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An optimal algorithm for preemptive scheduling on non-simultaneously available uniform machines*

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Abstract: We study preemptive scheduling on m uniform machines with non-simultaneous available times to minimize the makespan. Each machine has a different speed and a different available time. We first provide a lower bound on the optimal makespan of the problem by converting the real machines to virtual machines that guarantee that a machine with an earlier available time has a greater speed at any time. Then, we provide an $O(nm+m^2)$-time algorithm to find an optimal schedule with, at most, $0.5(m^2+3m)-2$ preemptions.

Key words: Optimal algorithm; Preemptive scheduling; Uniform machine; Non-simultaneous available times
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1 Introduction

In this paper, we consider the problem of preemptive scheduling on parallel machines with non-simultaneous machine available times. Our goal is to minimize the maximum completion time, i.e., the makespan. Unlike classical parallel-machine scheduling where all the machines are available simultaneously at time zero, the machines in our problem may not be available at zero time. Lee (1991) first considered such a scheduling model, which has many applications in real industry settings (Lee et al., 1997), e.g., machines may require different set-up times or warm-up times before they start processing jobs due to preventive maintenance and adjustment requirements.

We formally introduce the problem under study as follows: There are n independent jobs $\{J_1, J_2, ..., J_n\}$ to be processed on m parallel machines $\{M_1, M_2, ..., M_m\}$. Each job J_j has a processing time $p_j\,(p_j>0)$. Each machine M_i has a speed $s_i\,(s_i>0)$ and is available at time r_i, which means that M_i can only start processing jobs at time r_i, $i=1,2,...,m$. Without loss of generality, we assume that $r_1 \le r_2 \le \cdots \le r_m$. Note that the processing time of job J_j is $\frac{p_j}{s_i}$ if it is processed on machine M_i. Job preemption is allowed. Using the three-field notation for scheduling problems, we denote our problem as $Qm, r_i \mid pmtn \mid C_{\max}$.

For the problem of non-simultaneous paral-

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lel-machine scheduling, almost all the existing studies focus on the case where preemption is not permitted. Lee (1991) showed that the longest processing time (LPT) algorithm has a tight worst-case ratio of $\frac{2}{3}-\frac{1}{2m}$ for the case with m identical machines, i.e., $Pm,r_i\|C_{\max}$. They also provided a modified LPT (MLPT) algorithm with a worst-case ratio of $\frac{4}{3}$. He (1998) further showed that LPT is asymptotically optimal for the problem and provided a tight parametric worst-case bound for LPT. Lee et al. (2000) pointed out that there is a situation in which some machines do not process any job, so the lower bound on the makespan provided by Lee (1991) may not be valid. However, the bounds $\frac{2}{3}-\frac{1}{2m'}$ for LPT and $\frac{4}{3}$ for MLPT still hold, where m' is the number of used machines. Chang and Hwang (1999) developed the MULTIFIT algorithm for the problem and showed that its worst-case ratio is not greater than $\frac{9}{7}$, which Hwang and Lim (2014) improved to the tight bound of $\frac{24}{19}$. For scheduling on uniform non-simultaneously available machines, i.e., $Qm,r_i\|C_{\max}$, He (2000) showed that the worst-case ratio of LPT is at most $\frac{5}{3}$, and provided an improved algorithm with a worst-case ratio of $\frac{6}{5}$ for the two-machine case. Recently, Grigoriu and Friesen (2017) tackled the problem using MULTIFIT and showed that the worst-case ratio of MULTIFIT is at most 1.382 on general m machines and $\frac{\sqrt{6}}{2}$ on two machines. In addition, some different models on scheduling on non-simultaneously available machines have been studied. Wang et al. (2014) and Wang et al. (2015) investigated the scheduling of deteriorating jobs with non-simultaneous machine available times. Shen et al. (2013) considered parallel-machine scheduling with non-simultaneous machine available times to minimize a cost function. Huo (2019) considered a model with machine availability to minimize the makespan, subject to a total completion time constraint. Considering different types of jobs and performance measures, Kaabi and Harrath (2019) studied different versions of the problem on parallel uniform machines with an unavailability period. Mahjoub et al. (2021) applied a list scheduling (LS) algorithm to solve the problem of scheduling on identical machines with an arbitrary number of unavailability periods. Recently, Santoro and Junqueira (2023) introduced a new scheduling model for unrelated parallel machines with availability and eligibility constraints and presented mixed integer linear programming models for this problem with the objective of minimizing the makespan. For the problem of non-resumable jobs, Kurt and Çetinkaya (2024) developed an MILP model and a heuristic algorithm to solve the problem.

