy programming problems (Liu and Liu, 2002; Liu, 2004). GA has been used to successfully solved many complex industrial optimization problems which are hard to solve by analytic methods. In this work, a hybrid intelligent algorithm integrating fuzzy simulation and GA is adopted to solve two fuzzy cost-profit tradeoff models with regional constraints for a vehicle inspection station (Liu, 2006; 2010; Tian et al., 2011; Arish et al., 2014).

### 4.1 Fuzzy simulation (Tian et al., 2014b)

Let  $\xi = (\xi_1, \xi_2, \dots, \xi_m)$ , where  $m$  is the number of customer regions. We denote that  $\mu$  is the membership function of  $\xi$  and  $\mu_i$  is the membership function of  $\xi_i, i=1, 2, \dots, m$ . To solve the proposed models, we must handle the following three uncertainty functions:

$$U_1 : (x, y) \rightarrow E(C(x, y, \xi)), \tag{9}$$

$$U_2 : (x, y) \rightarrow \text{Cr}\{B(x, y, \xi) \geq \bar{B}\}, \tag{10}$$

$$U_3 : (x, y) \rightarrow \min\{\bar{C} \mid \text{Cr}\{C(x, y, \xi) \leq \bar{C}\} \geq \alpha\}. \tag{11}$$

To compute the uncertainty function  $U_1$ , the following simulation algorithm is introduced:

Step 1:  $E=0$ ;

Step 2: Randomly generate real numbers  $u_i$  of the  $\varepsilon$ -level sets of fuzzy variables  $\xi_i$  such that  $\mathbf{u}_j=(u_{ij}), i=1, 2, \dots, m$  and  $j=1, 2, \dots, N$ ;

Step 3: Set

$$a=C(x, y, u_1) \wedge C(x, y, u_2) \wedge \dots \wedge C(x, y, u_N),$$

$$b=C(x, y, u_1) \vee C(x, y, u_2) \vee \dots \vee C(x, y, u_N);$$

Step 4: Randomly generate  $r$  from  $[a, b]$ ;

Step 5: If  $r \geq 0$ , then  $E=E+\text{Cr}\{C(x, y, \xi) \geq r\}$ ;

Step 6: If  $r \leq 0$ , then  $E=E+\text{Cr}\{C(x, y, \xi) \leq r\}$ ;

Step 7: Repeat steps 4–6  $N$  times;

Step 8:  $E[C(x, y, \xi)]=a \vee 0 + b \wedge 0 + E \cdot (b-a)/N$ .

Similarly, to compute the uncertainty function  $U_2$  based on the concept of credibility measure, the credibility can be obtained approximately by

$$\begin{aligned} J = &\frac{1}{2} \left( \max_{1 \leq k \leq N} \{ \mu(\mathbf{u}_k) \mid B(x, y, \mathbf{u}_k) \geq \bar{B} \} \right. \\ &\left. + \min_{1 \leq k \leq N} \{ 1 - \mu(\mathbf{u}_k) \mid B(x, y, \mathbf{u}_k) > \bar{B} \} \right). \end{aligned} \tag{12}$$

Thus, the fuzzy simulation algorithm to compute  $U_2$  is described as follows:

Step 1:  $k=0$ ;

Step 2: Randomly generate real numbers  $u_i$  of the  $\varepsilon$ -level sets of fuzzy variables  $\xi_i$ ;

Step 3: Set  $\mathbf{u}_k=(u_1, u_2, \dots, u_m)$  and  $\mu(\mathbf{u}_k)=\mu_1(u_1) \wedge \mu_2(u_2) \wedge \dots \wedge \mu_m(u_m)$ ;

Step 4:  $k=k+1$ , go to step 2 if  $i \leq N$  and step 5, otherwise;

Step 5: Return  $J$ .

