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Transmission design for the XL-RIS-aided massive MIMO system with visibility regions

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Abstract: This study proposed a two-timescale transmission scheme for extremely extra-large (XL)-reconfigurable intelligent surfaces (RIS)-aided massive multi-input multi-output (MIMO) systems in the presence of visibility regions (VRs). The beamforming of base stations (BS) is designed based on rapidly changing instantaneous channel state information (CSI), while the phase shifts of RIS are configured based on slowly varying statistical CSI. Specifically, we first formulated a system model with spatially correlated Rician fading channels and introduced the concept of VRs. Then, we derived a closed-form approximate expression for the achievable rate and analyzed the impact of VRs on system performance and computational complexity. Then, we solved the problem of maximizing the minimum user rate by optimizing the phase shifts of RIS through an algorithm based on accelerated gradient ascent. Finally, we presented numerical results to validate the performance of the considered system from different aspects and revealed the low system complexity of deploying XL-RIS in massive MIMO systems with the help of VRs.

Key words: Reconfigurable intelligent surface; Massive MIMO; Two-timescale transmission scheme; Visibility regions

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1 Introduction

Recently, reconfigurable intelligent surface (RIS), also known as intelligent reflecting surface (IRS), has been extensively studied by academia and industry and has been recognized as an appealing complementary technology for next-generation communication systems (Pan et al., 2021; Liu et al., 2023; Zhang Y, 2023). It is an array composed of a large number of reconfigurable subwavelength units, which can achieve signal enhancement, attenuation, focusing, scattering, and other functions. Specifically, each RIS element can independently induce changes in the amplitude and phase of the incident signal, allowing the reflected signal to be constructively added to signals from other paths. Therefore, it can customize the wireless environment by manually controlling electromagnetic (EM) waves, thereby improving spectrum efficiency. Meanwhile, the RIS can significantly improve the performance, especially when direct links between users and base stations (BSs) are obstructed. Therefore, compared to existing multi-antenna systems, the RIS has the potential to achieve better performance while reducing costs and energy consumption.

In recent years, many researchers have studied the application of the RIS in wireless systems from different perspectives. The free-space path loss models for RIS-assisted wireless communications were developed for different scenarios by studying the physics and EM nature of RISs (Tang et al., 2021). Pan et al. (2022) provided a comprehensive

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overview of recent advances in RIS/IRS-aided wireless systems from the signal processing perspective. Xu et al. (2023) studied the energy efficiency of a RIS-assisted multi-cell communication system with a realistic RIS power consumption model and optimized the transmit beamforming vectors at the BS by a proposed alternative optimization algorithm. By jointly optimizing the beamforming matrix at amplify-and-forward (AF) relay and the phase-shift matrices at RIS, Wang XH et al. (2023) proposed a RIS-aided AF relay network and two schemes were put forward to address a maximizing signal-to-noise ratio (SNR) problem.

Recently, the supporting technologies of applying RIS in various scenarios have also been investigated in a wide range. Specifically, a novel cascaded channel estimation strategy with low pilot overhead was proposed by exploiting the sparsity and the correlation of multiuser cascaded channels in millimeter-wave (mmWave) multi-input single-output (MISO) systems (Zhou Get al., 2022). Peng et al. (2023) proposed a novel two-stage uplink channel estimation strategy with reduced pilot overhead and error propagation for RIS-aided multi-user (MU) mmWave multiple-antenna systems. Jiang et al. (2023) proposed a three-dimensional (3D) physics-based double RIS cooperatively assisted multi-input multi-output (MIMO) stochastic channel model for unmanned aerial vehicle (UAV)-to-ground communication scenarios. Yang L et al. (2020a) modeled the statistical distribution of the RIS-assisted ground-to-air (G2A) links and proved that the use of RISs can effectively improve the coverage and reliability of UAV communication systems. Ren et al. (2023) derived the approximate expression of the sum achievable security data rate (ASDR) with statistical channel state information (CSI) and demonstrated the effectiveness of using RIS in improving security performance.

Furthermore, researchers have conducted numerous studies and improvements on RIS-aided systems to meet the different needs of actual environments. A more efficient channel estimator was developed by leveraging the unitary approximate message passing (UAMP), which facilitates the applications of existing algorithms to a general RIS-aided MIMO system with a larger number of reflecting elements (Guo et al., 2023). Considering radio frequency damage and phase noise, Zhou SQ et al. (2020) conducted a theoretical study on the fundamental trade-off between spectrum and energy efficiency of RIS communication networks. In the study by Chen et al. (2024), a model-free cross-entropy (CE) algorithm was proposed to optimize the binary RIS configuration for improving the SNR at the receiver. Huang et al. (2020) demonstrated that holographic RIS can become an attractive way of achieving high spatial resolutions in satellite networks. Yang L et al. (2020b) studied the secrecy performance of a RIS-aided wireless communication system in the presence of an eavesdropping user. Zheng et al. (2021) presented a double-RIS cooperatively-assisted MIMO communication system where two distributed RISs are deployed near a user group and a multi-antenna BS, respectively. Assuming that the RIS with the highest instantaneous end-to-end SNR is selected to aid the communication, Yang L et al. (2021) considered a network assisted by multiple RISs and investigated the outage probability (OP) and average sumrate.

Although the advantages of the RIS have been demonstrated in the aforementioned studies, most of them were built on the assumption of estimating instantaneous CSI in each channel coherent interval. However, in practice, transmission design schemes based on instantaneous CSI face two challenges (Zhi et al., 2023). The RIS usually includes a large number of reflecting elements to achieve the desired gains. However, the pilot overhead of channel estimation schemes may increase in proportion to the number of RIS elements (Wang ZR et al., 2020), leading to excessive pilot overhead. Then, designing algorithms based on instantaneous CSI requires frequent beamforming calculations and information feedback, resulting in high computational complexity, feedback overhead, and energy consumption.

To address the above issues, researchers are currently considering adopting the two-timescale scheme to make the application of the RIS more practical. Specifically, in a two-timescale scheme, BS beamforming is designed based on instantaneous aggregated CSI, while optimization of RIS parameters is only decided based on long-term statistical CSI, such as position and angle information (Zhi et al., 2023). As a result, it is only necessary to update the phase shift of RIS components when the statistical CSI changes (Han et al., 2019). Specifically, Han et al. (2019) and Jia et al. (2020) studied RIS-aided single-user communication systems by taking into

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consideration interference and non-interference, respectively. Abrardo et al. (2021) considered a multi-RIS MU MIMO system and optimized the phase shifts of reconfigurable elements of RIS, as well as beamforming vectors at BS and user devices, without the need for instantaneous CSI or second-order channel statistics, to maximize the user rate. Gao et al. (2021) proposed an effective passive reflection beamforming design for RIS, which uses statistical CSI to analyze the achievable rate of the network while considering the impact of CSI estimation errors. Hu et al. (2021) proposed a two-timescale channel estimation framework and a dual-link pilot transmission scheme, where the BS transmits downlink pilots and receives uplink pilots reflected by the RIS. To maximize the average achievable rate of RIS-assisted MIMO systems, Cao et al. (2022) presented a new two-timescale beamforming approach, where the RIS is configured relatively infrequently based on statistical CSI, and the base station precoder and power allocation are updated frequently based on quickly outdated instantaneous CSI. In the presence of spatial correlation and electromagnetic interference (EMI), Zhi et al. (2023) investigated the two-timescale transmission scheme for RIS-aided MIMO systems. For cell-free systems, a low-complexity algorithm via the two-timescale transmission protocol was proposed by Gan et al. (2022), where the joint beamforming at BSs and RISs was designed via an alternating optimization framework. By using the active RIS architectures, a MU two-timescale channel estimation protocol was proposed to minimize the pilot overhead in the study by Yang SJ et al. (2023).

As massive MIMO technology was commercially deployed in 5G communication systems, a useful complementary is to deploy extra-large (XL) passive reflection arrays into massive MIMO systems (Han et al., 2022). This is because, thanks to the low-cost feature, the size of an RIS can be made very large to extend the service region, which can compensate for the double-fading effect (Zhi et al., 2022a), and also realize better performance in terms of localization and communications. Besides, XL-RIS can achieve unprecedented gains in spatial multiplexing, SNR, energy efficiency, and coverage probability with the help of a larger array aperture and a stronger ability to customize the wireless environment. However, with a larger aperture, the users could be easily located in the near field of the XL-RIS, based on the Rayleigh distance that characterizes the near-field region. As the result, the practical spherical EM wavefront can no longer be approximated as the tractable planar wavefront (Zhi et al., 2024). Moreover, the channel between users and XL-RIS will show spatial non-stationarity due to the non-stationary power distribution in the near-field channel because of the obstacles in the complicated environment (Carvalho et al., 2020). Specifically, as unequal path loss is experienced by different RIS elements, and some RIS elements suffer from blockage caused by obstacles, the received signal power from users on the XL-RIS will be concentrated on a portion of the RIS elements, which is indicated by the visibility region (VR) (Han et al., 2022). As a result, it is necessary to revise typical channel models and algorithms to capture spatially non-stationary fading and other near-field features in XL-RIS-aided massive MIMO systems (Li XR et al., 2015; Carvalho et al., 2020).

Taking the existence of VR into consideration, Cui and Dai (2022) proposed a polar simultaneous orthogonal matching pursuit (OMP) algorithm for near-field channel estimation in XL-MIMO systems. Ali et al. (2019) proposed a simple non-stationary channel model with VR and analyzed the performance of conjugate beamforming and zero-forcing precoding in the downlink MU XL-MIMO systems. Considering the non-stationarity between RIS and users, Han et al. (2022) proposed a recognition module that can estimate the VR of each user with line-of-sight (LoS) components between RIS and users. For XL-RIS-aided systems, Wei et al. (2022) proposed a low-cost near-field beam training scheme. Yu et al. (2023) studied the channel estimation problem of XL-RIS-aided millimeter wave communication systems considering near-field effects and spatial non-stationary effects. Gunasinghe and Amarasuriya (2023) investigated the achievable rate of XL-RIS-aided downlink massive MIMO in the presence of whole/partial VRs, spatial correlation, and imperfectly estimated CSI.

However, in the most of the aforementioned studies, the focus was on channel estimating and spatial non-stationary effects caused by XL-MIMO. Moreover, Jia et al. (2020) and Yang SJ et al. (2023) did not consider the employment of XL-RIS and ignored the VRs of users. To the best of the au-

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thors' knowledge, there are few studies on the performance analysis and phase shift design of spatially nonstationary XL-RIS-assisted massive MIMO systems based on two-timescale schemes, and therefore its performance and potential deserve further investigation.