Another closely related problem is classical preemptive scheduling on parallel machines. For the identical-machine case, McNaughton (1959) found an optimal schedule with no more than $m-1$ preemptions in linear time. For the uniform-machine case, Liu and Yang (1974) presented a lower bound on the optimal preemptive schedule and provided an algorithm that obtains an optimal schedule for the special case where the first machine has a speed of 1 and the others have a speed greater than 1. Horvath et al. (1977) adapted the critical path algorithm of Muntz and Coffman (1970) to address the problem with general m uniform machines, and obtained an optimal preemptive schedule with at most $n^2(m-1)$ preemptions in $O(mn^2)$ time. Gonzalez and Sahni (1978) provided an improved algorithm that generates an optimal schedule with at most $2(m-1)$ preemptions. In addition, Schmidt (1984) considered the problem of preemptive scheduling on m identical machines with different intervals of availability. They presented an $O(n+m\log m)$ -time algorithm and showed that the number of preemptions is proportional to the total number of processing intervals of all the machines. Błazewicz et al. (2003) considered the problem of scheduling on identical parallel machines available only in restricted intervals of time called time windows. For the objective of minimizing the makespan, they presented a preemptive optimal schedule, which can be seen as a general case of our problem for the identical-machine case.

In this paper, we consider preemptive scheduling

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on non-simultaneously available uniformly parallel machines. We first convert the real machines to virtual machines, which guarantee that a machine with an earlier available time has a greater speed at any time. Then, we provide a lower bound on the optimal makespan and propose a polynomial time solution algorithm that produces an optimal schedule. The number of preemptions of the algorithm is no more than $\frac{1}{2}(m^2+3m)-2$.

We organize the rest of the paper as follows: In Section 2, we provide a lower bound on the optimal makespan of the problems under study. In Section 3, we present a solution algorithm for our problem. Finally, in Section 4, we conclude the paper and suggest topics for future research.

2 Lower bound

In this section, we derive a lower bound on the optimal makespan of the problem under consideration. Let C_Q^* be the makespan of the optimal schedule for problem $Qm, r_i \mid pmtn \mid C_{\max}$. For the sake of simplicity, let $p_j^{\max}$ be the *j*-th largest job among all the jobs, i.e., $p_1^{\max} \ge p_2^{\max} \ge \cdots \ge p_n^{\max}$.

Since each machine has a different speed and is available at a different time, it is not easy to find a lower bound on the optimal makespan directly. So, we first convert all the m uniform machines to m virtual machines such that, at any time, the earlier the available time of a virtual machine is, the greater its speed.

Let $V=\{V_1,V_2,...,V_m\}$ be a set of virtual machines, where V_i is available at time r_i. We still assume that $r_1 \le r_2 \le \cdots \le r_m$. Let v_{ji}, $i \le j$, $1 \le j \le m$, represent the i th greatest speed in the set $\{s_1, s_2, ..., s_j\}$, where s_k is the speed of real machine M_k, $1 \le k \le j$. Let $v_i(t)$ be the speed of V_i at time t, which we define as follows:

$$v_i(t)=\begin{cases}0, & 0<t\le r_i;\\ v_{ji}, & r_j<t\le r_{j+1},\ j=i,...,m\text{-}1.\\ v_{mi}, & t>r_m.\end{cases}$$

Fig. 1 illustrates the conversion process and shows that, at any time t, the virtual machine V_i always has the i th greatest speed among all the available machines, i.e., $v_1(t) \ge v_2(t) \ge \cdots \ge v_m(t)$ at any time. In addition, it is easy to find that the total processed size on V_i from t_1 to t_2 is $\int_{t_1}^{t_2} v_i(\mathrm{t})\,dt$.