Similarly, for  $U_3$ , we have

$$L(r) = \frac{1}{2} \left( \max_{1 \leq k \leq N} \{ \mu(\mathbf{u}_k) \mid C(x, y, \mathbf{u}_k) \leq r \} + \min_{1 \leq k \leq N} \{ 1 - \mu(\mathbf{u}_k) \mid C(x, y, \mathbf{u}_k) > r \} \right). \quad (13)$$

Thus, the fuzzy simulation algorithm to compute  $U_3$  is described as follows:

Step 1:  $k=0$ ;

Step 2: Randomly generate real numbers  $u_i$  of the  $\varepsilon$ -level sets of fuzzy variables  $\xi_i$ ;

Step 3: Set  $\mathbf{u}_k=(u_1, u_2, \dots, u_m)$  and  $\mu(\mathbf{u}_k)=\mu_1(u_1) \wedge \mu_2(u_2) \wedge \dots \wedge \mu_m(u_m)$ ;

Step 4:  $k=k+1$ , go to step 2 if  $k \leq N$  and step 5 otherwise;

Step 5: Seek the minimum  $r$  satisfying  $L(r) \geq \alpha$ ;

Step 6: Return  $r$ .

In addition, the process of computing fuzzy function  $(x, y) \rightarrow E(B(x, y, \xi))$  can be obtained similarly based on the simulation process  $U_1$ , and thus is not described in detail here.

### 4.2 Hybrid algorithm

The hybrid algorithm integrating GA and fuzzy simulation is presented here. The basic knowledge of GA can refer to Ke and Liu (2010).

Step 1: Initialize pop\_size chromosomes, probability of mutation  $pr_m$ , probability of crossover  $pr_c$ , maximum generation  $g_{max}$ , and the number of simulation cycles. Note that the regional constraints should be checked in this step.

Step 2: Update the chromosomes by crossover and mutation operations.

Step 3: Calculate the objective value and fitness of all chromosomes via fuzzy simulation.

Step 4: Select the chromosomes by spinning the roulette wheel.

Step 5: Repeat steps 2–4 for a given number of

cycles.

Step 6: Report the best chromosome as the optimal solution.

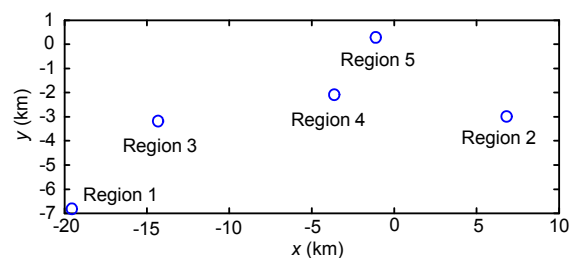
The above algorithm has been implemented in the Visual C++ 6.0 programming language.

## 5 Case study

A location problem for a vehicle inspection station in Fushun City, China, is considered a case study. This city is divided into five vehicle inspection regions, i.e., Development, Dongzhou, Wanghua, Xinfu, and Shuncheng regions. Their center coordinates of these regions are listed in Table 1. The simplified map of the city is shown in Fig. 1. In addition, the center of this city is located at (41 578 784.96, 4 638 592.97) m, which is measured via the Xi'an 80 coordinate method. To conveniently optimize the location problem, taking the center of this city as the relative origin, the coordinates of these five regions are transformed into numerical values (Table 1). Moreover, the number of inspection vehicles and the cost per kilometer of each demand region are listed in Table 1, and  $\xi_{ij}$  is treated as a triangular distribution in this work, i.e.,  $\xi_{ij}=(a, b, c)$ , where  $a, b$ , and  $c$  are the pessimistic, possible, and optimistic values of fuzzy numbers, respectively.

**Table 1 Typical parameters for Fushun City, China used in the established model**

Region No.	Region name	$l$ (m)	$z$ (m)	$\xi$ ( $\times 10^3$ )	$c$ (CNY/km)
1	Development	-19553.93	-6822.87	(1.5, 2.5, 3.5)	3
2	Dongzhou	6818.23	-2988.68	(2, 3, 4)	3
3	Wanghua	-14319.44	-3175.23	(0.5, 1.5, 2.5)	3
4	Xinfu	-3625.74	-2088.84	(2.5, 3.5, 4.5)	3
5	Shuncheng	-1109.74	285.12	(4, 5, 6)	3



**Fig. 1 The simplified map of the Fushun City, China and the five regions**

The parameters of the hybrid algorithm are set as follows: the population size  $pop\_size$  is 40, the probability of crossover  $pr_c$  is 0.3, the probability of mutation  $pr_m$  is 0.3, the maximum number of generations  $g_{max}$  is 800, and the number of simulation cycles is 3000.