To fill this research gap, in this study, we analyzed the uplink (UL) two-timescale transmission scheme of XL-RIS-assisted massive MIMO systems in the presence of VRs, considering the spatial nonstationary effects caused by XL-RIS, and applied the general Rician channel model to evaluate the impact of LoS and non-LoS (NLoS) channel components. Our study can be generalized to the case with multiple antenna users, and its difficulty lies mainly in the increase of the channel matrix dimension (Wang JH et al., 2021; Li RW et al., 2023), but the methods and algorithm are still applicable.

1. By employing the low-complexity maximal-ratio-combining (MRC) detector at the BS, we derived a closed-form expression of the uplink achievable rate under the correlated Rician channel model in the presence of VRs. This expression applies to any finite number of RIS elements and BS antennas and only depends on the statistical CSI. Then, we analyzed the impact of VRs and assessed how they help reduce computational complexity.

2. Based on the derived expression, we formulated a problem of maximizing the minimum user rate with statistical CSI. Then, an effective algorithm is proposed to design the phase shifts of the XL-RIS based on accelerated gradient ascent.

3. To unveil the property and benefit of integrating XL-RIS into massive MIMO networks, we provided simulation results about various important parameters. The results reveal that the existence of VR leads to a decrease in performance while the exploitation of its properties is beneficial to reduce the computational complexity of the system. Besides, it is shown that the proposed gradient algorithm can effectively solve the RIS phase optimization problem and has higher efficiency compared to the genetic algorithm (GA).

2 System model

As shown in Fig. 1, we considered a typical XL-RIS-aided massive MIMO communication system with one BS, one XL-RIS, and K single antenna users. The BS and the XL-RIS are equipped with M antennas and $N_1 \times N_2 = N$ passive reflective elements, using uniform linear array (ULA) and uniform planar array (UPA) models (N_1 rows and N_2 columns), respectively. The RIS is connected to the BS through a dedicated transmission link. Due to potential obstructions such as buildings and other objects, we assume that the direct link from the BS to users may be blocked (Zheng et al., 2021).

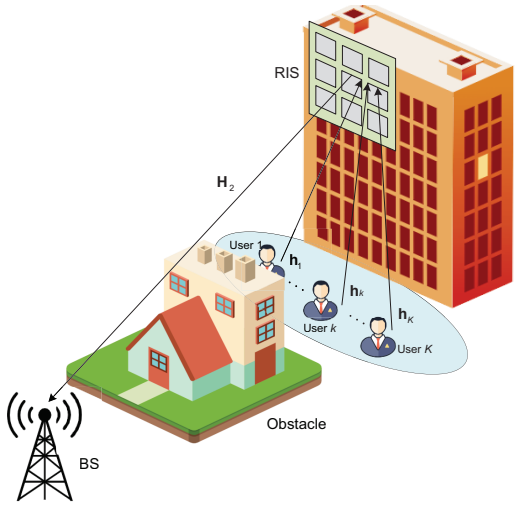

Fig. 1 An XL-RIS-aided massive MIMO system (MIMO: multi-input multi-output; RIS: reconfigurable intelligent surfaces)

Considering that the RIS can be mounted on tall building walls, it can help create channels dominated by LoS propagation with a few scatters (Zhi et al., 2022b). Thus, we adopted the Rician fading model for the channels between the BS and the RIS and the channel between users and the RIS (Jia et al., 2020). We denote $\mathbf{H}_2 \in \mathbb{C}^{M\times N}$ and $\mathbf{H}_1 = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_K] \in \mathbb{C}^{N\times K}$ as the channel between the BS and the RIS, and the channels between the RIS and users, respectively. Specifically, these channels can be modeled as follows:

$$\mathbf{h}_k = \sqrt{\alpha_k}\left(\sqrt{\frac{\varepsilon_k}{\varepsilon_k+1}}\mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k + \sqrt{\frac{1}{\varepsilon_k+1}}\mathbf{R}_{\mathrm{VR},k}^{1/2}\tilde{\mathbf{h}}_k\right), \tag{1}$$

$$\mathbf{H}_2 = \sqrt{\beta}\left(\sqrt{\frac{\delta}{\delta+1}}\bar{\mathbf{H}}_2 + \sqrt{\frac{1}{\delta+1}}\tilde{\mathbf{H}}_2\mathbf{R}_{\mathrm{ris}}^{1/2}\right), \tag{2}$$

where α_k and β represent the large-scale path loss factors of the user k-RIS link and BS-RIS link, respectively. ε_k and δ are the Rician factors of the user k-RIS link and the RIS-BS link, respectively.

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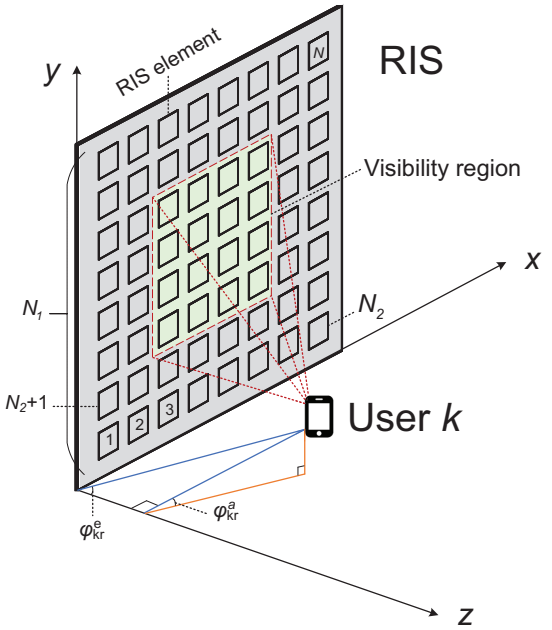

Fig. 2 3D geometry of an XL-RIS consisting of N_2 elements per row and N_1 elements per column, in the presence of VR (3D, three-dimensional; VR, visibility region)

$\mathbf{D}_k$ denotes the visibility indicator matrix. $\mathbf{R}_{\rm ris}$ and $\mathbf{R}_{{\rm VR},k}$ are the spatial correlation matrices of the RIS and the VRs, respectively. In addition, $\bar{\mathbf{h}}_k$ and $\bar{\mathbf{H}}_2$ denote the LoS components, while $\tilde{\mathbf{h}}_k$ and $\tilde{\mathbf{H}}_2$ denote the NLoS components.

For the NLoS components, $\tilde{\mathbf{h}}_k$ and $\tilde{\mathbf{H}}_2$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. First, we denote that $\varphi^a_{\rm kr}$ and $\varphi^e_{\rm kr}$ are the azimuth and elevation angles of arrival (AoA) at the RIS from user k, respectively, and φ^a_t and φ^e_t are the azimuth and elevation angles of departure (AoD) toward the BS from the RIS, respectively. φ^a_r and φ^e_r are the azimuth and elevation angles AoA at the BS from the RIS, respectively. Besides, we assume that these angles can be calculated by using the locations obtained from the global position system (GPS) (Zhi et al., 2022b). Then, for the LoS components, $\bar{\mathbf{h}}_k$ and $\bar{\mathbf{H}}_2$ are modeled as follows (Zhi et al., 2024):

$$\begin{aligned}&[\bar{\mathbf{h}}_k]_p = \sqrt{\varepsilon_k}\exp\{-j[\chi_k]_p\},\\&[\chi_k]_p \approx \frac{2\pi}{\lambda}\\&\times\sqrt{\left(x(p)d_H - u_{k,x}\right)^2 + \left(y(p)d_V - u_{k,y}\right)^2 + \left(u_{k,z}\right)^2},\end{aligned} \tag{3}$$

$$\bar{\mathbf{H}}_2 = \mathbf{a}_M(\phi^a_r,\phi^e_r)\mathbf{a}^H_N(\varphi^a_t,\varphi^e_t), \tag{4}$$

where d_H and d_V denote the width and height of a single RIS element, respectively, which can be assumed to be $d_H = d_V = d_{\rm ris}$. $x(p)$ and $y(p)$ are the horizontal and vertical indices of element n, respectively, on the two-dimensional grid. $u_k, x = r_{k,o}\sin\varphi^e_{\rm kr}\cos\varphi^a_{\rm kr}$, $u_{k,y} = r_{k,o}\sin\varphi^e_{\rm kr}\sin\varphi^a_{\rm kr}$, and $u_{k,z} = r_{k,o}\cos\varphi^e_{\rm kr}$ are the x-axis, y-axis, and z-axis coordinates of user k, respectively, where $r_{k,o}$ is the distance from user k to the XL-RIS. In addition, $\mathbf{a}_X(\vartheta^a,\vartheta^e) \in \mathbb{C}^{X\times 1}$ represents the array response vector, where the $x^{\rm th}$ term is as follows:

$$\begin{aligned}&[\mathbf{a}_M(\vartheta^a,\vartheta^e)]_x = \exp\left\{j2\pi\frac{d_{\rm bs}}{\lambda}(x-1)\sin\vartheta^e\sin\vartheta^a\right\},\\&[\mathbf{a}_N(\vartheta^a,\vartheta^e)]_x\\&=\exp\Big\{j2\pi\frac{d_{\rm ris}}{\lambda}\Big(\lfloor(x-1)/\sqrt{N}\rfloor\sin\vartheta^e\sin\vartheta^a\\&\quad+\Big((x-1)\bmod\sqrt{N}\Big)\cos\vartheta^e\Big)\Big\},\end{aligned} \tag{5}$$

where $d_{\rm bs}$, $d_{\rm ris}$, and λ represent the BS antenna spacing, RIS element spacing, and wavelength, respectively. To simplify the symbols, in the rest of the study, $\mathbf{a}_M(\phi^a_r,\phi^e_r)$ and $\mathbf{a}_N(\phi^a_t,\phi^e_t)$ are simply denoted as $\mathbf{a}_M$ and $\mathbf{a}_N$, respectively.

