Frontiers of Information Technology & Electronic Engineering
www.jzus.zju.edu.cn; engineering.cae.cn; www.springerlink.com
ISSN 2095-9184 (print); ISSN 2095-9230 (online)
E-mail: jzus@zju.edu.cn

Height estimation method based on a beamspace joint alternating iterative algorithm in MIMO radar*

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Received Jan. 13, 2025; Revision accepted June 10, 2025; Crosschecked

Abstract: This paper discusses the problem of low elevation target height estimation for multiple-input multiple-output (MIMO) radar in multipath environments. Beamspace compresses the data and is ideal for reducing the computational burden of elevation estimation. To accurately obtain the height parameter of the target, we propose a height estimation method based on a beamspace joint alternating iterative algorithm in MIMO radar. This method mainly converts the reduced-dimensional MIMO radar element space data into beamspace and whitens it to improve the reliability of the data. It then uses a simplified model to obtain the initial value of the elevation and combines the target reflection coefficient and the target elevation for alternate estimation. Finally, we calculate the target height using the obtained elevation information. Simulation verifies that the proposed algorithm has good estimation accuracy and robustness.

Key words: Multipath environment; Multiple-input multiple-output (MIMO) radar height estimation; Beamspace and whiten processing; Joint alternating iterative

https://doi.org/10.1631/FITEE.2500030 **CLC number:**

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1 Introduction

Meter-wave radar, also known as ultra-short wave or VHF radar, generally refers to radar operating in the meter-wave band (Gage and Green, 2016; Zheng et al., 2023). Because of its long wavelength and wide beam width, meter-wave radar has a strong ground reflection echo when measuring targets at low elevation. The direct signal of the reflected target and the coherent multipath signal reflected along the ground enter the main beam of the radar together (Zhu et al., 2017; Tan and Nie, 2018). The direct signal and multipath signal are usually difficult to separate in the time, Doppler and spatial domains. These problems seriously affect the accuracy of estimated elevation, and it is unrealistic to improve elevation estimation accuracy simply by increasing the physical aperture of the array. As a new radar system, MIMO radar (Li and Stoica, 2007; Zhang et al., 2012; Shi et al., 2021) has virtual aperture expansion characteristics and obvious advantages over traditional radar in terms of anti-interference, anti-clutter, low interception and angular resolution (Zhou et al., 2016). Therefore, it is more appropriate to use MIMO radar to handle target height estimation in a multipath environment, but how to find an accurate and robust method is an urgent problem that needs to be solved.

In recent years, there has been rapid development of array signal processing, leading to the emergence of many super-resolution methods (Carlin et al., 2013; Yang et al., 2020; Zheng et al., 2022). These methods can be broadly categorized into sub-

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* Project supported by the National Natural Science Foundation of China (No. 62271379), the National Radar Signal Processing Laboratory (No. KGJ202401) and the Fundamental Research Funds for the Central Universities, and the Innovation Fund of Xidian University (No. YJSJ24011)

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space algorithms and maximum likelihood (ML) algorithms. Subspace algorithms, such as MUSIC (Yin and Zhang, 2020) and ESPRIT (Feng et al., 2021), are commonly used. However, they require decoherence processing (Shi et al., 2016) to be completed first when estimating the target elevation, as they cannot directly process coherent signals. On the other hand, ML (Ziskind and Wax, 1988) type algorithms can directly process coherent signals and are widely used in height estimation problems. The angle and reflection surface height joint estimation (ARJE) algorithm (Wang et al., 2016) is a method based on eigenvalue decomposition for complex terrains. It offers enhanced robustness by searching more parameters. Beamspace (Xu and Zhao, 2019) reduces data transmission volume, storage and calculation, but sacrifices some degrees of freedom. To reduce the computational burden, the beamspace improved maximum likelihood (BIML) algorithm (Liu et al., 2010) for MIMO radar was proposed for elevation estimation. The BIML algorithm uses prior knowledge of geometric information to reduce search dimensions and beamspace processing to reduce data dimensions. The three-dimensional BML (3DBML) algorithm (Chen et al., 2022) converts element space data into beamspace and uses eigenvectors to find closed-form solutions. The calculation amount is low, but the accuracy is subject to certain limitations. In addition, the refined maximum likelihood (RML) estimation method (Lo and Litva, 1991) takes advantage of some a priori information, such as radar height, target range, location of the multipath reflection point, multipath attenuation coefficient and multipath mirror geometry. Although the a priori information contributes to the good performance of ML methods, these methods are usually applicable only to ideal smooth reflecting surfaces. In practice, it is difficult to obtain the relevant parameters.

In response to the above problems and research methods, in this paper we propose a height estimation method based on a beamspace joint alternating iterative (BJAI) algorithm in MIMO radar. This algorithm does not require prior information about the spatial distribution of the multipath. It first uses a simplified model in a multipath environment to obtain an initial estimation of the elevation of the target and performs reduced-dimensional processing on the MIMO radar. Then, based on the initial elevation estimation (Tang et al., 2024), the element space data are converted to the beamspace and whitened. Finally, the reflection coefficient and the target elevation are combined for alternating iterative estimation, which ensures effective estimation accuracy while reducing the computational complexity of the algorithm. Simulation results show that this method has strong robustness and high estimation accuracy.

The main contributions of this paper are as follows:

1. A new method of height estimation based on a beamspace joint alternating iterative algorithm in MIMO radar is proposed. This method does not require any prior information about terrain and reflection paths and enhances the robustness of the multipath propagation model in complex terrains.

2. Addressing the challenge of MIMO radar target height estimation in complex terrains involves initially using dimensionality reduction transformation and beam space processing. Subsequently, the issue is resolved by alternately and iteratively estimating the attenuation coefficient and target elevation angle. The multivariable optimization problem is decomposed into multiple single variable optimization sub-problems.

3. Simulation analysis and real data results show that the proposed algorithm has good robustness and estimation accuracy in complex terrains.