**Example 1** A decision-maker hopes to build a vehicle inspection station such that the following expected profit constraint  $E(B) \geq 85000$  CNY and regional constraint  $x^2 + y^2 \leq 1.7 \times 10^7$  are satisfied. Set the obtained benefit to inspect one vehicle as  $b_j = 98$  CNY, and the fixed cost to build a vehicle inspection station as  $e_j = 1.1 \times 10^6$  CNY. This problem can be translated to the following model:

$$\min E(C) \tag{14}$$

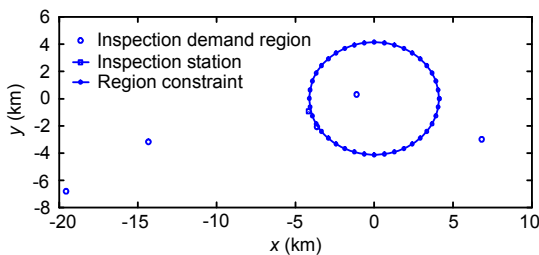
subject to

$$\begin{cases} E\left(\sum_i \sum_j b_j \xi_{ij} - \sum_j e_j\right) \geq 85\ 000, \\ x^2 + y^2 \geq 1.7 \times 10^7, \\ x \in (-19\ 553.93, 6818.23), \\ y \in (-6822.87, 285.12). \end{cases} \tag{15}$$

After the algorithm is executed, the following results can be obtained:

$$(x, y) = (-4165.35, -941.23), E(C) = 3.26839 \times 10^5.$$

The results denote that the vehicle inspection customers have the lowest total transportation cost, i.e.,  $3.26839 \times 10^5$  CNY when the vehicle inspection station is located at  $(-4165.35, -941.23)$  m. In addition, the plane distribution graph for locating a vehicle inspection station is obtained (Fig. 2).



**Fig. 2** Plane distribution graph for locating a vehicle inspection station

**Example 2** A decision-maker wishes to build a vehicle inspection station such that the following profit constraint  $B \geq 7500$  CNY and regional constraint  $1.8 \times 10^7 \leq x^2 + y^2 \leq 3.5 \times 10^7$  are satisfied. Set the obtained benefit to inspect one vehicle as  $b_j = 98$  CNY, the fixed cost to build a vehicle inspection station as  $e_j = 1.1 \times 10^6$  CNY,  $\alpha = 0.9$ , and  $\beta = 0.8$ . This problem can be translated to the following model:

$$\min \bar{C} \tag{16}$$

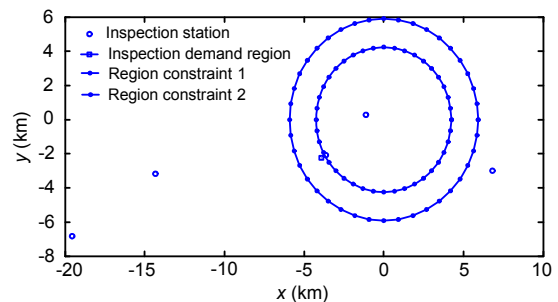
subject to

$$\begin{cases} Cr\left\{\sum \sum \xi_{ij} c_{ij} d_{ij} \leq \bar{C}\right\} \geq 0.9, \\ Cr\left\{\sum \sum b_j \xi_{ij} - \sum e_j \geq 75\ 000\right\} \geq 0.8, \\ 1.8 \times 10^7 \leq x^2 + y^2 \leq 3.5 \times 10^7, \\ x \in (-19\ 553.93, 6818.23), \\ y \in (-6822.87, 285.12). \end{cases} \tag{17}$$

After the algorithm is executed, the following results can be obtained:

$$(x, y) = (-3914.9566, -2242.4633), \bar{C} = 3.93693 \times 10^5.$$

The results denote that the vehicle inspection customers have the lowest total transportation cost, i.e.,  $3.93693 \times 10^5$  CNY when the vehicle inspection station is located at  $(-3914.9566, -2242.4633)$  m. The plane distribution graph for the location of a vehicle inspection station is shown in Fig. 3.