As shown in Fig. 2, the elements of the RIS are indexed row-by-row by $n \in [1,N]$. We denote $\mathbf{u}_p$ as the location of the $p^{\rm th}$ element with respect to the origin in Fig. 2, which can be represented by the following model (Björnson and Sanguinetti, 2021):

$$\mathbf{u}_p = [x(p)d_H, y(p)d_V, 0]^{\rm T}, \tag{6}$$

$$x(p) = {\rm mod}(p-1, N_2), \tag{7}$$

$$y(p) = \lfloor p-1/N_2\rfloor. \tag{8}$$

Therefore, the elements inside the spatial correlation matrix of the RIS can be defined as follows:

$$[\mathbf{R}_{\rm ris}]_{p,q} = {\rm sinc}\left(\frac{2\,||\mathbf{u}_p-\mathbf{u}_q||}{\lambda}\right)\ \text{for}\ p,q=1,2,\ldots,N, \tag{9}$$

where λ is the wavelength, and $||\mathbf{u}_p-\mathbf{u}_q||$ represents the distance between the $p^{\rm th}$ and $q^{\rm th}$ elements of the RIS. As illustrated in Fig. 2, we used VRs to capture spatially non-stationary fading characteristics. The visibility indicator matrix $\mathbf{D}_k \in \mathbb{C}^{N\times N}$ is defined as follows:

$$\mathbf{D}_k = {\rm diag}(\mathbf{d}_k), \tag{10}$$

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$$[\mathbf{d}_k]_n = \begin{cases} 1, & \text{if} \quad n \in \boldsymbol{\emptyset}_k, \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

$\mathbf{d}_k \in \mathbb{C}^N$ is the visibility indicator vector. $\boldsymbol{\emptyset}_k$ is a set containing the indices of RIS elements, which are visible to the kth user. When n belongs to $\boldsymbol{\emptyset}_k$, we set $[\mathbf{d}_k]_n$ to 1 and 0 otherwise, which did not change the range of n values. Then, the RIS covariance matrix for the kth user with partial visibility is defined as follows (Ali et al., 2019):

$$\mathbf{R}_{\text{VR},k} = \mathbf{D}_k^{1/2} \mathbf{R}_{\text{ris}} \mathbf{D}_k^{1/2}. \tag{12}$$

With the help of XL-RIS, the received signal at the BS can be expressed as follows:

$$\mathbf{y} = \sqrt{p}\mathbf{Q}\mathbf{x} + \mathbf{n}, \tag{13}$$

where p is the average transmit power of each user. $\mathbf{x} = [x_1, x_2, \ldots, x_K]^{\text{T}}$ represents the information sequence from K users, satisfying $\mathbb{E}\{|x_k|^2\} = 1$. $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$ is the additional white Gaussian noise (AWGN). $\mathbf{Q} \triangleq \mathbf{H}_2 \boldsymbol{\Phi} \mathbf{H}_1 \in \mathbb{C}^{M \times K}$ is the total aggregation channel from K users to the BS.

We adopted the low-complexity MRC technique, due to its simple construction and near-optimal performance for massive MIMO (Marzetta et al., 2016). The BS performs MRC by multiplying the received signal $\mathbf{y}$ with $\mathbf{Q}^H$, as follows:

$$\mathbf{r}_{\text{VR}} = \mathbf{Q}^H \mathbf{y} = \sqrt{p}\mathbf{Q}^H \mathbf{Q}\mathbf{x} + \mathbf{Q}^H \mathbf{n}. \tag{14}$$

Before calculating the achievable rate, the signal of user k can be written as follows:

$$r_{\text{VR},k} = \sqrt{p}\, \mathbf{q}_k^H \mathbf{q}_k x_k + \sqrt{p} \sum_{i=1, i \neq k}^{K} \mathbf{q}_k^H \mathbf{q}_i x_i + \mathbf{q}_k^H \mathbf{n}, \tag{15}$$

where $\mathbf{q}_k \triangleq \mathbf{H}_2 \boldsymbol{\Phi} \mathbf{h}_k \in \mathbb{C}^{M \times 1}$ is the kth column of $\mathbf{Q}$ denoting the cascaded channel from user k to the BS. Specifically, $\mathbf{q}_k$ can be represented as follows:

$$\begin{aligned} \mathbf{q}_k =& \mathbf{H}_2 \boldsymbol{\Phi} \mathbf{h}_k \\ =& \sqrt{c_k \delta \varepsilon_k} \left(\bar{\mathbf{H}}_2 \boldsymbol{\Phi} \mathbf{D}_k^{\frac{1}{2}} \bar{\mathbf{h}}_k \right) + \sqrt{c_k \delta} \left(\bar{\mathbf{H}}_2 \boldsymbol{\Phi} \mathbf{R}_{\text{VR},k}^{\frac{1}{2}} \tilde{\mathbf{h}}_k \right) \\ &+ \sqrt{c_k \varepsilon_k} \left(\tilde{\mathbf{H}}_2 \mathbf{R}_{\text{ris}}^{\frac{1}{2}} \boldsymbol{\Phi} \mathbf{D}_k^{\frac{1}{2}} \bar{\mathbf{h}}_k \right) \\ &+ \sqrt{c_k} \left(\tilde{\mathbf{H}}_2 \mathbf{R}_{\text{ris}}^{\frac{1}{2}} \boldsymbol{\Phi} \mathbf{R}_{\text{VR},k}^{\frac{1}{2}} \tilde{\mathbf{h}}_k \right), \end{aligned} \tag{16}$$

where $c_k \triangleq \frac{\beta \alpha_k}{(\delta+1)(\varepsilon_k+1)}$, $\boldsymbol{\Phi} = \text{diag}\left\{ e^{j\theta_1}, e^{j\theta_2}, ..., e^{j\theta_N} \right\}$ denotes the phase shift matrix of the RIS and $\theta_n \in [0, 2\pi]$ denotes the phase shift introduced by the nth RIS element.

Due to the ergodic channel, we can express the uplink achievable rate of user k as $R_{\text{VR},k} = \mathbb{E}\left\{\log_2\left(1 + \text{SINR}_k\right)\right\}$, and the Signal to interference plus noise ratio (SINR) of user k is given as follows (Zhang Q et al., 2013):

$$\text{SINR}_k = \frac{p \parallel \mathbf{q}_k \parallel^4}{\sum_{i=1, i \neq k}^{K} p |\mathbf{q}_k^H \mathbf{q}_i|^2 + \sigma^2 \parallel \mathbf{q}_k \parallel^2}. \tag{17}$$

3 Achievable rate analysis

Considering the VRs, the approximate closed-form expression of the achievable rate in the XL-RIS-aided massive MIMO system is derived and analyzed in this section. We will also present the expression with reduced dimensions using the features of VRs.

Theorem 1 In the XL-RIS-aided massive MIMO systems, the uplink ergodic achievable rate of user k can be approximated as follows:

$$R_{\text{VR},k} \approx \mathbb{E}\left\{ \log_2 \left\{ \frac{p E_{\text{VR},k}^{\text{signal}}(\boldsymbol{\Phi})}{p \sum_{i=1, i \neq k}^{K} I_{\text{VR},ki}(\boldsymbol{\Phi}) + \sigma^2 E_{\text{VR},k}^{\text{noise}}(\boldsymbol{\Phi}) + 1} \right\} \right\}, \tag{18}$$

where the first, second, and third items are the desired signal received by the BS, the MU interference, and the thermal noise, respectively. The values of $E_{\text{VR},k}^{\text{noise}}(\boldsymbol{\Phi})$, $E_{\text{VR},k}^{\text{signal}}(\boldsymbol{\Phi})$ and $I_{\text{VR},ki}(\boldsymbol{\Phi})$ are given in Eqs. (19)–(22) at the bottom of the next page.

Proof Please refer to appendix in supplementary materials.

3.1 VR analysis

Considering the VRs, the derivation process is difficult due to the visibility indicator matrix $\mathbf{D}_k$ and the spatial correlation matrices $\mathbf{R}_{\text{ris}}$ and $\mathbf{R}_{\text{VR},k}$. Nevertheless, the expression (18) in Theorem 1 can be calculated quickly without relying on inverse matrices and integrals, resulting in lower computational complexity for evaluating the system performance. Besides, Theorem 1 does not rely on the instantaneous CSI, i.e., $\tilde{\mathbf{h}}_k$, and $\tilde{\mathbf{H}}_2$, but it only depends on the statistical CSI, i.e., the AoA and AoD in $\bar{\mathbf{h}}_k$ and $\bar{\mathbf{H}}_2$, the locations of the RIS, the BS, and users, which change slowly and therefore could remain invariant in a long term. As a result, by using the

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theoretical expression of the rate in Eq. (18) as an objective function for a two-timescale transmission scheme, we can optimize the phase shifts of the XL-RIS only determined by statistical CSI.

On the contrary, by analyzing the derived achievable user rate expression, it can be found that some zero terms exist in the diagonal elements of the visibility indicator matrix $\mathbf{D}_k \in \mathbb{C}^{N\times N}$, which indicates that the RIS elements corresponding to these terms have no contribution to the user k. Due to this special property, the dimension of matrix $\mathbf{D}_k$ can be further compressed into $\mathbf{D}_{k_*} = \mathbf{I}_{N_{k_*}\times N_{k_*}}$, assuming that the VR of the kth user contains N_{k*} RIS elements ($N_{k*} \le N$). Based on expressions (1)–(12), we can also reduce the dimension of other matrices, thereby decreasing the computational complexity of the entire system. Specifically, we used $\mathbf{a}_{N,k_*} \in \mathbb{C}^{1\times N_{k_*}}$, $\bar{\mathbf{h}}_{k_*} \in \mathbb{C}^{N_{k_*}\times 1}$, $\bar{\mathbf{H}}_{2,k_*} \in \mathbb{C}^{M\times N_{k_*}}$, $\mathbf{R}_{\text{VR},k_*} \in \mathbb{C}^{N_{k_*}\times N_{k_*}}$, $\mathbf{R}_{\text{ris},\text{k}_*,\text{i}_*} \in \mathbb{C}^{N_{k_*}\times N_{i_*}}$, and $\mathbf{\Phi}_{k_*} \in \mathbb{C}^{N_{k_*}\times N_{k_*}}$ to represent the dimension-reduced matrices of $\mathbf{a}_N$, $\bar{\mathbf{h}}_k$, $\bar{\mathbf{H}}_2$, $\mathbf{R}_{\text{VR}}$, $\mathbf{R}_{\text{ris}}$, and $\mathbf{\Phi}$, respectively. The detailed results are given as follows:

$$\begin{cases} [\mathbf{a}_{N,k_*}]_q = [\mathbf{a}_N]_{[\boldsymbol{\o}_k]_q}, \\ [\bar{\mathbf{h}}_{k_*}]_q = [\bar{\mathbf{h}}_k]_{[\boldsymbol{\o}_k]_q}, \\ [\bar{\mathbf{H}}_{2,k_*}]_{(:,q)} = [\bar{\mathbf{H}}_2]_{(:,[\boldsymbol{\o}_k]_q)}, \\ [\mathbf{R}_{\text{VR},k_*}]_{(p,q)} = [\mathbf{R}_{\text{VR}}]_{([\boldsymbol{\o}_k]_p,[\boldsymbol{\o}_k]_q)}, \\ [\mathbf{R}_{\text{ris},\text{i}_*,\text{k}_*}]_{(p,q)} = [\mathbf{R}_{\text{ris}}]_{([\boldsymbol{\o}_i]_p,[\boldsymbol{\o}_k]_q)}, \\ [\mathbf{\Phi}_{k_*}]_{(:,q)} = [\mathbf{\Phi}]_{(:,[\boldsymbol{\o}_k]_q)}, \end{cases} \tag{23}$$

where $q \in [1, N_{k*}]$ and $p \in [1, N_{i*}]$. Besides, $[\boldsymbol{\o}_k]_q$ presents the qth element of the set $\boldsymbol{\o}_k$. As a result, the expressions in Eq. (22) can be rewritten as follows:

$$E_{\text{VR},k}^{\text{noise}}(\mathbf{\Phi}) = c_k\big(M\delta\varepsilon_k|f_k(\mathbf{\Phi})|^2 + \delta f_{k,1,1}(\mathbf{\Phi}) + \varepsilon_k f_{kk,2}(\mathbf{\Phi})M + f_{k,3,1}(\mathbf{\Phi})M\big), \tag{19}$$

$$\begin{aligned} E_{\text{VR},k}^{\text{signal}}(\mathbf{\Phi}) &= M^2(c_k\delta\varepsilon_k)^2|f_k(\mathbf{\Phi})|^4 + (c_k\delta)^2\left(f_{kk,1,2}(\mathbf{\Phi}) + |f_{k,1,1}(\mathbf{\Phi})|^2\right) + (c_k\varepsilon_k)^2M(M+1)|f_{kk,2}(\mathbf{\Phi})|^2 \\ &+ c_k^2M(M+1)\left(f_{kk,3,2}(\mathbf{\Phi}) + |f_{k,3,1}(\mathbf{\Phi})|^2\right) + 4M(c_k\delta)^2\varepsilon_k|f_k(\mathbf{\Phi})|^2f_{k,1,1}(\mathbf{\Phi}) \\ &+ 2M(M+1)(c_k\varepsilon_k)^2\delta|f_k(\mathbf{\Phi})|^2f_{kk,2}(\mathbf{\Phi}) + 2M(M+1)c_k^2\delta\varepsilon_k|f_k(\mathbf{\Phi})|^2f_{k,3,1}(\mathbf{\Phi}) + 2M(M+1)c_k^2\varepsilon_kf_{kk,4}(\mathbf{\Phi}) \\ &+ 2M(M+1)c_k^2\varepsilon_kf_{kk,2}(\mathbf{\Phi})f_{k,3,1}(\mathbf{\Phi}) + 2(M+1)c_k^2\delta\left(f_{k,1,1}(\mathbf{\Phi})f_{k,3,1}(\mathbf{\Phi}) + f_{kk,5}(\mathbf{\Phi})\right) \\ &+ 2(M+1)c_k^2\delta\varepsilon_kf_{k,1,1}(\mathbf{\Phi})f_{kk,2}(\mathbf{\Phi}) + 4(M+1)c_k^2\delta\varepsilon_k\text{Re}\{f_{kk,6}(\mathbf{\Phi})\}, \end{aligned} \tag{20}$$

$$\begin{aligned} I_{\text{VR},ki}(\mathbf{\Phi}) &= c_kc_i\delta^2\varepsilon_k\varepsilon_iM^2|f_k(\mathbf{\Phi})|^2|f_i(\mathbf{\Phi})|^2 + c_kc_i\delta^2f_{ki,1,2}(\mathbf{\Phi}) + c_kc_i\varepsilon_k\varepsilon_i\left(Mf_{kk,2}(\mathbf{\Phi})f_{ii,2}(\mathbf{\Phi}) + M^2\left|f_{ki,2}(\mathbf{\Phi})\right|^2\right) \\ &+ c_kc_iM\left(f_{k,3,1}(\mathbf{\Phi})f_{i,3,1}(\mathbf{\Phi}) + Mf_{ki,3,2}(\mathbf{\Phi})\right) + c_kc_i\delta\varepsilon_k\varepsilon_iM\left(|f_k(\mathbf{\Phi})|^2f_{ii,2}(\mathbf{\Phi}) + |f_i(\mathbf{\Phi})|^2f_{kk,2}(\mathbf{\Phi})\right) \\ &+ c_kc_i\delta^2M\left(\varepsilon_k|f_k(\mathbf{\Phi})|^2f_{i,1,1}(\mathbf{\Phi}) + \varepsilon_i|f_i(\mathbf{\Phi})|^2f_{k,1,1}(\mathbf{\Phi})\right) + c_kc_i\delta\varepsilon_k\left(M|f_k(\mathbf{\Phi})|^2f_{i,3,1}(\mathbf{\Phi}) + f_{i,1,1}(\mathbf{\Phi})f_{kk,2}(\mathbf{\Phi})\right) \\ &+ c_kc_i\delta\varepsilon_i\left(\boldsymbol{M}|f_i(\mathbf{\Phi})|^2f_{k,3,1}(\mathbf{\Phi}) + f_{k,1,1}(\mathbf{\Phi})f_{ii,2}(\mathbf{\Phi})\right) + c_kc_i\delta\left(f_{k,1,1}(\mathbf{\Phi})f_{i,3,1}(\mathbf{\Phi}) + f_{i,1,1}(\mathbf{\Phi})f_{k,3,1}(\mathbf{\Phi})\right) \\ &+ c_kc_i\varepsilon_kM\left(Mf_{ki,4}(\mathbf{\Phi}) + f_{kk,2}(\mathbf{\Phi})f_{i,3,1}(\mathbf{\Phi})\right) + c_kc_i\varepsilon_iM\left(Mf_{ik,4}(\mathbf{\Phi}) + f_{ii,2}(\mathbf{\Phi})f_{k,3,1}(\mathbf{\Phi})\right) + \varepsilon_i\text{Re}\big\{f_{ik,6}(\mathbf{\Phi})\big\}\big) \\ &+ c_kc_i\delta\varepsilon_k\varepsilon_iM^2\left(f_{ki,7}(\mathbf{\Phi})f_{ik,2}(\mathbf{\Phi}) + f_{ki,2}(\mathbf{\Phi})f_{ik,7}(\mathbf{\Phi})\right) + 2c_kc_i\delta M\text{Re}\{f_{ki,5}(\mathbf{\Phi})\} + 2c_kc_i\delta M\big(\varepsilon_k\text{Re}\big\{f_{ki,6}(\mathbf{\Phi})\big\}, \end{aligned} \tag{21}$$

where

$$\begin{aligned} &f_k(\mathbf{\Phi}) \triangleq \mathbf{a}_N^H\mathbf{\Phi}\mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k,\ f_{k,1,1}(\mathbf{\Phi}) \triangleq \text{Tr}(\bar{\mathbf{H}}_2\mathbf{\Phi}\mathbf{R}_{\text{VR},k}\mathbf{\Phi}^H\bar{\mathbf{H}}_2^H), \\ &f_{ki,1,2}(\mathbf{\Phi}) \triangleq \text{Tr}\{(\bar{\mathbf{H}}_2\mathbf{\Phi}\mathbf{R}_{\text{VR},k}\mathbf{\Phi}^H\bar{\mathbf{H}}_2^H)(\bar{\mathbf{H}}_2\mathbf{\Phi}\mathbf{R}_{\text{VR},i}\mathbf{\Phi}^H\bar{\mathbf{H}}_2^H)\},\ f_{ki,2}(\mathbf{\Phi}) \triangleq \bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\mathbf{\Phi}^H\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{D}_i^{1/2}\bar{\mathbf{h}}_i, \\ &f_{k,3,1}(\mathbf{\Phi}) \triangleq \text{Tr}\big(\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{R}_{\text{VR},k}\mathbf{\Phi}^H\big),\ f_{ki,3,2}(\mathbf{\Phi}) \triangleq \text{Tr}\{\big(\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{R}_{\text{VR},k}\mathbf{\Phi}^H\big)\left(\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{R}_{\text{VR},i}\mathbf{\Phi}^H\right)\}, \\ &f_{ki,4}(\mathbf{\Phi}) \triangleq \bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\mathbf{\Phi}^H\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{R}_{\text{VR},i}\mathbf{\Phi}^H\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k,\ f_{ki,5}(\mathbf{\Phi}) \triangleq \text{Tr}\big(\bar{\mathbf{H}}_2\mathbf{\Phi}\mathbf{R}_{\text{VR},k}\mathbf{\Phi}^H\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{R}_{\text{VR},i}\mathbf{\Phi}^H\bar{\mathbf{H}}_2^H\big), \\ &f_{ki,6}(\mathbf{\Phi}) \triangleq \bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\mathbf{\Phi}^H\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2\mathbf{\Phi}\mathbf{R}_{\text{VR},i}\mathbf{\Phi}^H\mathbf{R}_{\text{ris}}\mathbf{\Phi}\mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k,\ f_{ki,7}(\mathbf{\Phi}) \triangleq \bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\mathbf{\Phi}^H\mathbf{a}_N\mathbf{a}_N^H\mathbf{\Phi}\mathbf{D}_i^{1/2}\bar{\mathbf{h}}_i. \end{aligned} \tag{22}$$