Now we consider a lower bound on the optimal makespan. Let $w_Q = \max\{T_1, T_2, ..., T_m\}$, here T_k, $1 \le k \le m-1$ is the solution of the equation

$$\sum_{i=1}^{k}\int_{r_i}^{T_k} v_i(t)\,dt = \sum_{i=1}^{k} p_i^{\max}$$

and T_m is the solution of the equation

$$\sum_{i=1}^{m}\int_{r_i}^{T_m} v_i(t)\,dt = \sum_{i=1}^{n} p_i^{\max}.$$

We show that w_Q is a lower bound of the optimal makespan in the following lemma.

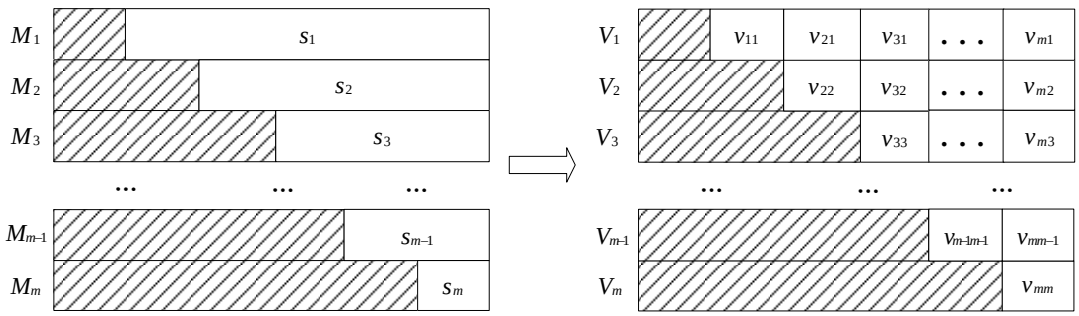

Fig. 1 From real machines to virtual machines.

Lemma 1 For problem $Qm, r_i \mid pmtn \mid C_{\max}$, we have $C_Q^* \ge w_Q$.

Proof. Given $v_1(t) \ge v_2(t) \ge \cdots \ge v_m(t)$ and $r_1 \le r_2 \le \cdots \le r_m$, we derive that $C_Q^* \ge T_1$ because the completion time will be at least T_1 by scheduling the

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largest job p_1 on the fastest machine V_1. Similarly, for the first k jobs, $1<k\le m-1$, it is sufficient to use the first k machines and the completion time is at least T_k by the definition of T_k, so we have $C_Q^* \ge T_k$. Finally, after scheduling all the n jobs on the m virtual machines, we must have $C_Q^* \ge T_m$ clearly. □

3 An Optimal Solution Algorithm

In this section, we provide a polynomial algorithm that can find an optimal schedule for $Qm, r_i \mid pmtn \mid C_{\max}$ with at most $\frac{1}{2}(m^2+3m)-2$ preemptions. We denote by l_i the completion time of virtual machine V_i and refer to it as a "not full" virtual machine if $l_i < w_Q$. We denote by $S_{NFVM}=\{V_{j1}, V_{j2}, ..., V_{jq_{NFVM}}\}$ the set of all the "not full" virtual machines, where $l_{j1} \le l_{j2} \le \cdots \le l_{jq_{NFVM}}$. If $l_i = w_Q$, the virtual machine will be removed from S_{NFVM}. For simplicity, let q_{NFVM} denote the index of the last virtual machine in S_{NFVM}. Let $W_i = \int_{l_i}^{w_Q} v_i(t)\,dt$ be the total size of the jobs processed on V_i from time l_i to w_Q.

We now present our algorithm in the following. The main idea of the algorithm is to schedule the current job so as to make a virtual machine with as small a completion time as possible to be a full machine.

Algorithm

Step 1 Let $l_i = r_i$, $i=1,2,...,m$, $S_{NFVM}=\{V_1, V_2, ..., V_m\}$, and $q_{NFVM}=m$

Step 2 For any p_j, $j=1,2,...,n$:

Step 3 If $p_j \le W_{q_{NFVM}}$:

Schedule p_j on virtual machine $V_{q_{NFVM}}$ as early as possible. Find a time T such that $\int_{l_{q_{NFVM}}}^{T} v_{q_{NFVM}}(t)\,dt = p_j$.