**Fig. 3** Plane distribution graph for locating a vehicle inspection station

From Figs. 2 and 3, we see that the vehicle inspection station is legitimately located. Moreover, it meets the regional constraints. The results mean that the proposed approach is feasible and effective in

solving fuzzy cost-profit tradeoff models of the location problem for a vehicle inspection station.

To test the effectiveness of the algorithm, the results for Example 2 with different cases and GA parameters are shown in Table 2. The errors are computed, and the relative error is defined as (actual value–best value)/(best value)×100%, where ‘best value’ is the minimum value of solution results among cases while ‘actual value’ is the solution result of the model when the algorithm is run in each case.

**Table 2 Comparison of solutions for Example 2 in five different cases**

Parameter	Value				
	Case 1	Case 2	Case 3	Case 4	Case 5
pop_size	40	25	30	40	40
pr <sub>c</sub>	0.3	0.2	0.2	0.2	0.2
pr <sub>m</sub>	0.3	0.4	0.3	0.4	0.2
Actual value	393 693	391 259	390 124	394 787	392 948
Best value	390 124	390 124	390 124	390 124	390 124
Relative error (%)	0.91	0.29	0.00	1.20	0.72

From Table 2, the relative error does not exceed 1.2%. This indicates that the proposed algorithm is highly effective when it is used to solve the proposed models.

To further test the algorithm and the proposed method, the results of the proposed method and the deterministic optimization method (Dai, 2010) are compared using the same GA parameters. The results of optimal solutions in both cases with different parameters for Example 1 are listed in Table 3. Note that when the deterministic optimization method is executed,  $\xi_i$  ( $i=1, 2, \dots, 5$ ) are set to the possible values of the fuzzy number.

Table 3 shows that the optimal solutions of both methods are consistent and basically close. This shows that the proposed method is reasonable and feasible when it is used to solve a vehicle inspection station location problem. In addition, the proposed method can better describe the uncertainty of data obtained due to expert opinions, while the deterministic optimization method cannot. Thus, it cannot better describe the actual condition of locating a vehicle inspection station than the proposed one. That is, the proposed method is feasible for solving the vehicle inspection station location problem.

**Table 3 Comparison of solutions of Example 1 for different cases**

Case	pop_size	pr <sub>c</sub>	pr <sub>m</sub>	Quasi-optimal solution	
				Proposed method	Deterministic method
1	40	0.3	0.3	326 839	319 213
2	25	0.2	0.4	327 703	321 316
3	35	0.2	0.3	327 047	319 213
4	40	0.2	0.3	326 515	319 244
Average				327 026	319 746

## 6 Conclusions

Transportation facility and automotive service enterprise location is an interesting and important issue. This type of FLA can encounter some uncertainty problems, e.g., customer demands and allocations. Even locations of customers and facilities are non-deterministic. To handle this, uncertain FLA problems have been presented. However, the current uncertain automotive service enterprise location research focuses mainly on the analysis of fuzzy transportation cost or distance of customers. To do so, considering regional constraints and balance between the obtained profit of investors and the transportation cost of target customers, and taking the vehicle inspection station as a typical automotive service enterprise and an example, we have established the fuzzy cost-profit tradeoff models with regional constraints for the first time. A hybrid algorithm integrating fuzzy simulation and GA is adopted to solve the established models. Compared with the standard method (deterministic optimization), the proposed method can better describe the uncertainty of data obtained due to expert opinions with a small sacrifice of optimality. The results reveal that it is feasible and effective in solving the proposed models. The results can be used to guide decision-makers in making better decisions when a vehicle inspection station is located. In addition, the extension of the proposed approach can be used to analyze the fuzzy profit-cost tradeoff issue of FLA.