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$$
\begin{aligned}
f_{k_*}(\mathbf{\Phi}) &\triangleq \mathbf{a}_{N,k_*}^H \mathbf{\Phi}_{k_*} \bar{\mathbf{h}}_{k_*}, \\
f_{k_*,1,1}(\mathbf{\Phi}) &\triangleq \mathrm{Tr}\big(\bar{\mathbf{H}}_{2,k_*} \mathbf{\Phi}_{k_*} \mathbf{R}_{\mathrm{VR},k_*} \mathbf{\Phi}_{k_*}^H \bar{\mathbf{H}}_{2,k_*}^H\big), \\
f_{k_*,i_*,1,2}(\mathbf{\Phi}) &\triangleq \mathrm{Tr}\left\{\big(\bar{\mathbf{H}}_{2,k_*} \mathbf{\Phi}_{k_*} \mathbf{R}_{\mathrm{VR},k_*} \mathbf{\Phi}_{k_*}^H \bar{\mathbf{H}}_{2,k_*}^H\big)\right. \\
&\quad \left.\times\big(\bar{\mathbf{H}}_{2,i_*} \mathbf{\Phi}_{i_*} \mathbf{R}_{\mathrm{VR},i_*} \mathbf{\Phi}_{i_*}^H \bar{\mathbf{H}}_{2,i_*}^H\big)\right\}, \\
f_{k_*,i_*,2}(\mathbf{\Phi}) &\triangleq \bar{\mathbf{h}}_{k_*}^H \mathbf{\Phi}_{k_*}^H \mathbf{R}_{\mathrm{ris,k_*,i_*}} \mathbf{\Phi}_{i_*} \bar{\mathbf{h}}_{i_*}, \\
f_{k_*,3,1}(\mathbf{\Phi}) &\triangleq \mathrm{Tr}(\mathbf{R}_{\mathrm{ris,k_*,k_*}} \mathbf{\Phi}_{k_*} \mathbf{R}_{\mathrm{VR},k_*} \mathbf{\Phi}_{k_*}^H), \\
f_{k_*,i_*,3,2}(\mathbf{\Phi}) &\triangleq \mathrm{Tr}\left\{(\mathbf{R}_{\mathrm{ris,i_*,k_*}} \mathbf{\Phi}_{k_*} \mathbf{R}_{\mathrm{VR},k_*} \mathbf{\Phi}_{k_*}^H)\right. \\
&\quad \left.\times(\mathbf{R}_{\mathrm{ris,k_*,i_*}} \mathbf{\Phi}_{i_*} \mathbf{R}_{\mathrm{VR},i_*} \mathbf{\Phi}_{i_*}^H)\right\}, \\
f_{k_*,i_*,4}(\mathbf{\Phi}) &\triangleq \bar{\mathbf{h}}_{k_*}^H \mathbf{\Phi}_{k_*}^H \mathbf{R}_{\mathrm{ris,k_*,i_*}} \mathbf{\Phi}_{i_*} \mathbf{R}_{\mathrm{VR},i_*} \mathbf{\Phi}_{i_*}^H \\
&\quad \times \mathbf{R}_{\mathrm{ris,i_*,k_*}} \mathbf{\Phi}_{k_*} \bar{\mathbf{h}}_{k_*}, \\
f_{k_*,i_*,5}(\mathbf{\Phi}) &\triangleq \mathrm{Tr}\left(\bar{\mathbf{H}}_{2,k_*} \mathbf{\Phi}_{k_*} \mathbf{R}_{\mathrm{VR},k_*} \mathbf{\Phi}_{k_*}^H \mathbf{R}_{\mathrm{ris,k_*,i_*}}\right. \\
&\quad \left.\times \mathbf{\Phi}_{i_*} \mathbf{R}_{\mathrm{VR},i_*} \mathbf{\Phi}_{i_*}^H \bar{\mathbf{H}}_{2,i_*}^H\right), \\
f_{k_*,i_*,6}(\mathit{\Phi}) &\triangleq \bar{\mathbf{h}}_{k_*}^H \mathbf{\Phi}_{k_*}^H \bar{\mathbf{H}}_{2,k_*}^H \bar{\mathbf{H}}_{2,i_*} \mathbf{\Phi}_{i_*} \mathbf{R}_{\mathrm{VR},i_*} \mathbf{\Phi}_{i_*}^H \\
&\quad \times \mathbf{R}_{\mathrm{ris,i_*,k_*}} \mathbf{\Phi}_{k_*} \bar{\mathbf{h}}_{k_*}, \\
f_{k_*,i_*,7}(\mathit{\Phi}) &\triangleq \bar{\mathbf{h}}_{k_*}^H \mathbf{\Phi}_{k_*}^H \mathbf{a}_{N,k_*} \mathbf{a}_{N,i_*}^H \mathbf{\Phi}_{i_*} \bar{\mathbf{h}}_{i_*}. \qquad (24)
\end{aligned}
$$

Due to the lower dimensionality of the participating matrices, the computations in Eq. (24) are comparatively simpler than those in Eq. (22). Specifically, the complexity reduction is attributed to the VR where a certain user k can only observe a subset of RIS elements ($N_{k*} \leq N$). By substituting Eq. (24) into Eqs. (18)–(21), the achievable user rate can be conveniently computed.

4 RIS phase shifts design

In this section, we optimized the phase shifts of the RIS to maximize the achievable rate derived from Eq. (18). In a MU scenario, it is necessary to ensure fairness among different users. Therefore, we considered the minimum achievable user rate maximization problem. Specifically, this problem can be formulated as follows:

$$
\begin{aligned}
&\max_{\mathbf{\Phi}} \quad \min_{k} \quad R_{\mathrm{VR},k}, \\
&\text{s.t.} \quad \left|[\mathbf{\Phi}]_{(n,n)}\right| = 1, \forall n, \qquad (25)
\end{aligned}
$$

where $R_{\mathrm{VR},k}$ is the achievable user rate obtained from Eq. (18).

From Eqs. (18)–(22), it can be observed that the objective function of the optimization problem becomes more intricate when spatial correlation and VRs are present. Besides, when N is large, the dimension of matrices in calculation can be extremely high, which leads to low algorithm efficiency. Fortunately, we have investigated the beneficial effects of VRs in Section 3 and found that they can decrease the computational complexity of the entire system. Furthermore, we also need a new algorithm to better address the considered minimum user rate maximization problem.

In this section, we proposed a gradient-based algorithm with respect to real variable θ_n instead of complex variable $e^{j\theta_n}$, where θ_n is the phase of the nth RIS element. Since the objective function in Eq. (25) contains a non-differentiable minimum function, to conduct the gradient algorithm, we first employed the Lagrangian duality and the Jaynes' maximum entropy principle (Jaynes, 1957) to approximate the objective function in Eq. (25) as follows:

$$
\begin{aligned}
\min_{k} \; R_{\mathrm{VR},k}(\boldsymbol{\theta}) &\approx -\frac{1}{\mu} \ln \left\{ \sum_{k=1}^{K} \exp\{-\mu R_{\mathrm{VR},k}(\boldsymbol{\theta})\} \right\} \\
&\triangleq f_{\mathrm{VR}}(\boldsymbol{\theta}), \qquad (26)
\end{aligned}
$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_N]^{\mathrm{T}}$ is the new optimization variable and μ is a constant value that controls the approximation accuracy, which can be proved that the approximation error is less than $\frac{\ln \mathrm{K}}{\mu}$ (Li XS, 1992). It also can be shown that $f_{\mathrm{VR}}(\boldsymbol{\theta})$ will approach $R_{\mathrm{VR},k}$ as the parameter μ tends to infinity. Therefore, Problem (25) can be rewritten as follows:

$$
\begin{aligned}
&\max_{\mathbf{\Phi}} \qquad f_{\mathrm{VR}}(\boldsymbol{\theta}), \\
&\text{s.t.} \quad \theta_n \in [0, 2\pi), \forall n. \qquad (27)
\end{aligned}
$$

Due to the periodicity of the objective function $f_{\mathrm{VR}}(\boldsymbol{\theta})$ relative to $\boldsymbol{\theta}$, the constraints in Eq. (27) can be ignored, which effectively tackles the unit-modulus constraint in Eq. (25). To update variable $\boldsymbol{\theta}$, the gradient of $f_{\mathrm{VR}}(\boldsymbol{\theta})$ needs to be calculated as follows:

$$
\frac{\partial f_{\mathrm{VR}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \frac{\Sigma_{k=1}^{K} \left\{ \frac{\exp\{-\mu R_{\mathrm{VR},k}(\boldsymbol{\theta})\}}{1+\mathrm{SINR}_{\mathrm{VR},k}(\boldsymbol{\theta})} \frac{\partial \mathrm{SINR}_{\mathrm{VR},k}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right\}}{(\ln 2) \left(\sum_{k=1}^{K} \exp\{-\mu R_{\mathrm{VR},k}(\boldsymbol{\theta})\} \right)}, \qquad (28)
$$

where

$$
\begin{aligned}
\frac{\partial \mathrm{SINR}_{\mathrm{VR},k}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} &= \frac{p \frac{\partial E_{\mathrm{VR},k}^{\mathrm{signal}}}{\partial \boldsymbol{\theta}}}{p \sum_{i=1, i\neq k}^{K} I_{\mathrm{VR},ki} + \sigma^2 E_{\mathrm{VR},k}^{\mathrm{noise}}} \\
&\quad - p E_{\mathrm{VR},k}^{\mathrm{signal}} \frac{p \sum_{i=1, i\neq k}^{K} \frac{\partial I_{\mathrm{VR},ki}}{\partial \boldsymbol{\theta}} + \sigma^2 \frac{\partial E_{\mathrm{VR},k}^{\mathrm{noise}}}{\partial \boldsymbol{\theta}}}{\left(p \sum_{i=1, i\neq k}^{K} I_{\mathrm{VR},ki} + \sigma^2 E_{\mathrm{VR},k}^{\mathrm{noise}} \right)^2}. \qquad (29)
\end{aligned}
$$