2 Multipath signal model

In this study we considered a narrowband configured monostatic MIMO radar consisting of a uniform linear array. The radar was assumed to use the pulsed system with a transceiver co-located antenna design and evenly distributed vertically, and the detection targets were mostly far-field targets, which could easily meet the rayleigh conditions on smooth

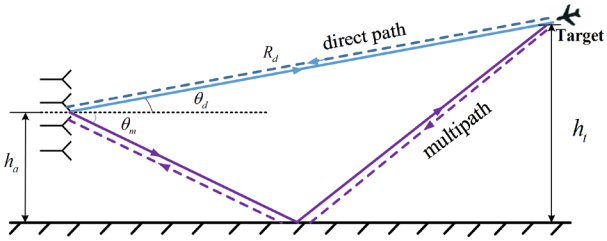

Fig. 1 Reflection model of the MIMO radar in the low-elevation area

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ground. The geometry of the target height estimation scenario for monostatic MIMO radar is shown in Fig. 1. In the MIMO radar system, there are M transmitting sensors and receiving sensors. The height of the antenna center is h_a. The element spacing of the antenna is denoted by d, which is less than or equal to $\lambda/2$. λ is the wavelength and the target-antenna element distance is R_d. In practical applications, both receiving and transmitting two-way multipath signals should be taken into consideration, so the echo signals received by the antenna elements should come from four different paths. The monostatic MIMO radar transmitter emits a set of mutually orthogonal signals.

After matched filtering, the received signal can be written as (Wang et al., 2015):

$$\boldsymbol{X}(t) = \boldsymbol{B}_{tr} s(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{B}_{tr} = \boldsymbol{b}_r(\theta_d, \theta_m) \otimes \boldsymbol{b}_t(\theta_d, \theta_m) \in \mathbb{C}^{M^2 \times 1}$, and $\otimes$ represents the Kronecker product. The composite transmitter and receiver steering vectors are denoted $\boldsymbol{b}_t(\theta_d, \theta_m) \in \mathbb{C}^{M \times 1}$ and $\boldsymbol{b}_r(\theta_d, \theta_m) \in \mathbb{C}^{M \times 1}$, respectively. Here, θ_d and θ_m represent the arrival angles of the direct signal and multipath signal, respectively, and $\boldsymbol{n}(t) \in \mathbb{C}^{M^2 \times 1}$ is zero-mean Gaussian white noise, which is uncorrelated with the target signal.

The composite steering vector includes direct and multipath signals:

$$\boldsymbol{b}_t(\theta_d, \theta_m) = \boldsymbol{a}_t(\theta_d) + \rho e^{-j2\pi\Delta R/\lambda} \boldsymbol{a}_t(\theta_m) \tag{2}$$

$$\boldsymbol{b}_r(\theta_d, \theta_m) = \boldsymbol{a}_r(\theta_d) + \rho e^{-j2\pi\Delta R/\lambda} \boldsymbol{a}_r(\theta_m) \tag{3}$$

where $a_t(\theta)$ and $a_r(\theta)$ take the form

$$\boldsymbol{a}_t(\theta) = \left[1, e^{-j2\pi d \sin\theta/\lambda}, \cdots, e^{-j2\pi(M-1)d \sin\theta/\lambda}\right]^T \tag{4}$$

$$\boldsymbol{a}_r(\theta) = \left[1, e^{-j2\pi d \sin\theta/\lambda}, \cdots, e^{-j2\pi(M-1)d \sin\theta/\lambda}\right]^T \tag{5}$$

where j denotes imaginary units, and $\Delta R = 2h_a(\sin\theta_d + h_a/R_d)$ is the path-difference between the direct and multipath signal. $[\boxed{?}]^T$ denotes the transpose operator, ρ is the surface reflection coefficient (Lo and Litva, 1991), and $\boldsymbol{a}_t(\theta)$ and $\boldsymbol{a}_r(\theta)$ are the transmitter steering vector and receiver steering vector, respectively.

Assuming that the number of snapshots of the received data is L, then the observation data matrix can be written as

$$\boldsymbol{X} = \boldsymbol{B}_{tr}\boldsymbol{S} + \boldsymbol{N} \tag{6}$$

where $\boldsymbol{X} = [\boldsymbol{x}(t_1), \boldsymbol{x}(t_2), \cdots, \boldsymbol{x}(t_L)] \in \mathbb{C}^{M^2 \times L}$, $\boldsymbol{S} = [s(t_1), s(t_2), \cdots, s(t_L)] \in \mathbb{C}^{1 \times L}$ together with $\boldsymbol{N} = [\boldsymbol{n}(t_1), \boldsymbol{n}(t_2), \cdots, \boldsymbol{n}(t_L)] \in \mathbb{C}^{M^2 \times L}$.

3 Beamspace joint alternating iterative algorithm for height estimation

In this section we present the proposed BJAI algorithm appropriate for target height estimation in a MIMO radar setup. The BJAI algorithm first uses a simplified model to roughly estimate the target elevation angle and then reduces the dimension of the received signal based on reduced-dimensional transformation and beamspace processing. After that, the BJAI algorithm estimates the target height using the joint alternating iterative algorithm (Liu et al., 2018).

3.1 Initial angle estimation on a simplified model

According to the MIMO radar multipath signal reflection model and the geometric relationship of the low-elevation area, the elevation expression of the reflection path can be obtained as:

$$\theta_m = -arcsin\left(\sin\theta_d + 2h_a/R_d\right) \tag{7}$$

In general, $R_d \gg h_a$, taking the receiver as an example, combined with the steering vector, we can obtain (Chen et al., 2022)

$$\boldsymbol{a}_r(\theta_m) \approx \boldsymbol{a}_r(-\theta_d) = \boldsymbol{TF}(\theta_d)\boldsymbol{a}_r(\theta_d) \tag{8}$$

where $\boldsymbol{TF}(\theta_d)$ are diagonal matrices, and the expression is

$$\boldsymbol{TF}(\theta_d) = \mathrm{diag}\left(1, \cdots, e^{j4\pi(M-1)d\sin\theta_d/\lambda}\right) \tag{9}$$

From the equation above, the composite receiver steering vector can be expressed as

$$\boldsymbol{b}_r(\theta_d, \theta_m) = [\boldsymbol{I}_M + \gamma\boldsymbol{TF}(\theta_d)]\boldsymbol{a}_r(\theta_d) \tag{10}$$

where $\gamma = \rho e^{-j2\pi\Delta R/\lambda}$, $\boldsymbol{I}_M$ is an identity matrix with dimension M.