There exist some limitations with the proposed method. For example, we consider merely the circular regional constraints. In reality, however, the regional constraint can be irregular in shape, and effective handling of this needs further work. In addition, a

fuzzy multi-objective model integrating additional parameters, e.g., customer satisfaction degree, needs further studies (Ani et al., 2013; Huang and Lin, 2014; Tian and Liu, 2014). Also, the proposed method can be extended to the other fields, e.g., reverse logistics (Tian et al., 2012; 2014a; Li et al., 2014).

## References

- Ani, O.B., Xu, H., Shen, Y.P., et al., 2013. Modeling and multiobjective optimization of traction performance for autonomous wheeled mobile robot in rough terrain. *J. Zhejiang Univ.-Sci. C (Comput. & Electron.)*, **14**(1):11-29. [doi:10.1631/jzus.C12a0200]
- Arabani, B., Farahani, R.Z., 2012. Facility location dynamics: an overview of classifications and applications. *Comput. Ind. Eng.*, **62**(1):408-420. [doi:10.1016/j.cie.2011.09.018]
- Arish, S., Amiri, S., Noori, K., 2014. FICA: fuzzy imperialist competitive algorithm. *J. Zhejiang Univ.-Sci. C (Comput. & Electron.)*, **15**(5):363-371. [doi:10.1631/jzus.C1300088]
- Arostegui, M.A.Jr., Kadipasaoglu, S.N., Khumawala, B.M., 2006. An empirical comparison of tabu search, simulated annealing, and genetic algorithms for facilities location problems. *Int. J. Prod. Econ.*, **103**(2):742-754. [doi:10.1016/j.ijpe.2005.08.010]
- Badri, M.A., 1999. Combining the analytic hierarchy process and goal programming for global facility location-allocation problem. *Int. J. Prod. Econ.*, **62**(3):237-248. [doi:10.1016/S0925-5273(98)00249-7]
- Church, R., ReVelle, C., 1974. The maximal covering location problem. *Papers Reg. Sci. Assoc.*, **32**(1):101-118. [doi:10.1007/BF01942293]
- Dai, J.Y., 2010. Study on Optimization Problem for Vehicle Detection Station Network Layout. MS Thesis, Jilin University, China (in Chinese).
- Daskin, M.S., 1995. Network and Discrete Location: Models, Algorithms, and Applications. Wiley, New York, NY. [doi:10.1002/9781118032343]
- Drezner, Z., Hamacher, H., 2002. Facility Location: Applications and Theory. Springer-Verlag, Berlin. [doi:10.1007/978-3-642-56082-8]
- Ernst, A.T., Krichnamoorthy, M., 1999. Solution algorithms for the capacitated single allocation hub location problem. *Ann. Oper. Res.*, **86**:141-159. [doi:10.1023/A:1018994432663]
- Escavy, J.I., Herrero, M.J., 2013. The use of location-allocation techniques for exploration targeting of high place-value industrial minerals: a market-based prospectivity study of the Spanish gypsum resources. *Ore Geol. Rev.*, **53**:504-516. [doi:10.1016/j.oregeorev.2013.02.010]
- Farahani, R.Z., Hekmatfar, M., 2009. Facility Location: Concepts, Models, Algorithms and Case Studies. Physica-Verlag, Heidelberg.