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According to Eqs. (28) and (29), the gradient of $f_{\mathrm{VR}}(\boldsymbol{\theta})$ can be obtained by calculating the gradients of $E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi})$, $E_{\mathrm{VR},k}^{\mathrm{signal}}(\boldsymbol{\Phi})$, and $\sum_{i=1,i\neq k}^{K} I_{\mathrm{VR},ki}(\boldsymbol{\Phi})$. By using the conclusion of [Lemma 1](Zhi et al., 2023), we defined the following:

$$\begin{aligned}\boldsymbol{f}_a(\mathbf{A},\mathbf{B}) &\triangleq \frac{\partial \operatorname{Tr}\{\mathbf{A}\boldsymbol{\Phi}\mathbf{B}\boldsymbol{\Phi}^H\}}{\partial\boldsymbol{\theta}} \\ &= j\boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{A}^{\mathrm{T}}\odot\mathbf{B})\boldsymbol{c}^* - j\boldsymbol{\Phi}^H(\mathbf{A}\odot\mathbf{B}^{\mathrm{T}})\boldsymbol{c},\end{aligned} \tag{30}$$

where $\mathbf{A}$ and $\mathbf{B}$ are definite matrices. Then, the gradients of the auxiliary functions in Eq. (22) can be calculated as follows:

$$\begin{aligned}
&f'_k(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\mathbf{a}_N\mathbf{a}_N^H, \mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}),\\
&f'_{k,1,1}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2, \mathbf{R}_{\mathrm{VR},k}),\\
&f'_{ki,1,2}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^H\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2, \mathbf{R}_{\mathrm{VR},k})\\
&+\boldsymbol{f}_a(\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^H\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2, \mathbf{R}_{\mathrm{VR},i})\\
&f'_{ki,2}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\mathbf{R}_{\mathrm{ris}}, \mathbf{D}_i^{1/2}\bar{\mathbf{h}}_i\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}),\\
&f'_{k,3,1}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\mathbf{R}_{\mathrm{ris}}, \mathbf{R}_{\mathrm{VR},k}),\\
&f'_{ki,3,2}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a\big(\mathbf{R}_{\mathrm{ris}}\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^H\mathbf{R}_{\mathrm{ris}}, \mathbf{R}_{\mathrm{VR},k}\big)\\
&+\boldsymbol{f}_a(\mathbf{R}_{\mathrm{ris}}\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^H\mathbf{R}_{\mathrm{ris}}, \mathbf{R}_{\mathrm{VR},i})\\
&f'_{ki,4}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a\big(\mathbf{R}_{\mathrm{ris}}\boldsymbol{\Phi}\mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\boldsymbol{\Phi}^H\mathbf{R}_{\mathrm{ris}}, \mathbf{R}_{\mathrm{VR},i}\big)\\
&+\boldsymbol{f}_a(\mathbf{R}_{\mathrm{ris}}\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^H\mathbf{R}_{\mathrm{ris}}, \mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2})\\
&f'_{ki,5}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^H\mathbf{R}_{\mathrm{ris}}, \mathbf{R}_{\mathrm{VR},i})\\
&+\boldsymbol{f}_a(\mathbf{R}_{\mathrm{ris}}\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^H\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2, \mathbf{R}_{\mathrm{VR},k})\\
&f'_{ki,6}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\mathbf{R}_{\mathrm{ris}}\boldsymbol{\Phi}\mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\boldsymbol{\Phi}^H\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2, \mathbf{R}_{\mathrm{VR},i})\\
&+\boldsymbol{f}_a(\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^H\mathbf{R}_{\mathrm{ris}}, \mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2})\\
&f^*_{ik,6}{}'(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a(\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2\boldsymbol{\Phi}\mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\boldsymbol{\Phi}^H\mathbf{R}_{\mathrm{ris}}, \mathbf{R}_{\mathrm{VR},i})\\
&+\boldsymbol{f}_a\left(\mathbf{R}_{ris}\boldsymbol{\Phi}\mathbf{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^H\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2, \mathbf{D}_k^{1/2}\bar{\mathbf{h}}_k\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\right)\\
&f'_{ki,7}(\boldsymbol{\theta}) \triangleq \boldsymbol{f}_a\big(\mathbf{a}_N\mathbf{a}_N^H, \mathbf{D}_i^{1/2}\bar{\mathbf{h}}_i\bar{\mathbf{h}}_k^H\mathbf{D}_k^{1/2}\big).
\end{aligned} \tag{31}$$

Then, $\frac{\partial E_{\mathrm{VR},k}^{\mathrm{noise}}}{\partial\boldsymbol{\theta}}$, $\frac{\partial E_{\mathrm{VR},k}^{\mathrm{signal}}}{\partial\boldsymbol{\theta}}$, and $\frac{\partial I_{\mathrm{VR},ki}}{\partial\boldsymbol{\theta}}$ can be derived using the above results and the chain rule. The results are given in Eqs. (32)–(34) at the bottom of the next page.

The complete process for solving Problem (27) is provided in Algorithm 1. To improve the convergence performance, the Nesterov accelerated gradient method as shown in steps 6 and 7 is used in the algorithm.

In Algorithm 1, $\boldsymbol{\theta}^*$ is the optimized solution, from which the RIS phase shift matrix $\boldsymbol{\Phi}$ can be calculated. After obtaining $\boldsymbol{\Phi}$, it can be applied to the original Problem (25) to obtain the maximum value of the minimum achievable rate. At the same time, the results show that the complexity of expression (28) is $\mathcal{O}(MN^2+M^2N+N^3)$, which depends only on the fundamental system parameters. Therefore, relying on simple matrix operations, the two-timescale transmission scheme based on the gradient algorithm can significantly reduce the computational complexity.

Algorithm 1 gradient algorithm

1: Randomly initialize $\boldsymbol{\theta}_0$ and set $i=0, e_0=1, \boldsymbol{x}_{-1}=\boldsymbol{\theta}_0$
2: **while** 1 **do**
3: Calculate the gradient vector $f'_{\mathrm{VR}}(\boldsymbol{\theta}_i)$
4: Obtain the step size κ_i based on the backtracking line search
5: $\boldsymbol{x}_i=\boldsymbol{\theta}_i+\kappa_i f'_{\mathrm{VR}}(\boldsymbol{\theta}_i)$
6: $e_{i+1}=(1+\sqrt{4e_i^2+1})/2$
7: $\boldsymbol{\theta}_{i+1}=\boldsymbol{x}_i+(e_i-1)(\boldsymbol{x}_i-\boldsymbol{x}_{i-1})/e_{i+1}$
8: **if** $f_{\mathrm{VR}}(\boldsymbol{\theta}_{i+1})-f_{\mathrm{VR}}(\boldsymbol{\theta}_i)<10^{-4}$ **then**
9: $\boldsymbol{\theta}^*=\boldsymbol{\theta}_{i+1}$, break
10: **end if**
11: $i=1+1$
12: **end while**

5 Simulation and results

In this section, we evaluated the performance of XL-RIS-aided massive MIMO systems and validated the correctness of our analysis. We also illustrated the impact of key parameters in this section. First, a typical XL-RIS-aided scenario is considered where the XL-RIS is deployed near some cell-edge users. We assumed that $k=4$ users are evenly located on a semicircle centered at the XL-RIS with a radius of $d_{\mathrm{UI}}=15$ m. The distance between the BS and the XL-RIS is $d_{\mathrm{IB}}=800$ m. All AoA and AoD of BS, XL-RIS, and users are randomly generated from $[0,2\pi]$, and these parameters will be fixed as known after initial generation. Besides, we set the distance-dependent large-scale path-loss factors equal to $\alpha_k=10^{-3}d_{\mathrm{UI}}^{\alpha_{k,\mathrm{UR}}}, \forall k$ and $\beta=10^{-3}d_{\mathrm{IB}}^{\beta_{\mathrm{RB}}}$, and the path-loss exponents are $\alpha_{k,\mathrm{UR}}=2, \forall k$ and $\beta_{\mathrm{RB}}=2.5$. Due to the dual consideration of time cost and accuracy, we set the approximation factor mu to 100. Considering the impact of VRs, we assume that each user can only see the XL-RIS elements within a certain region of the entire UPA model. To simulate the complexity

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of the actual environment, we randomly set the size and location of the VRs for different users. The other simulation parameters are presented in Table 1.

Figs. 3 and 4 show the relationship between M or N and the minimum user rate when considering the VR of RIS, respectively. The comparison between theoretical and simulation results demonstrates that the obtained approximate achievable rate expression in Eq. (18) is in good agreement with the Monte Carlo simulation results, confirming the accuracy of the mathematical derivations. In addition, as shown in Fig. 3, it can be confirmed that as the value of antennas M increases, the minimum user rate gradually increases. Although the rate eventually converges to a theoretical limit, which is due to the inter-user interference, it can be significantly improved by increasing the number of N. On the contrary, deploying RIS results in promising throughput with a moderate number of antennas, which reduces power consumption and hardware costs. For instance, 60 antennas with 400 RIS elements could perform better than 120 antennas with 200 RIS elements, thanks to the RIS passive beamforming gain.

In Figs. 3 and 4, "partial visibility" indicates that each user can only see partial non-overlapping RIS elements, while "full visibility" indicates that each user can see all RIS elements, i.e., a special case without spatial non-stationary. It can be observed that the minimum user rate is relatively lower when considering VR. This is due to a reduction in the number of available RIS elements per user, resulting in a corresponding decline in array effective aperture and signal strength. In addition, although the VRs lead to performance degradation, it can be compen-

$$\frac{\partial E_{\mathrm{VR},k}^{\mathrm{noise}}}{\partial \boldsymbol{\theta}} = c_k \left\{ M\delta\varepsilon_k f_k'(\boldsymbol{\theta}) + \delta f_{k,1,1}'(\boldsymbol{\theta}) + \varepsilon_k f_{kk,2}'(\boldsymbol{\theta})M + f_{k,3,1}'(\boldsymbol{\theta})M \right\}, \tag{32}$$