If $T == w_Q$:

Update $S_{NFVM} = S_{NFVM} \setminus \{V_{q_{NFVM}}\}$ and $q_{NFVM} = \max_{V_i \in S_{NFVM}} \{i\}$; $j=j+1$, go back to Step 2.

Else:

Update $l_{q_{NFVM}} = T$ and $W_{q_{NFVM}} = W_{q_{NFVM}} - p_j$; $j=j+1$, go back to Step 2.

Step 4 Else:

Find two adjacent virtual machines V_k and V_h in S_{NFVM} such that $W_k < p_j \le W_h$.

If $\int_{l_h}^{l_k} v_h(t)\,dt + \int_{l_k}^{w_Q} v_k(t)\,dt \ge p_j$:

Partition p_j into two parts p_j^1 and p_j^2 such that $p_j^1 = W_k$ and $p_j^2 = p_j - W_k$. Find a time T such that $p_j^2 = \int_{l_h}^{T} v_h(t)\,dt$. Schedule p_j^1 on V_k in the time interval $[l_k, w_Q)$ and p_j^2 on V_h in $[l_h, T)$.

Update $S_{NFVM} = S_{NFVM} \setminus \{V_k\}$, $q_{NFVM} = \max_{V_i \in S_{NFVM}} \{i\}$, $l_h = T$, and $W_h = W_h + W_k - p_j$; $j=j+1$, go back to Step 2.

Step 5 Else:

Find a time T such that $\int_{l_h}^{T} v_h(t)\,dt + \int_{T}^{w_Q} v_k(t)\,dt = p_j$. Partition p_j into two parts p_j^1 and p_j^2 such that $p_j^1 = \int_{T}^{w_Q} v_k(t)\,dt$ and $p_j^2 = p - p_j^1 = \int_{l_h}^{T} v_h(t)\,dt$. Schedule p_j^1 on V_k in the time interval $[T, w_Q)$ and p_j^2 on V_h $[l_h, T)$. Change the time slot $[l_k, T]$ of the two virtual machines; we have a new virtual machine V_h with $l_h = l_k$ and a "full" virtual machine V_k.

Update $S_{NFVM} = S_{NFVM} \setminus \{V_k\}$, $q_{NFVM} = \max_{V_i \in S_{NFVM}} \{i\}$, $l_h = l_k$, and $W_h = W_h + W_k - p_j$; $j=j+1$, go back to Step 2.

It is easy to derive that the time complexity of algorithm A is $O(mn)$. Converting the uniform ma-

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chines to virtual machines takes $O(m^2)$ time. So, we can obtain an optimal schedule in $O(mn+m^2)$ time. Fig. 2 shows the changes of machines before and after processing the jobs in Steps 3, 4, or 5. We have the following observation about algorithm A:

Observation 1. Suppose that $S_{NFVM}=\{V_{j1},V_{j2},...,V_{jqNFVM}\}$ after scheduling job p_j, $1\le j\le n$, we have the following:

(i) $l_{j1}\le l_{j2}\le\cdots\le l_{jqNFVM}$;

(ii) If a job p_j is scheduled in Step 4 or 5, the assignment is feasible and there is exactly one virtual machine to be removed in S_{NFVM}.

Proof. (i) We prove it by induction. For $j=0$, by the definition of S_{NFVM} and Step 1, the result holds obviously.

Suppose that the result holds for $j-1$, $j\ge 1$. We consider the assignment of job p_j. It is clearly true if the job is processed on the last virtual machine in S_{NFVM} in Step 3. Therefore, we focus on the case where job p_j is processed in Step 4 or 5. In this case, p_j is partitioned into two parts with sizes p_j^1 and p_j^2, which are scheduled on the virtual machine V_k and the adjacent virtual machine V_h in S_{NFVM}, respectively.

If job p_j is processed in Step 4, we have $p_j^1=W_k=\int_{l_k}^{w_Q}v_k(t)dt$ and $p_j^2=\int_{l_h}^{T}v_h(t)dt$. By $\int_{l_h}^{l_k}v_h(t)dt+\int_{l_k}^{w_Q}v_k(t)dt\ge p_j$, we have

$$T\le l_k. \tag{2}$$

It means that the completion time of V_h after processing p_j is not greater than that of V_k before processing p_j, as shown in Fig. 2 (ii).