- Farahani, R.Z., SteadieSeifi, M., Asgari, N., 2010. Multiple criteria facility location problems: a survey. *Appl. Math. Model.*, **34**(7):1689-1709. [doi:10.1016/j.apm.2009.10.005]
- Farahani, R.Z., Asgari, N., Heidari, N., et al., 2012. Covering problems in facility location: a review. *Comput. Ind. Eng.*, **62**(1):368-407. [doi:10.1016/j.cie.2011.08.020]
- Gong, D., Gen, M., Xu, W., et al., 1995. Hybrid evolutionary method for obstacle location-allocation problem. *Int. J. Comput. Ind. Eng.*, **29**(1-4):525-530. [doi:10.1016/0360-8352(95)00128-N]
- Huang, Y., Lin, L., 2014. Designing a location update strategy for free-moving and network-constrained objects with varying velocity. *J. Zhejiang Univ.-Sci. C (Comput. & Electron.)*, **15**(8):675-686. [doi:10.1631/jzus.C1300337]
- Ke, H., Liu, B., 2010. Fuzzy project scheduling problem and its hybrid intelligent algorithm. *Appl. Math. Model.*, **34**(2):301-308. [doi:10.1016/j.apm.2009.04.011]
- Klose, A., Drexl, A., 2005. Facility location models for distribution system design. *Eur. J. Oper. Res.*, **162**(1):4-29. [doi:10.1016/j.ejor.2003.10.031]
- Kuenne, R.E., Soland, R.M., 1972. Exact and approximate solutions to the multisource Weber problem. *Math. Program.*, **3**(1):193-209. [doi:10.1007/BF01584989]
- Li, Z., Tian, G.D., Cheng, G., et al., 2014. An integrated cultural particle swarm algorithm for multi-objective reliability-based design optimization. *Proc. Inst. Mech. Eng. C*, **228**(7):1185-1196. [doi:10.1177/0954406213502589]
- Liu, B., 2004. Uncertainty Theory: an Introduction to Its Axiomatic Foundations. Springer-Verlag, Berlin. [doi:10.1007/978-3-540-39987-2]
- Liu, B., 2006. A survey of credibility theory. *Fuzzy Optim. Dec. Making*, **5**(4):387-408. [doi:10.1007/s10700-006-0016-x]
- Liu, B., 2010. Uncertainty Theory: a Branch of Mathematics for Modeling Human Uncertainty. Springer, Berlin. [doi:10.1007/978-3-642-13959-8]
- Liu, B., Liu, Y.K., 2002. Expected value of fuzzy variable and fuzzy expected value models. *IEEE Trans. Fuzzy Syst.*, **10**(4):445-450. [doi:10.1109/TFUZZ.2002.800692]
- Logendran, R., Terrell, M.P., 1988. Uncapacitated plant location-allocation problems with price sensitive stochastic demands. *Comput. Oper. Res.*, **15**(2):189-198. [doi:10.1016/0305-0548(88)90011-1]
- Lu, Z.Q., Bostel, N., 2007. A facility location model for logistics systems including reverse flows: the case of remanufacturing activities. *Comput. Oper. Res.*, **34**(2):299-323. [doi:10.1016/j.cor.2005.03.002]
- Min, H., Melachrinoudis, E., Wu, X., 1997. Dynamic expansion and location of an airport: a multiple objective approach. *Transp. Res. Part A*, **31**(5):403-417. [doi:10.1016/S0965-8564(96)00037-7]
- Mousavi, S.M., Niaki, S.T.A., 2013. Capacitated location allocation problem with stochastic location and fuzzy demand: a hybrid algorithm. *Appl. Math. Model.*, **37**(7):