$\boldsymbol{b}_r(\theta_d, \theta_m)$ is a function of θ_d. Digital beamforming (DBF) scanning is performed based on the element space received data to obtain an initial ele-

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vation estimation $\hat{\theta}_d^0$. In addition, $\hat{\theta}_d^0$ can be used as the initial iteration of the algorithm and to determine the angle search range.

3.2 Reduced-dimensional transformation

In each column of the steering matrix $\boldsymbol{B}_{tr}$ in MIMO radar, $(M+M-1)$ elements are unique. Exploiting this feature, a reduced dimensional transformation is adopted to lower the dimension of the received data $\boldsymbol{X}$ (Liu et al., 2017).

$$\boldsymbol{C}(\theta_d,\theta_m)=\boldsymbol{C}(\theta_d)+\rho e^{-j2\pi\Delta R/\lambda}\boldsymbol{C}(\theta_m) \quad (11)$$

where $\boldsymbol{C}(\theta_d,\theta_m)\in\mathbb{C}^{(M+M-1)\times 1}$ is the virtual composite steering vector.

$$\boldsymbol{C}(\theta)=\left[1,\cdots,e^{-j2\pi(M+M-2)d\sin\theta/\lambda}\right]^T \quad (12)$$

$\boldsymbol{J}$ is a transformation matrix and can be expressed as

$$\boldsymbol{J}=[\boldsymbol{\Xi}_0^T,\boldsymbol{\Xi}_1^T,\cdots,\boldsymbol{\Xi}_{M-1}^T]^T \quad (13)$$

where

$\boldsymbol{\Xi}_m^T=\left[\boldsymbol{0}_{M\times m},\boldsymbol{I}_M,\boldsymbol{0}_{M\times(M-m-1)}\right]\in\mathbb{C}^{M\times(M+M-1)}$ for $m=0,1,2,\cdots,M-1$. Therefore, we can obtain

$$\boldsymbol{B}_{tr}=\boldsymbol{J}\boldsymbol{C}(\theta_d,\theta_m) \quad (14)$$

According to the above relationship, a reduced-dimensional matrix can be defined as

$$\boldsymbol{T}=\boldsymbol{P}^{-1}\boldsymbol{J}^H \quad (15)$$

where $\boldsymbol{T}\in\mathbb{C}^{(M+M-1)\times M^2}$, $[\cdot]^H$ denotes the conjugate-transpose operator. $\boldsymbol{P}=(\boldsymbol{J}^H\boldsymbol{J})^{1/2}$ is a diagonal matrix directly calculated as

$$\boldsymbol{P}=\mathrm{diag}\left[1,\sqrt{2},\cdots,\sqrt{M},\cdots,\sqrt{2},1\right] \quad (16)$$

Consequently, using the reduced dimensional transformation of the data matrix, $\boldsymbol{X}$ is implemented as follows:

$$\overline{\boldsymbol{X}}=\boldsymbol{T}(\boldsymbol{B}_{tr}\boldsymbol{S}+\boldsymbol{N})=\boldsymbol{P}\boldsymbol{C}(\theta_d,\theta_m)\boldsymbol{S}+\overline{\boldsymbol{N}} \quad (17)$$

where $\overline{\boldsymbol{X}}\in\mathbb{C}^{(M+M-1)\times L}$ and $\overline{\boldsymbol{N}}\in\mathbb{C}^{(M+M-1)\times L}$ are the new received data matrix and noise matrix, respectively. The reduced-dimensional data $\overline{\boldsymbol{X}}$ is equivalent to the received data of a virtual array of $(M+M-1)$ array elements.

3.3 Beamspace multipath signals and whitening the data

In Section 3.1, based on the DBF scanning of the array space data, a rough estimation of the arrival angle of the direct signal, $\hat{\theta}_d^0$, can be obtained. A rough estimation of the arrival angle of the multipath signal is $-\hat{\theta}_d^0$.

According to the multipath signal propagation model, consider a $(M+M-1)\times 3$ beamspace conversion matrix $\boldsymbol{T}_B$, which aims to convert array element space data to beamspace.

$$\boldsymbol{T}_B=\left[\boldsymbol{c}(\hat{\theta}_d^0)\quad \boldsymbol{c}(0)\quad \boldsymbol{c}(-\hat{\theta}_d^0)\right] \quad (18)$$

Then, the target data in the beamspace can be expressed as

$$\boldsymbol{X}_B=\boldsymbol{T}_B^H\overline{\boldsymbol{X}}=\boldsymbol{T}_B^H\boldsymbol{P}\boldsymbol{C}(\theta_d,\theta_m)\boldsymbol{S}+\boldsymbol{T}_B^H\overline{\boldsymbol{N}} \quad (19)$$

where $\boldsymbol{X}_B\in\mathbb{C}^{3\times L}$. The three rows of the beamspace data $\boldsymbol{X}_B$ are the outputs of the beam forming in the three directions $\hat{\theta}_d^0$, 0 and $-\hat{\theta}_d^0$ respectively.

To keep the noise of the beamspace data as Gaussian white noise, it should be whitened. We combine the beamspace conversion matrix $\boldsymbol{T}_B$ with whitening to obtain a new beamspace conversion matrix:

$$\boldsymbol{T}_{WB}=(\boldsymbol{T}_B^H\boldsymbol{T}_B)^{-1/2}\boldsymbol{T}_B^H \quad (20)$$

where $\boldsymbol{T}_{WB}^H\boldsymbol{T}_{WB}=\boldsymbol{I}_3$. Then, the whitened beamspace data can be expressed as

$$\boldsymbol{X}_{WB}=\boldsymbol{T}_{WB}\overline{\boldsymbol{X}}=\boldsymbol{T}_{WB}\boldsymbol{P}\boldsymbol{C}(\theta_d,\theta_m)\boldsymbol{S}+\boldsymbol{T}_{WB}\overline{\boldsymbol{N}} \quad (21)$$

where $\boldsymbol{X}_{WB}\in\mathbb{C}^{3\times L}$. The noise of the whitened beamspace data is Gaussian white noise, which is uncorrelated. At present, most beamspace angle estimation or height estimation algorithms convert only array element space data into beamspace, and do not consider whether the noise after conversion is still Gaussian white noise, so they are not optimal.