If job p_j is processed in Step 5, there is a time T such that $\int_{l_h}^{T}v_h(t)dt+\int_{T}^{w_Q}v_k(t)dt=p_j$. As shown in Fig. 2 (iii), two parts of the job $p_j^1=\int_{T}^{w_Q}v_k(t)dt$ and $p_j^2=\int_{l_h}^{T}v_h(t)dt$ are scheduled on two virtual machines V_k and V_h, respectively. By exchanging the time slot $[l_k,T]$ of the two virtual machines, we have a new virtual machine V_h with $l_h=l_k$, and a new "full" virtual machine V_k.

By removing the new "full" machine V_k, and with the assumption that all the virtual machines in S_{NFVM} are in non-decreasing order at $j-1$, we conclude that the result is still true at j.

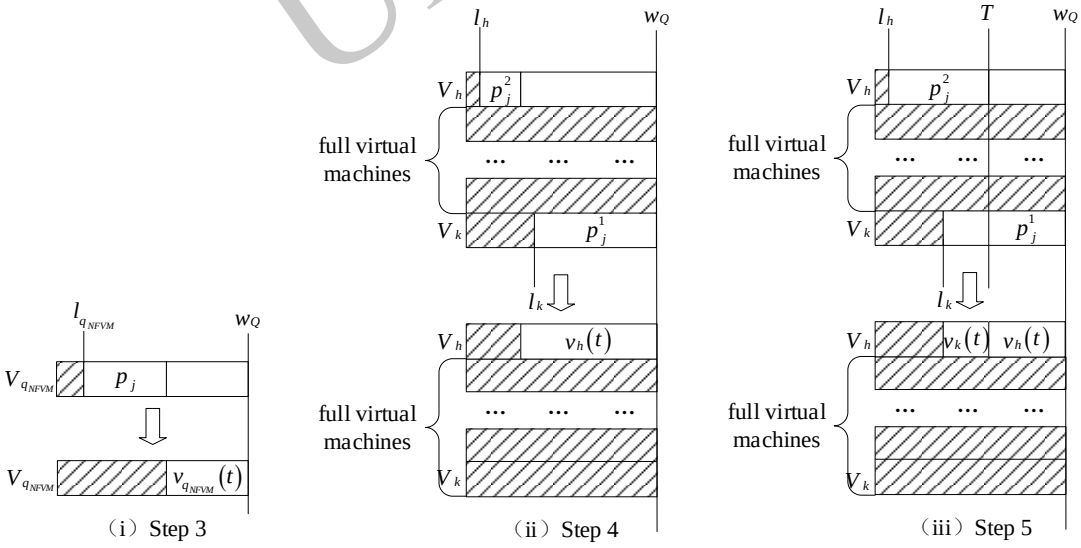

Fig. 2 Jobs scheduled in Steps 3, 4, and 5.

(ii) If job p_j is scheduled in Step 4, we obtain that the time slots $[l_k,w_Q)$ for p_j^1 and $[l_h,T)$ for p_j^2

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do not overlap by (2), so the assignment of p_j is feasible. If job p_j is scheduled in Step 5, it is easy to see that the time slots assigned to the two parts of p_j do not overlap. Clearly, there is a machine to be removed from S_{NFVM} in Step 4 or 5. □

Lemma 2. For any job p_j, we have $p_j \le W_e = \int_{l_e}^{w_Q} v_e(t)\,dt$, where $e = \min_{V_i \in S_{NFVM}}\{i\}$.

Proof. If $e > 1$, i.e., the machines $V_1, V_2, ..., V_{e-1}$ have been removed from S_{NFVM}. According to Lemma 3.1(ii), once a job is processed in accordance with Step 4 or Step 5, it involves two machines, and the machine with the larger subscript becomes a full machine and is removed. Therefore, together with the definition of w_Q, the fact that the first $e-1$ machines are removed indicates that the first $e-1$ largest jobs, i.e., $p_1^{\max}, p_2^{\max}, ..., p_{e-1}^{\max}$, are exactly processed on the first $e-1$ virtual machines before scheduling p_j. Moreover, if $l_e = r_e$, i.e., there is no job on V_e before scheduling p_j, we must have $p_j \le W_e$, since the first e largest jobs can be processed on the first e virtual machines by the definitoin of w_Q. So, we consider the case where $l_e > r_e$, i.e., there are some jobs (or parts of the jobs) on V_e before scheduling p_j.