$$\begin{aligned}
\frac{\partial E_{\mathrm{VR},k}^{\mathrm{signal}}}{\partial \boldsymbol{\theta}} &= M^2(c_k\delta\varepsilon_k)^2|f_k(\boldsymbol{\Phi})|^2 f_k'(\boldsymbol{\theta}) + (c_k\delta)^2\left(f_{kk,1,2}'(\boldsymbol{\theta}) + 2f_{k,1,1}(\boldsymbol{\Phi})f_{k,1,1}'(\boldsymbol{\theta})\right)\\
&+ 2(c_k\varepsilon_k)^2 M(M+1) f_{kk,2}(\boldsymbol{\Phi}) f_{kk,2}'(\boldsymbol{\theta}) + c_k^2 M(M+1)\left(f_{kk,3,2}'(\boldsymbol{\theta}) + 2f_{k,3,1}'(\boldsymbol{\theta}) f_{k,3,1}(\boldsymbol{\Phi})\right)\\
&+ 4M(c_k\delta)^2\varepsilon_k\left(f_{k,1,1}'(\boldsymbol{\theta})|f_k(\boldsymbol{\Phi})|^2 + f_{k,1,1}(\boldsymbol{\Phi})f_k'(\boldsymbol{\theta})\right) + 2M(M+1)(c_k\varepsilon_k)^2\delta f_{kk,2}'(\boldsymbol{\theta})|f_k(\boldsymbol{\Phi})|^2\\
&+ 2M(M+1)(c_k\varepsilon_k)^2\delta f_{kk,2}(\boldsymbol{\Phi})f_k'(\boldsymbol{\theta}) + 2M(M+1)c_k^2\delta\varepsilon_k f_{k,3,1}'(\boldsymbol{\theta})|f_k(\boldsymbol{\Phi})|^2 + 2M(M+1)c_k^2\delta\varepsilon_k f_{k,3,1}(\boldsymbol{\Phi})f_k'(\boldsymbol{\theta})\\
&+ 2(M+1)c_k^2\delta\varepsilon_k f_{kk,2}'(\boldsymbol{\theta})f_{k,1,1}(\boldsymbol{\Phi}) + 2(M+1)c_k^2\delta\varepsilon_k f_{kk,2}(\boldsymbol{\Phi})f_{k,1,1}'(\boldsymbol{\theta}) + 2(M+1)c_k^2\delta f_{k,3,1}'(\boldsymbol{\theta})f_{k,1,1}(\boldsymbol{\Phi})\\
&+ 2(M+1)c_k^2\delta\left(f_{k,3,1}(\boldsymbol{\Phi})f_{k,1,1}'(\boldsymbol{\theta}) + f_{kk,5}'(\boldsymbol{\theta})\right) + 2M(M+1)c_k^2\varepsilon_k f_{k,3,1}'(\boldsymbol{\theta})f_{kk,2}(\boldsymbol{\Phi})\\
&+ 2M(M+1)c_k^2\varepsilon_k\left(f_{k,3,1}(\boldsymbol{\Phi})f_{kk,2}'(\boldsymbol{\theta}) + f_{kk,4}'(\boldsymbol{\theta})\right) + 2(M+1)c_k^2\delta\varepsilon_k\left(f_{kk,6}'(\boldsymbol{\theta}) + f_{kk,6}^*(\boldsymbol{\theta})\right),
\end{aligned} \tag{33}$$

$$\begin{aligned}
\frac{\partial I_{\mathrm{VR},ki}}{\partial \boldsymbol{\theta}} &= c_kc_i\delta^2\varepsilon_k\varepsilon_i M^2\left(f_k'(\boldsymbol{\theta})|f_i(\boldsymbol{\Phi})|^2 + |f_k(\boldsymbol{\Phi})|^2 f_i'(\boldsymbol{\theta})\right) + c_kc_i\delta^2 f_{ki,1,2}'(\boldsymbol{\theta})\\
&+ c_kc_i\varepsilon_k\varepsilon_i M\left(f_{kk,2}(\boldsymbol{\Phi})f_{ii,2}'(\boldsymbol{\theta}) + f_{kk,2}'(\boldsymbol{\theta})f_{ii,2}(\boldsymbol{\Phi})\right) + c_kc_i\varepsilon_k\varepsilon_i M^2\left(f_{ik,2}(\boldsymbol{\Phi})f_{ki,2}'(\boldsymbol{\theta}) + f_{ki,2}(\boldsymbol{\Phi})f_{ik,2}'(\boldsymbol{\theta})\right)\\
&+ c_kc_i M\left(f_{k,3,1}(\boldsymbol{\Phi})f_{i,3,1}'(\boldsymbol{\theta}) + f_{k,3,1}'(\boldsymbol{\theta})f_{i,3,1}(\boldsymbol{\Phi})\right) + c_kc_i\delta\varepsilon_k\varepsilon_i M\left(|f_k(\boldsymbol{\Phi})|^2 f_{ii,2}'(\boldsymbol{\theta}) + f_k'(\boldsymbol{\theta})f_{ii,2}(\boldsymbol{\Phi})\right)\\
&+ c_kc_i\delta\varepsilon_k\varepsilon_i M\left(|f_i(\boldsymbol{\Phi})|^2 f_{kk,2}'(\boldsymbol{\theta}) + f_i'(\boldsymbol{\theta})f_{kk,2}(\boldsymbol{\Phi})\right) + c_kc_i\delta^2\varepsilon_k M\left(|f_k(\boldsymbol{\Phi})|^2 f_{i,1,1}'(\boldsymbol{\theta}) + f_k'(\boldsymbol{\theta})f_{i,1,1}(\boldsymbol{\Phi})\right)\\
&+ c_kc_i\delta^2\varepsilon_i M\left(|f_i(\boldsymbol{\Phi})|^2 f_{k,1,1}'(\boldsymbol{\theta}) + f_i'(\boldsymbol{\theta})f_{k,1,1}(\boldsymbol{\Phi})\right) + c_kc_i\delta\varepsilon_k M\left(|f_k(\boldsymbol{\Phi})|^2 f_{i,3,1}'(\boldsymbol{\theta}) + f_k'(\boldsymbol{\theta})f_{i,3,1}(\boldsymbol{\Phi})\right)\\
&+ c_kc_i\delta\varepsilon_k\left(f_{i,1,1}(\boldsymbol{\Phi})f_{kk,2}'(\boldsymbol{\theta}) + f_{i,1,1}'(\boldsymbol{\theta})f_{kk,2}(\boldsymbol{\Phi})\right) + c_kc_i\delta\varepsilon_i M\left(|f_i(\boldsymbol{\Phi})|^2 f_{k,3,1}'(\boldsymbol{\theta}) + f_i'(\boldsymbol{\theta})f_{k,3,1}(\boldsymbol{\Phi})\right)\\
&+ c_kc_i\delta\varepsilon_i\left(f_{ii,2}'(\boldsymbol{\theta})f_{k,1,1}(\boldsymbol{\Phi}) + f_{ii,2}(\boldsymbol{\Phi})f_{k,1,1}'(\boldsymbol{\theta})\right) + c_kc_i\delta\left(f_{i,3,1}'(\boldsymbol{\theta})f_{k,1,1}(\boldsymbol{\Phi}) + f_{i,3,1}(\boldsymbol{\Phi})f_{k,1,1}'(\boldsymbol{\theta})\right)\\
&+ c_kc_i\delta\left(f_{i,1,1}(\boldsymbol{\Phi})f_{k,3,1}'(\boldsymbol{\theta}) + f_{i,1,1}'(\boldsymbol{\theta})f_{k,3,1}(\boldsymbol{\Phi})\right) + c_kc_i\varepsilon_k M\left(f_{i,3,1}'(\boldsymbol{\theta})f_{kk,2}(\boldsymbol{\Phi}) + f_{i,3,1}(\boldsymbol{\Phi})f_{kk,2}'(\boldsymbol{\theta})\right)\\
&+ c_kc_i\varepsilon_i M\left(f_{k,3,1}'(\boldsymbol{\theta})f_{ii,2}(\boldsymbol{\Phi}) + f_{k,3,1}(\boldsymbol{\Phi})f_{ii,2}'(\boldsymbol{\theta})\right) + c_kc_i\delta\varepsilon_k\varepsilon_i M^2\left(f_{ik,2}'(\boldsymbol{\theta})f_{ki,7}(\boldsymbol{\Phi}) + f_{ik,2}(\boldsymbol{\Phi})f_{ki,7}'(\boldsymbol{\theta})\right)\\
&+ c_kc_i\varepsilon_i M^2 f_{ik,4}'(\boldsymbol{\theta}) + c_kc_i\delta\varepsilon_k\varepsilon_i M^2\left(f_{ki,2}'(\boldsymbol{\theta})f_{ik,7}(\boldsymbol{\Phi}) + f_{ki,2}(\boldsymbol{\Phi})f_{ik,7}'(\boldsymbol{\theta})\right) + c_kc_i\delta M\left(f_{ki,5}'(\boldsymbol{\theta}) + f_{ik,5}'(\boldsymbol{\theta})\right)\\
&+ c_kc_i M^2 f_{ki,3,2}'(\boldsymbol{\theta}) + c_kc_i\varepsilon_k M^2 f_{ki,4}'(\boldsymbol{\theta}) + 2c_kc_i\delta\varepsilon_k M\left(f_{ki,6}'(\boldsymbol{\theta}) + f_{ki,6}^*(\boldsymbol{\theta})\right) + 2c_kc_i\delta\varepsilon_i M\left(f_{ik,6}'(\boldsymbol{\theta}) + f_{ik,6}^*(\boldsymbol{\theta})\right).
\end{aligned} \tag{34}$$

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Table 1 Simulation parameters

BS antennas	RIS element spacing	RIS elements	Antenna spacing	Rician factors	Transmit power
$M = 64$	$d_{\text{ris}} = \lambda/2$	$N = 200$	$d_{\text{bs}} = \lambda/2$	$\delta = 1,\ \varepsilon_k = 10, \forall k$	$p = 30$ dBm

BS: base stations; RIS: reconfigurable intelligent surfaces;

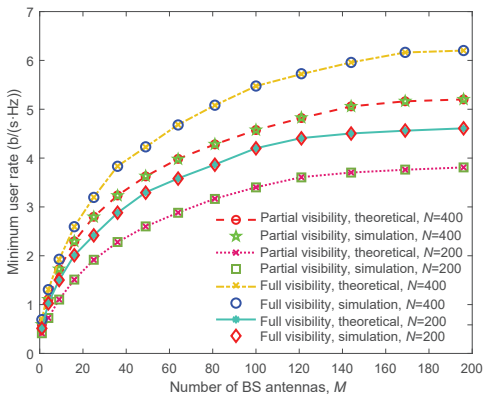

Fig. 3 Minimum user rate versus BS antennas number M (BS: base stations)

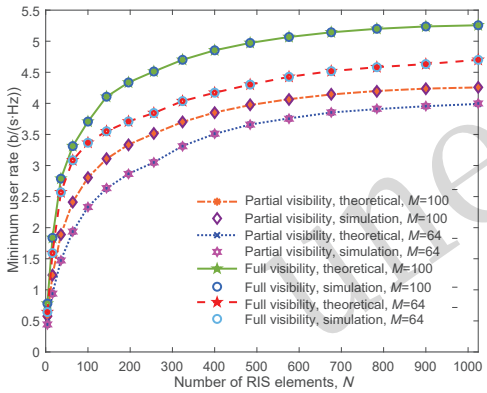

Fig. 4 Minimum user rate versus RIS element number N (RIS: reconfigurable intelligent surfaces)

sated by increasing N. For example, by analyzing Fig. 4 we can find that a partial visibility system with 400 RIS elements can outperform a full visibility system with about 100 RIS elements, assuming that the two systems have the same number of antennas. Therefore, with the help of increasing N, we can achieve the same rate as full visibility systems while still maintaining network capacity requirements.