The angle search interval selected here is

$$\theta\in\left(\hat{\theta}_d^0-\theta_{3\mathrm{dB}}/2,\quad \hat{\theta}_d^0+\theta_{3\mathrm{dB}}/2\right) \quad (22)$$

where we define $\theta_{3\mathrm{dB}}=\frac{50.8\lambda}{(M+M-1)d}$ as the 3-dB beam width.

3.4 Joint alternating iterative estimation

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The purpose of low-elevation estimation is to obtain the target elevation θ_d from the beamspace data $\boldsymbol{X}_{WB}$. First, the covariance matrix of the beamspace can be estimated as

$$\widehat{\boldsymbol{R}}_{WB}(\bar{\rho},\theta_d)=\boldsymbol{X}_{WB}(\bar{\rho},\theta_d)\boldsymbol{X}_{WB}^H(\bar{\rho},\theta_d)/L \quad (23)$$

where, $\widehat{\boldsymbol{R}}_{WB}\in\mathbb{C}^{3\times3}$, $\bar{\rho}=\rho e^{-j2\pi\Delta R/\lambda}$.

For the symmetrical direct and reflected signals, to improve the angle estimation accuracy, $\widehat{\boldsymbol{R}}_{WB}$ can be replaced by (Zoltowski and Lee, 1991)

$$\boldsymbol{R}_{WB}=\frac{1}{2}\left(\widehat{\boldsymbol{R}}_{WB}+\widehat{\boldsymbol{I}}_3\widehat{\boldsymbol{R}}_{WB}\widehat{\boldsymbol{I}}_3\right) \quad (24)$$

where the matrix

$$\widehat{\boldsymbol{I}}_3=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix} \quad (25)$$

The algorithm proposed in this paper involves the joint alternating estimation of $\bar{\rho}$ and θ_d. Subsequently, by performing singular value decomposition (SVD) on the covariance matrix $\boldsymbol{R}_{WB}$, we can obtain

$$\boldsymbol{R}_{WB}=\boldsymbol{E}_s\boldsymbol{\Lambda}_s\boldsymbol{E}_s^H+\boldsymbol{E}_n\boldsymbol{\Lambda}_n\boldsymbol{E}_n^H \quad (26)$$

where $\boldsymbol{E}_s$ and $\boldsymbol{E}_n$ constitute the signal subspace and noise subspace, and $\boldsymbol{\Lambda}_s$ and $\boldsymbol{\Lambda}_n$ represent two diagonal matrices, whose diagonal elements correspond to the signal subspace eigenvalues and noise subspace eigenvalues, respectively.

Based on the arrival angle of the direct signal $\widehat{\theta}_d^0$ obtained in Section 3.1, the arrival angle of the multipath signal can be calculated from Eq. (7). At this point, only the reflection coefficient $\bar{\rho}$ is unknown. By solving the corresponding optimization problem, we can obtain $\widehat{\rho}^{k+1}$.

Claim 1. *The closed-form solution for $\widehat{\rho}^{k+1}$ can be calculated using the Lagrange multipliers:*

$$\boldsymbol{\xi}^{k+1}=\frac{\left[\boldsymbol{Q}^H\boldsymbol{P}^H\boldsymbol{T}_{WB}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{T}_{WB}\boldsymbol{P}\boldsymbol{Q}\right]^{-1}\boldsymbol{\omega}}{\boldsymbol{\omega}^H\left[\boldsymbol{Q}^H\boldsymbol{P}^H\boldsymbol{T}_{WB}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{T}_{WB}\boldsymbol{P}\boldsymbol{Q}\right]^{-1}\boldsymbol{\omega}} \quad (27)$$

where $\boldsymbol{\xi}^{k+1}=[1,\widehat{\rho}^{k+1}]^T$, $\boldsymbol{Q}=[\boldsymbol{c}(\theta_d),\boldsymbol{c}(\theta_m)]$, $\boldsymbol{\omega}=[1,0]^T$, and $[\cdot]^{-1}$ denotes inverse operator.

Proof. See Appendix.

Then, use the above estimated parameter $\widehat{\rho}^{k+1}$ to solve the following optimization problem, thereby obtaining the parameter $\widehat{\theta}_d^{k+1}$ to estimate the target elevation. The optimization problem can be expressed as

$$\widehat{\theta}_d^{k+1}=arg\max_{\theta_d}\frac{\left\|\boldsymbol{T}_{WB}\boldsymbol{P}\boldsymbol{C}(\widehat{\rho}^{k+1})\right\|_2^2}{\boldsymbol{F}_{capon}\times\boldsymbol{F}_{music}} \quad (28)$$

where

$\boldsymbol{F}_{capon}=\boldsymbol{C}^H(\widehat{\rho}^{k+1})\boldsymbol{P}^H\boldsymbol{T}_{WB}^H\boldsymbol{R}_{WB}^{-1}\boldsymbol{T}_{WB}\boldsymbol{P}\boldsymbol{C}(\widehat{\rho}^{k+1})$,

$\boldsymbol{F}_{music}=\boldsymbol{C}^H(\widehat{\rho}^{k+1})\boldsymbol{P}^H\boldsymbol{T}_{WB}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{T}_{WB}\boldsymbol{P}\boldsymbol{C}(\widehat{\rho}^{k+1})$,

and $\|\boxed{?}\|_2$ denotes the L_2 norm. The Capon-based cost function needs to calculate the inverse operation of the matrix, but the covariance matrix of the beamspace is a third-order matrix, so the inverse operation does not involve a large amount of calculation.