- 5109-5119. [doi:10.1016/j.apm.2012.10.038]
- Murray, A.T., Church, R.L., 1996. Applying simulated annealing to location-planning models. *J. Heur.*, **2**(1):31-53. [doi:10.1007/BF00226292]
- Nickel, S., Puetro, J., 2005. Location Theory: a Unified Approach. Springer-Verlag Berlin Heidelberg.
- O'Kelly, M.E., 1987. A quadratic integer program for the location of interacting hub facilities. *Eur. J. Oper. Res.*, **32**(3):393-404. [doi:10.1016/S0377-2217(87)80007-3]
- Qiang, T.G., Tian, G.D., Chu, J.W., et al., 2013. Location analysis of vehicle inspection station based on NN-GA. *Adv. Inf. Sci. Serv. Sci.*, **5**(4):622-629. [doi:10.4156/AISS.vol5.issue4.76]
- Rajagopalan, H.K., Saydam, C., Xiao, J., 2008. A multi-period set covering location model for dynamic redeployment of ambulances. *Comput. Oper. Res.*, **35**(3):814-826. [doi:10.1016/j.cor.2006.04.003]
- Sahin, G., Sural, H., 2007. A review of hierarchical facility location models. *Comput. Oper. Res.*, **34**(8):2310-2331. [doi:10.1016/j.cor.2005.09.005]
- Taaffe, K., Geunes, J., Romeijn, H.E., 2010. Supply capacity acquisition and allocation with uncertain customer demands. *Eur. J. Oper. Res.*, **204**(2):263-273. [doi:10.1016/j.ejor.2009.10.030]
- Tian, G.D., Liu, Y., 2014. Energy-efficient models of sustainable location for a vehicle inspection station with emission constraints. *IEEE Trans. Autom. Sci. Eng.*, [doi:10.1109/TASE.2014.2360673]
- Tian, G.D., Chu, J.W., Liu, Y.M., et al., 2011. Expected energy analysis for industrial process planning problem with fuzzy time parameters. *Comput. Chem. Eng.*, **35**(12):2905-2912. [doi:10.1016/j.compchemeng.2011.05.012]
- Tian, G.D., Zhou, M.C., Chu, J.W., et al., 2012. Probability evaluation models of product disassembly cost subject to random removal time and different removal labor cost. *IEEE Trans. Autom. Sci. Eng.*, **9**(2):288-295. [doi:10.1109/TASE.2011.2176489]
- Tian, G.D., Chu, J.W., Hu, H.S., et al., 2014a. Technology innovation system and its integrated structure for automotive components remanufacturing industry development in China. *J. Cleaner Prod.*, **85**(15):419-432. [doi:10.1016/j.jclepro.2014.09.020]
- Tian, G.D., Zhou, M.C., Chu, J.W., et al., 2014b. Stochastic cost-profit tradeoff model for locating an automotive service enterprise. *IEEE Trans. Autom. Sci. Eng.*, (99):1-8. [doi:10.1109/TASE.2013.2297623]
- Vasko, F.J., Newhart, D.D., Stott, K.L., et al., 2003. A large-scale application of the partial coverage uncapacitated facility location problem. *J. Oper. Res. Soc.*, **54**(1):11-20. [doi:10.1057/palgrave.jors.2601469]
- Wang, K.J., Makond, B., Liu, S.Y., 2011. Location and allocation decisions in a two-echelon supply chain with stochastic demand: a genetic-algorithm based solution. *Expert Syst. Appl.*, **38**(5):6125-6131. [doi:10.1016/j.eswa.2010.11.008]
- Wang, S., Watada, J., 2010. Recourse-based facility location problems in hybrid uncertain environment. *IEEE Trans. Syst. Man Cybern. B*, **40**(4):1176-1187. [doi:10.1109/TSMCB.2009.2035630]
- Wang, S., Watada, J., 2012. A hybrid modified PSO approach to VaR-based facility location problems with variable capacity in fuzzy random uncertainty. *Inf. Sci.*, **192**:3-18. [doi:10.1016/j.ins.2010.02.014]
- Wang, S., Watada, J., Pedrycz, W., 2009. Value-at-risk-based two-stage fuzzy facility location problems. *IEEE Trans. Ind. Inf.*, **5**(4):465-482. [doi:10.1109/TII.2009.2022542]
- Wen, M., Iwamura, K., 2008. Fuzzy facility location-allocation problem under the Hurwicz criterion. *Eur. J. Oper. Res.*, **184**(2):627-635. [doi:10.1016/j.ejor.2006.11.029]
- Zhou, J., Liu, B., 2003. New stochastic models for capacitated location-allocation problem. *Comput. Ind. Eng.*, **45**(1):111-125. [doi:10.1016/S0360-8352(03)00021-4]
- Zhou, J., Liu, B., 2007. Modeling capacitated location-allocation problem with fuzzy demands. *Comput. Ind. Eng.*, **53**(3):454-468. [doi:10.1016/j.cie.2006.06.019]