Fig. 5 shows the influence of the Rician factors, indicating that the minimum user rate is a decreasing function of δ, but it is also an increasing function of ε_k. By analyzing Fig. 5, it can be concluded that as δ gradually increases, the impact of the LoS component $\bar{\mathbf{H}}_2$ in the RIS-BS channel is more significant.

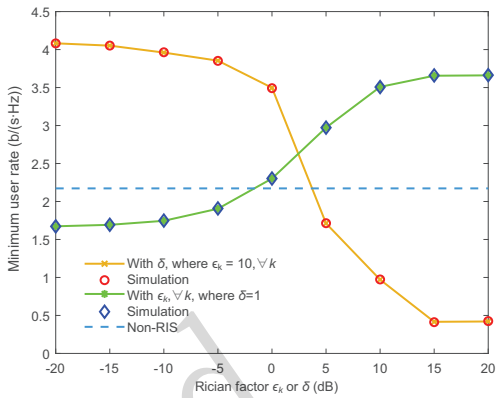

Fig. 5 Max-min achievable rate versus δ or ε_k

It increases the correlation of channels between different users and intensifies MU interference, thereby reducing spatial multiplexing gain. On the contrary, positioning the RIS at a higher elevation compared to the ground level can increase the value of ε_k. This, in turn, enhances the proportion of the LoS component $\bar{\mathbf{h}}_k$ in the user k-RIS channel, achieving higher system throughput as shown in Fig. 5.

In Figs. 6 and 7, we compared the results and runtime of the gradient algorithm and the GA, respectively. The analysis shows that the two algorithms have similar computational results, which proves the accuracy of the gradient algorithm. However, the runtime of the GA is always longer than that of the gradient algorithm, especially when considering the VRs. This is because in the presence of spatial correlation and VRs, the objective function of the optimization problem becomes more complex and the local search ability of the GA is poor, resulting in long time consumption and low search efficiency in the later stage of evolution. In this case, the GA can no longer provide perfect operation results, while the gradient algorithm can still play a good role. Moreover, the efficiency of the gradient algorithm does not decrease as dramatically as the GA when N is large. The reason is that the proposed gradient algorithm considers real variables of angles as optimization variables, thus avoiding performance losses caused by projection operations, and it is able

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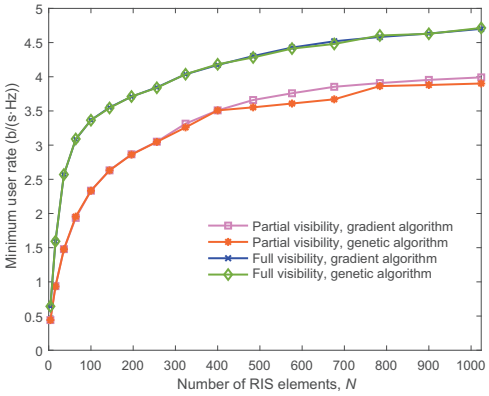

Fig. 6 Comparison of results between different optimization algorithms

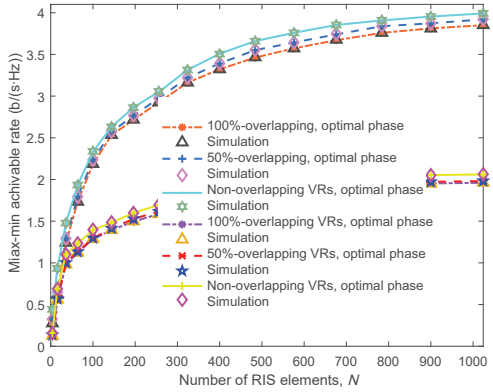

Fig. 8 Max-min achievable rate versus N for overlapping and non-overlapping VRs (VRs: visibility regions)

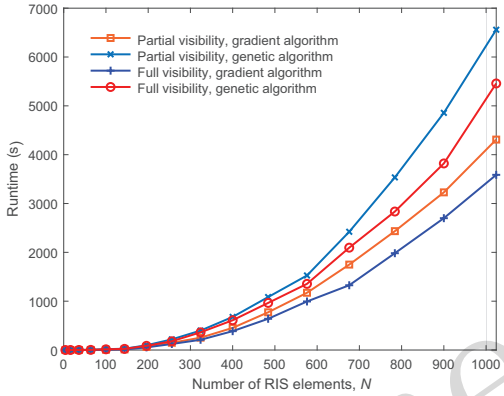

Fig. 7 Comparison of runtime between different optimization algorithms

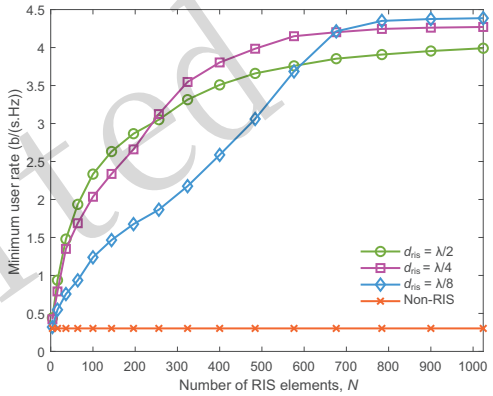

Fig. 9 Max-min achievable rate versus N for different values of the RIS element spacing d_{ris} (RIS: reconfigurable intelligent surfaces)

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to provide more stable results.

In Fig. 8, the influence of overlapping VRs was analyzed. First, it is assumed that non-overlapping means that each user's VR does not overlap with others' VR, while overlapping means that each user's VR will overlap with others' VR. It can be seen that the minimum user rate of a 100%-overlapping system is reduced by 0.1408 bits/s/Hz (b/(s·Hz)) compared with a non-overlapping system, assuming $N = 1024$. Therefore, the impact of overlapping VRs on the minimum user rate is relatively small and can be almost ignored when the rate is large. It indicates that by using the proposed phase shifts optimization algorithm, the interference at overlapping VRs can be reduced. As a result, it can significantly minimize the performance gap between overlapping and non-overlapping systems, and ensure fairness in the user rate of the entire system.

Note that in this study, the objective function of the optimization problem is $f_{\mathrm{VR}}(\boldsymbol{\theta})$, and therefore spatial correlation is considered in the RIS phase shifts design. From Fig. 9, it can be observed that as the spacing between RIS components decreases ($d_{\mathrm{ris}} = \lambda/2, \lambda/4, \lambda/8$), the influence of spatial correlation on the minimum user rate cannot be ignored. Specifically, for cases where the value of N is small, the rate decreases as the element spacing decreases, which is attributed to the decrease in channel rank. For cases where the value of N is relatively large, the channel rank still decreases. However, by increasing the number of RIS elements and using channel correlation to enhance the customized wireless channel capability provided by the RIS, the beamforming gain provided by the optimized RIS can exceed the negative impact of spatial correlation, thereby achieving higher user rates.

In Fig. 10, it can be observed that when considering the influence of RIS visibility, as N increases,

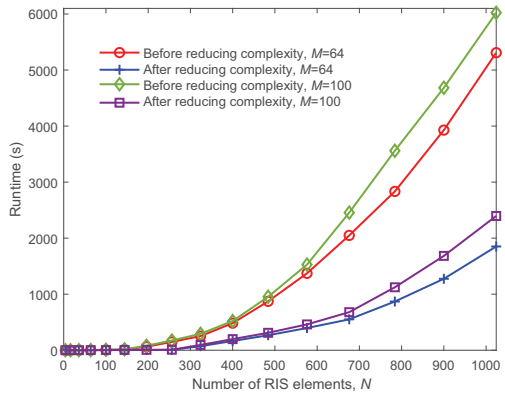

Fig. 10 Comparison of runtime before and after reducing complexity

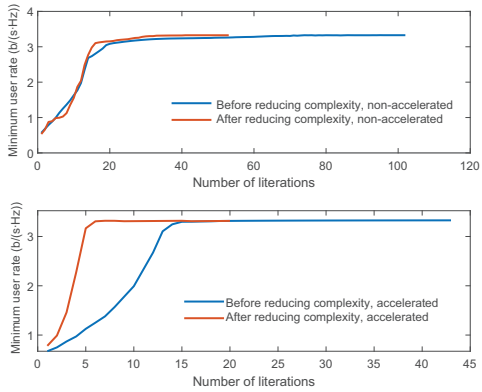

Fig. 11 Comparison of system solution iteration times before and after reducing complexity

the time for solving the max-min problem with the gradient algorithm also gradually increases. However, as the solution method for low complexity rate expressions is much simpler, the runtime can be significantly reduced. Especially when N is large in XL-RIS scenarios, such as $N = 1024$, reducing complexity can increase computational speed by nearly three times.

On the contrary, Fig. 11 investigated the convergence behavior of the proposed low-complexity gradient algorithm compared to the ordinary method. By applying the proposed complexity reduction method, it can be observed that when $M = 64$ and $N = 400$, the algorithm converges within 20 iterations. Compared with the 43 iterations that use the usual gradient algorithm, there is a slight improvement in convergence speed. Besides, although there is not much difference in the number of iterations before and after reducing complexity, the efficiency is significantly increased and the total time consumption is decreased due to the greatly reduced complexity of single iteration calculations. Moreover, the convergence speed can be further effectively improved by applying the proposed acceleration method. Hence, the above results confirm that redundant components can be removed through the proposed method in Eq. (23). As a result, the achievable rate expression with dimension-reduced matrices can effectively improve system efficiency and save resources.

6 Conclusions

In this study, we studied the two-timescale design of XL-RIS-aided massive MIMO communication systems, taking into account the influence of VRs. First, we considered a spatially-correlated channel model in the presence of VRs. A closed-form expression of the achievable user rate was derived and the optimization problem was clarified. At the same time, we analyzed the impact of the VR of the RIS on system complexity and simplified the user rate expression. Second, we proposed a gradient algorithm different from the GA and validated the potential of applying it to the phase shift optimization problem. Finally, we conducted simulations and analysis about XL-RIS-aided massive MIMO systems, and the impact of VRs on system performance was verified from different perspectives.

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Contributors

Luchu LI designed the research and drafted the paper. Cunhua PAN and Kangda ZHI revised and finalized the paper.

Conflict of interest

All the authors declare that they have no conflict of interest.

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