The optimization problem combines the cost functions of Capon and Music, which reduces the beam width of the scanning curve, enhances the sparsity of the signal, improves the estimation accuracy of the target elevation, and makes the algorithm more robust. At this point, we have completed the optimization update in the $k+1$ iteration and can obtain the updated parameter vector $\left(\widehat{\rho}^{k+1},\widehat{\theta}_d^{k+1}\right)$. Using that vector, repeat the above-mentioned optimization iterative process to complete the next iterative update of the parameter vector until the algorithm converges.

The convergence condition of the proposed algorithm can be defined as when the parameter estimates obtained in two adjacent iterations have not changed significantly, and when $\left|\widehat{\theta}_d^{k+1}-\widehat{\theta}_d^k\right|/\widehat{\theta}_d^k\le\varepsilon$, the algorithm stops the iterative process, where ε is the given convergence threshold. In addition, the maximum number of iterations K of the algorithm can also be given. When the number of iterations k is greater than the given maximum number of iterations, the algorithm iterations are terminated.

Finally, when the algorithm meets the convergence conditions, we can obtain the estimated value of the target elevation $\widehat{\theta}_d$. The height information of the target can be calculated using the target elevation and the structural relationship between the target and the radar. For a target in the near distance, its height can be calculated by the following formula

$$\widehat{H}_t=h_a+R_d\widehat{sin\,\theta_d} \quad (29)$$

where $\widehat{H}_t$ denotes the height of target. For a long-

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range target, the spherical-earth model must be considered, and the target height based on the beamspace data can be subsequently obtained:

$$\widehat{H}_t \simeq h_a + R_d \widehat{\sin\theta}_d + \frac{R_d^2}{2R_e} \tag{30}$$

where R_e is the effective radius of the earth. Therefore, we iterate the estimation process to obtain the final estimate. The estimation steps of the proposed algorithm for beamspace data are summarized in Algorithm 1.

Algorithm 1 Proposed algorithm

Step 1: Set the number of iterations $K, k = 0, \varepsilon$.
Step 2: Obtain initial angle estimation $\hat{\theta}_d^0$ by DBF scanning in simplified model.
Step 3: Use reduced-dimensional matrices to process data by Eq. (17).
Step 4: Obtain whitened beamspace data of $\boldsymbol{X}_{WB}$ by Eq. (21), and obtain angle search interval.
Step 5: Estimate the reflection coefficient $\hat{\rho}^{k+1}$by Eq. (27), and update the parametric.
Step 6: Estimate the target elevation $\hat{\theta}_d^{k+1}$ by Eq. (28), and update the parametric.
Step 7: If the maximum number of the iterations or convergence condition is satisfied, stop the iteration. Otherwise, set $k = k + 1$and go to Step 5.
Step 8: Calculate the target height $\widehat{H}_t$ by Eq. (29) or Eq. (30).

4 Simulation analysis and real data validation

4.1 Simulation analysis

In this section, because different reflective terrains exist, several simulations are presented to compare the performance of the proposed method based on different scenarios. In the following simulations, an array vertically and uniformly distributed narrowband configuration monostatic MIMO radar was considered. The radar emits orthogonal waveform signals, and the number of sampling points in the pulse is 400. The noise type is zero-mean Gaussian white noise. We set $d = \lambda/2 = 0.5m$, $L = 64$, $h_a = 8m$, $M = 12$, $K = 30$, and $\varepsilon = 0.002$. Experiments were run in the MATLAB R2021b environment of an Intel Core i9-10900 CPU computer. We define the signal-to-noise ratio (SNR) as $\mathrm{SNR} = 10\,lg\left(\frac{\eta}{\sigma_n^2}\right)$ where η is the signal average power, and σ_n^2 is the noise power.

The first simulation considered a scenario with an ideal flat reflecting terrain. We compared the angle estimation performance of the BIML, 3D-BML, ARJE, BRML, BJAI-NW and BJAI-W algorithms against the cramer-rao bound (CRB) (Boman and Stoica, 2001). The conventional beamspace transformation matrix T_B corresponds to the BJAI-NW algorithm and the matrix T_{WB}, based on whitening processing, corresponds to the BJAI-W algorithm. The BJAI algorithm proposed in this paper refers to the BJAI-W algorithm. We assumed that$R_d = 100$km, $H_t = 4500$m, $\rho = 0.9e^{j\pi}$ and the number of monte carlo experiments was 1000.

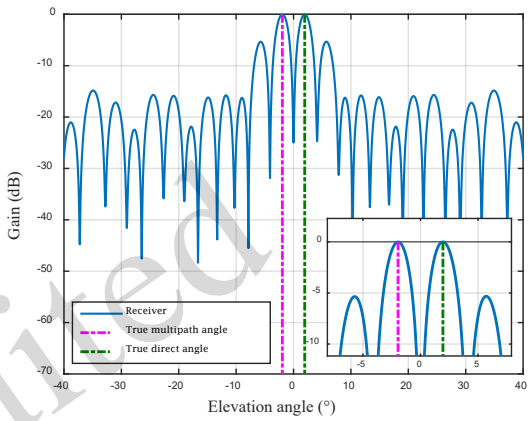

Fig. 2 Initial angle estimation by DBF scanning in simplified model

First, we present a rough estimation of the target angle as shown in Fig. 2. Assume that $M = 12$ and $\mathrm{SNR} = 10$dB, by using the symmetry of the direct signal and the multipath signal, the estimated arrival angle of the target direct signal and multipath signal obtained through digital beamforming scanning is relatively accurate. The measured value of the beam scanning is within half the beam width of the true values of the target direct angle and multipath angle. This estimation provides a reasonable initial value and search interval for the subsequent algorithm iteration process.