If there is only one virtual machine V_e in S_{NFVM}, i.e., all the other virtual machines have been removed with loads of w_Q, we must have $p_j \le W_e$, as all the jobs can be processed on the m virtual machines before time w_Q.

Now, we focus on the case where there are at least two virtual machines in S_{NFVM}. It is easy to derive that all the jobs (or parts) on V_e must be scheduled in Step 4 or Step 5. Otherwise, if a job (or parts) on V_e is scheduled in Step 3, then V_e must be the only one in S_{NFVM}, a contradiction. By the rule of Step 4 or Step 5, we obtain that V_{e+1} must be removed. Let V_{k+2} be the current adjacent virtual machine of V_e in S_{NFVM}, $e \le k \le m-2$, i.e., all the $k-e+1$ virtual machines between V_e and V_{k+2} have been removed from S_{NFVM}. Because $V_{k+2} \in S_{NFVM}$, there is no job processed on both V_{k+2} and any one of the first $k+1$ virtual machines. Moreover, by Steps 4 and 5, and Observation 1, we conclude that there are exactly k jobs, i.e., $p_{j1}, p_{j2}, ..., p_{jk}$, on the first $k+1$ machines before scheduling p_j. By the definition of w_Q, we have

$$\sum_{i=1}^{k+1}\int_{r_i}^{w_Q} v_i(t)dt \ge \sum_{i=1}^{k+1}\int_{r_i}^{T_{k+1}} v_i(t)dt = \sum_{i=1}^{k+1} p_i^{\max} \ge \sum_{i=1}^{k} p_{ji} + p_j ,$$

which yields $p_j \le \int_{l_e}^{w_Q} v_e(t)dt = W_e$, given that $\sum_{i=1}^{k} p_{ji} > \sum_{i=1}^{e-1}\int_{r_i}^{w_Q} v_i(t)dt + \sum_{i=e+1}^{k+1}\int_{r_i}^{w_Q} v_i(t)dt$. □

By Observation 1, and Lemmas 1 and 2, we obtain the following result.

Theorem 1. Algorithm A generates an optimal schedule.

We now consider the number of preemptions. There are two types of preemptions in the algorithm. One is produced by partitioning a job into two parts and scheduling them on two different virtual machines like in Steps 4 and 5. The other is produced by the different speeds on the virtual machines. Clearly, there are at most $m-1$ peemptions for the first type of preemptions.

For the second type, it suffices to calculate the number of speeds of each virtual machines. Initially, the number of speeds of virtual V_i is at most $m+1-i$ after converting all the m uniform machines to m virtual machines. It follows that there are at most $m-i$ preemptions on V_i, so the total number of preemptions is at most $\sum_{i=1}^{m}(m-i) = \frac{m^2-m}{2}$. Note that the speeds on the two virtual machines V_h and V_k are unchanged in Step 4, so we have no new preemptions. However, when a job is processed in Step 5, one more preemption is introduced because the time slot $[l_k, T]$ of the new virtual machine V_h is from the virtual machine V_k. This produces a maximum of $m-1$ preemptions. Hence, the total number of preemptions is at most

$$\frac{m^2-m}{2} + 2(m-1) = \frac{m^2+3m}{2} - 2 .$$

To illustrate Algorithm A, we present an example as follows:

Example. There are four jobs and three ma-

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chines as shown in Table 1.

Table 1 An example

j	1	2	3	4
p_j	12	11	2	4
r_j	1	4	5	
s_j	2	3	4	

It is easy to find that $w_Q = 7$ for the above instance. Fig. 3 shows the virtual machines and the schedule of the jobs. The first job p_1 is assigned to V_1 and V_2 in Step 4, and V_2 is removed from S_{NFVM}. Job p_2 is assigned to V_1 and V_3 in Step 5, and the reminding time slots of V_1 and V_3 are spliced together into one time slot of the new virtual machine V_1. Finally, p_3 and p_4 are scheduled on V_1 in Step 3.

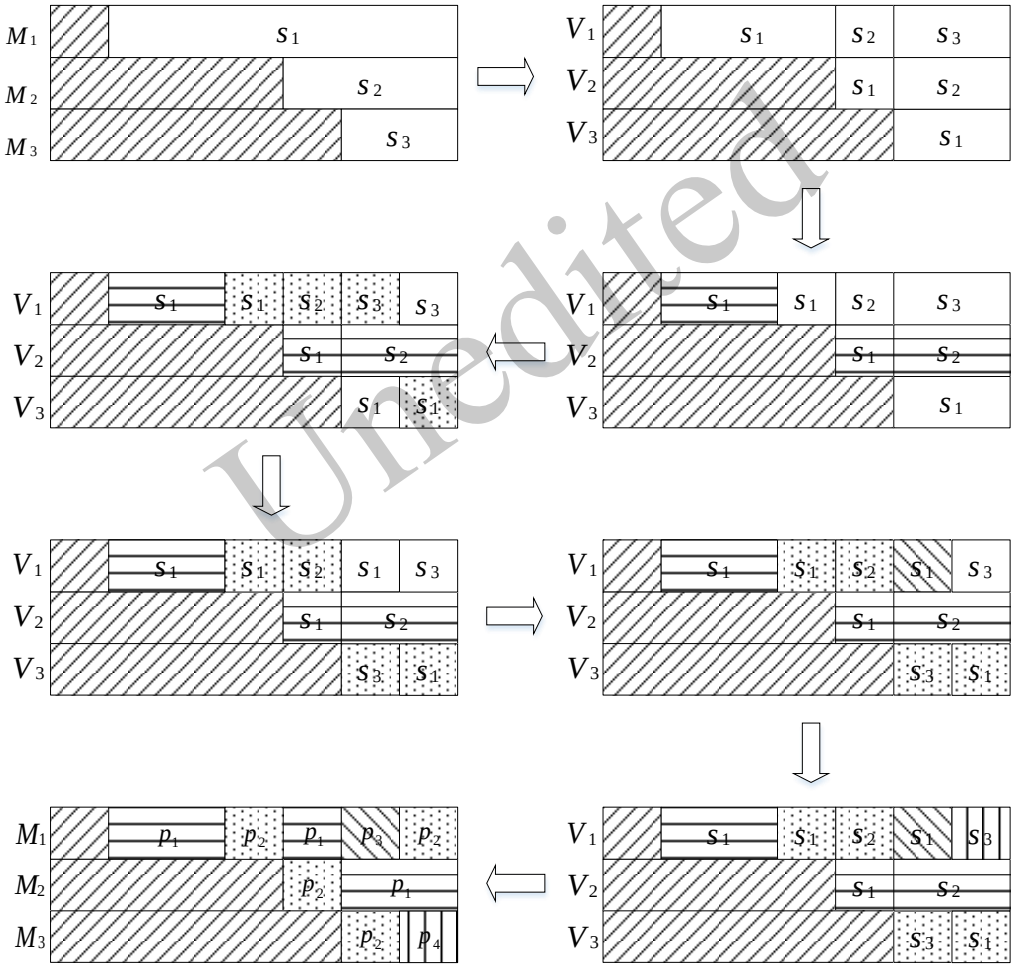

Fig. 3. The scheduling process of the example.

4 Conclusions

In this paper, we studied preemptive scheduling on parallel uniform machines with non-simultaneously available time. Our objective was to minimize the makespan. We provided a lower bound on the optimal makespan by transforming the m uniform machines into m virtual machines, ensuring that a machine with an earlier available time has a greater speed at any time. We provided a solution algorithm for the problem. For future research, it is worth considering the online version of the problem for different parallel-machine settings, including identical machines and uniform machines.

Contributors

Hao ZHOU designed the research. Li-ping CAO and Qi WEI designed and analyzed the algorithm. Hao ZHOU and Li-ping CAO drafted the manuscript. Zhen-yu SHU helped organize the manuscript. Yi-wei JIANG revised and finalized

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the paper.

Compliance with ethics guidelines

Hao ZHOU, Li-ping CAO, Qi WEI, Zhen-yu SHU and Yi-wei JIANG declare that they have no conflict of interest.

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