Secondly, the change curves of the root mean square error (RMSE) for target elevation and height estimation by each algorithm with SNR are shown in Figs. 3 and 4, respectively, when the target is at an altitude of 4500 m. The figures show that the accuracy of angle and height estimation increases as the SNR increases. The estimation accuracy of the BIML, 3DBML, and ARJE algorithms is relatively low due to their limited use of information. The

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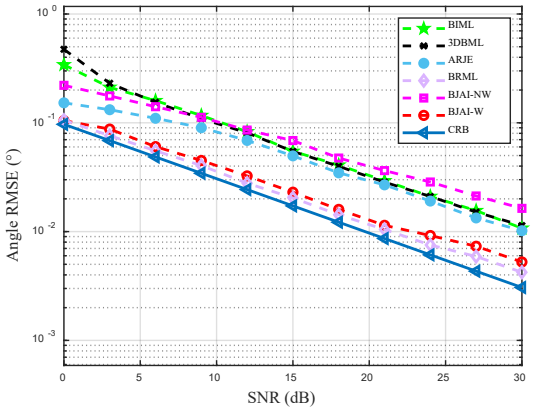

Fig. 3 Target elevation RMSE against SNR for $H_t = 4500$ m

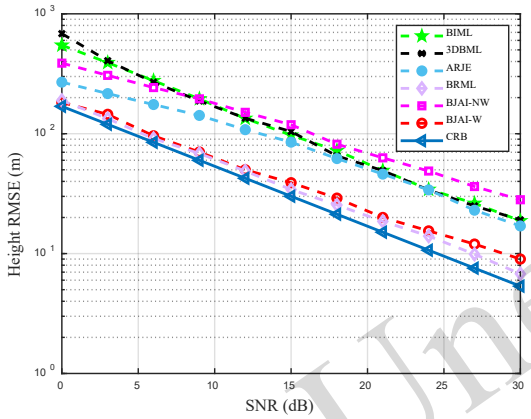

Fig. 4 Target height RMSE against SNR for $H_t = 4500$ m

BRML algorithm relies on prior information of the terrain. Under an ideal flat terrain, it can obtain terrain-related parameters based on the determined geometric structure and has high angle estimation performance. The BJAI-NW algorithm relies mainly on $\boldsymbol{X}_B$ for estimation, which is suboptimal when the noise of the beamspace data is not Gaussian white noise. On the other hand, the BJAI-W algorithm, which is based on the noise subspace and whitens the beamspace data, exhibits angle and height estimation accuracy that is relatively close to the CRB. This algorithm also enhances robustness and estimate accuracy by estimating the reflection coefficient $\bar{\rho}$ using additional information.

Table 1 Running time comparison

Methods	Running time (s)
BIML	0.0274
3DBML	0.0017
ARJE	0.8164
BRML	0.0635
BJAI-NW	0.0497
BJAI-W	0.0525

Finally, Table 1 shows the average running time of 200 Monte Carlo trials. The ARJE algorithm takes the most time because it is based on the element space, and the joint estimation and eigen decomposition take up a lot of time. The 3DBML algorithm completes the angle estimation by finding a closed-form solution, so it takes the least time. The BIML algorithm can complete the target elevation angle estimation through a one-dimensional search. The BRML algorithm needs to calculate intermediate variables based on geometric structure and prior information, so the computational complexity is higher than that of the BIML algorithm. The BJAI-NW and BJAI-W algorithms complete the angle estimation through joint alternating iterations, and the iterative processing brings additional computational complexity. Additionally, by converting the element space to beamspace and using the initial estimate value to determine the angle search range, the computational complexity is also reduced. In future research, we will explore low-complexity algorithms to make the proposed method more efficient.

The second simulation considered a complex scenario with an undulating reflecting terrain. We assumed that the target had a height of 9000 m moving from 150 to 250 km in the horizontal direction. Due to the undulating terrain, we assumed the $\mathrm{SNR} = 15\mathrm{dB}$, $\rho = 0.9e^{j\pi} \cdot \Delta\gamma$, where $\Delta\gamma$ represents the perturbation parameter, which reflects the degree to which the multipath signal is affected by the irregular reflection of the complex position and is an unknown variable. The form of perturbation parameter can be expressed as $\Delta\gamma = (1 + \Delta\gamma_a)e^{j\pi(180^\circ + \Delta\gamma_p)/180^\circ}$, where $\Delta\gamma_a$ and $\Delta\gamma_p$ represent the amplitude and phase perturbation of the perturbation parameter, respectively. We assumed the amplitude perturbation satisfies $\Delta\gamma_a \sim U(-0.3, 0.3)$ and the phase perturbation satisfies $\Delta\gamma_p \sim U(-30, 30)$. A target point trace was generated every 10 km, and the target elevation angle

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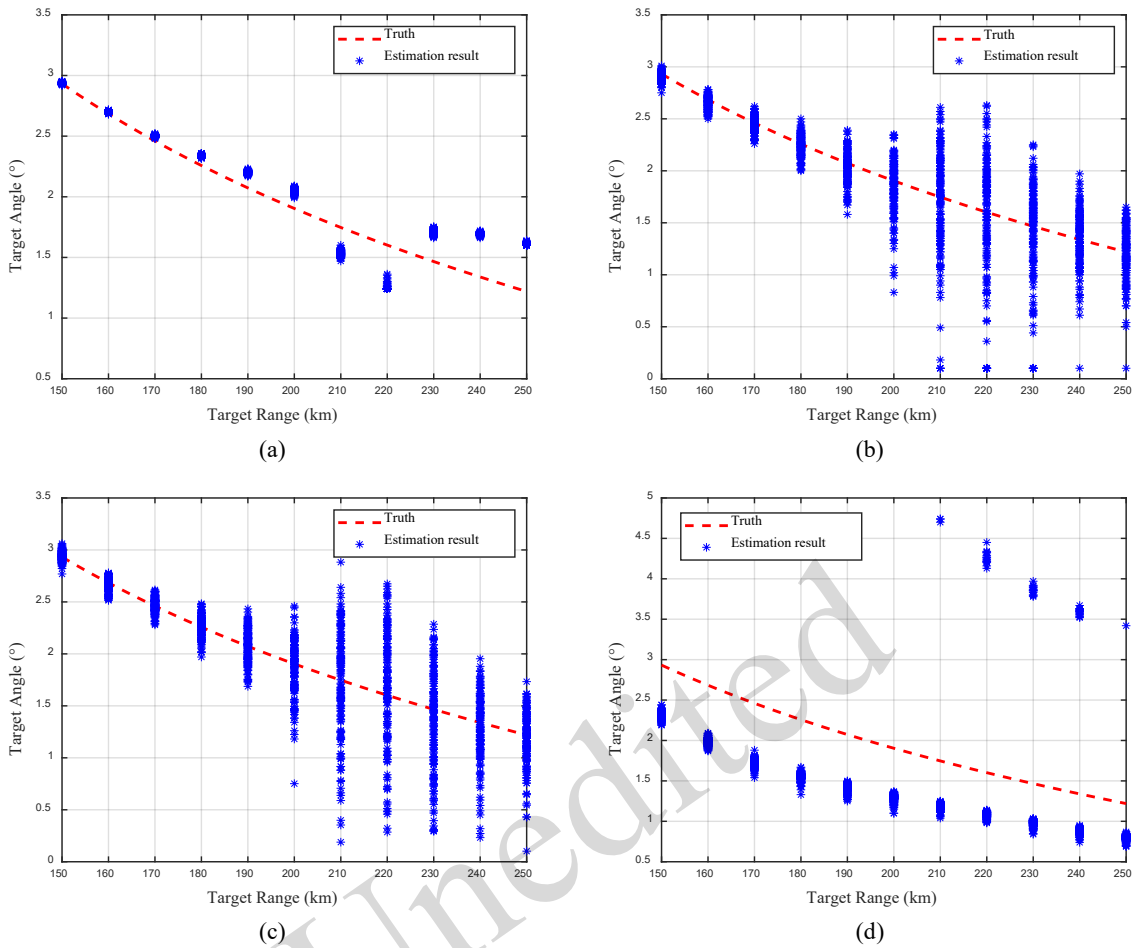

Fig. 5 Comparison of estimation elevation angles for simulated target of BJAI-W (a), BIML (b), 3DBML (c), and BRML (d) algorithms

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corresponding to each point trace was estimated. The number of Monte Carlo experiments was 1000.

Fig. 5 shows the angle estimation results of each point trace of the simulated target for each of the four beamspace algorithms, where the red dotted line is the real elevation angle of the simulated target and the blue point trace is the angle estimation result of the first 150 Monte Carlo tests. Fig. 6 shows the height estimation error results of the simulated target. The proposed BJAI-W algorithm has more accurate angle estimation results than other algorithms. When the target is far away and the elevation angle is low, affected by multipath perturbation, the BIML and 3DBML algorithms cannot effectively distinguish between direct and multipath signals, and their performance is seriously degraded. The BRML algorithm relies on prior information of the terrain. When the terrain is undulating and the terrain information is unknown, the BRML algorithm has a large angle estimation error, and the algorithm fails. Therefore, the algorithm proposed in this paper is suitable for dealing with low-elevation target height estimation

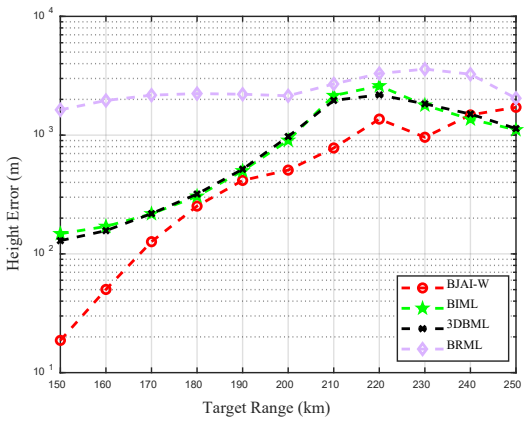

Fig. 6 Comparison of estimation height error for simulated target

problems under undulating terrain conditions.

By analyzing the algorithm proposed in this study, we observed that we combine the reflection coefficient $\bar{\rho}$ and the target elevation θ_d for alternating iterative estimation. However, due to the complexity and variability of the actual surface environment, the position of the multipath reflection point and the reflection coefficient will change as the target moves. This can lead to degradation in the performance of existing algorithms that rely on deterministic scene geometry structure information, and even failure in severe cases. Therefore, by using the information of the reflection coefficient $\bar{\rho}$, the proposed algorithm shows promising potential even under undulating terrain.

4.2 Real data validation

To further verify the performance of the proposed height estimation algorithm under complex terrain scenarios, real data were applied. The array radar was equipped with a uniform linear array and worked in the meter-band. The polarization mode was horizontal polarization. After the original data were processed by pulse compression, because of moving target indication (MTI) and constant false alarm detection, etc., due to the limitation of the radar system, only one snapshot of measured data could be obtained for height estimation. Fig. 7 shows the flight trajectory of the target relative to the radar position, where the arrow indicates the flight direction. Fig. 8 shows the change of the distance and azimuth of the target relative to the radar against the frame time. The radar was set up at the junction of land and sea, and the flight trajectory of the target was located over the land, which included irregular reflective surfaces such as undulating hills, fish ponds, and paddy fields.

Since the ARJE algorithm performs poorly under a single snapshot, we verified the proposed algorithm with BIML and 3DBML algorithms using real data. Figs. 9 and 10 show the processing results of the real data. The angle and height estimation performance of the proposed algorithm is better than that of the other two algorithms. The estimation results of the 3DBML algorithm have large errors, mainly because the algorithm uses a closed-form solution obtained under an accurate model. As the complexity of the terrain increases, the performance

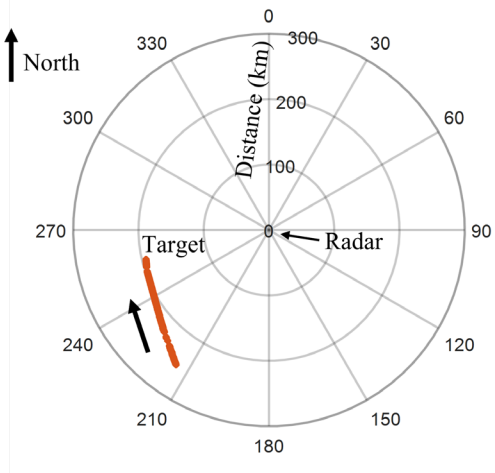

Fig. 7 Navigational trace map of the target

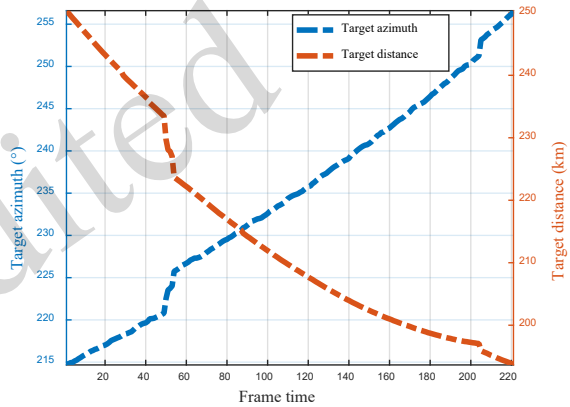

Fig. 8 Target azimuth and distance against frame time

of the algorithm seriously deteriorates. The BIML algorithm has a large estimation error when the target elevation angle is low, because the target elevation angle and the multipath angle are within a beam width, and the angle difference is about 1/3 of the beam width at that time. In addition, for some frames, the estimation results of the three algorithms all jump, which is due to low SNR or problems with the original data. The algorithm proposed in this paper uses covariance matrix estimation, so single snapshot data and a low SNR will cause errors in the estimation of the attenuation coefficient. In general, since the proposed algorithm takes the effect of the complex terrain into consideration and applies iterative estimation, it can resolve the attenuation coefficient and target elevation angle. The effectiveness of the proposed algorithm was validated by experimental results using real data.

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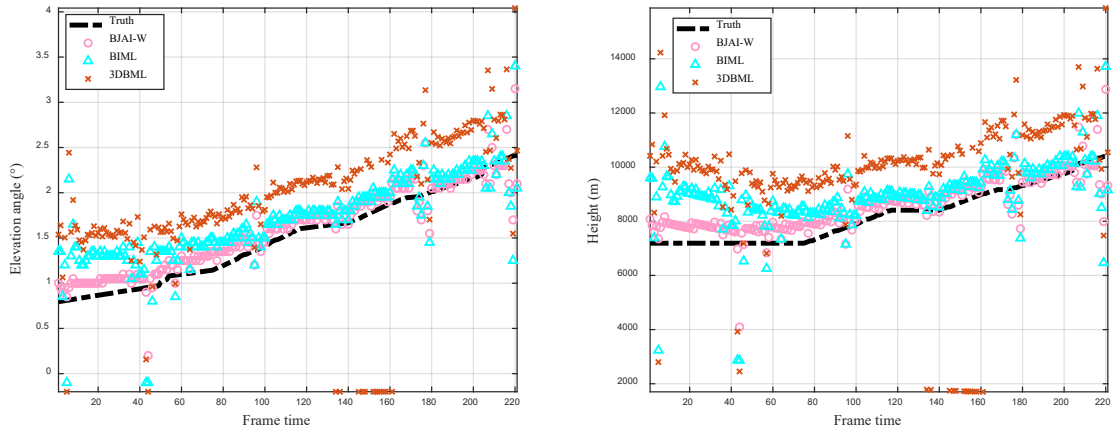
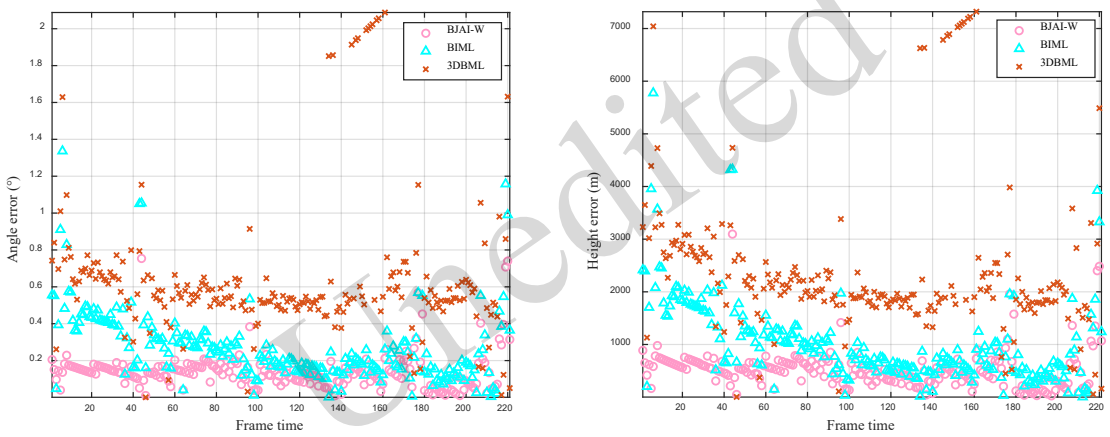


(a) (b)

Fig. 9 Estimated results for real data. (a) Estimation of the target elevation angle versus frame time, (b) Estimation of the target height versus frame time



(a) (b)

Fig. 10 Estimated error for real data. (a) Estimation of the elevation angle error versus frame time, (b) Estimation of the height error versus frame time

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5 Conclusions

In this paper we present a solution to the problem of estimating the target height for MIMO radar in a multipath environment. The proposed method based on whitened beamspace data, achieves high estimate accuracy through the attenuation coefficient and target elevation angle joint alternating iterative estimation. The proposed algorithm improves the performance of target height estimation in complex multipath environments without any prior information about the reflected path. Numerical results based on simulated data and real data show the efficiency of our proposed algorithm under complex scenarios.

The multipath reflection coefficient model is dynamic. The model works well in simple undulating terrain scenarios, but is difficult to match with the actual situation of more complex scenarios, which leads to the failure of the model. At present, it is difficult to build a model that can accurately match complex scenarios, but it is feasible to estimate the reflection coefficient. For more complex terrain conditions, we are now studying more matched multipath reflection models to improve the estimation performance for low-elevation targets.

Contributors

Derui TANG designed the research. Derui TANG and Yongbo ZHAO processed the data. Derui TANG drafted the manuscript. Shuaijie ZHANG helped organize the manuscript. Derui TANG and Yongbo ZHAO revised and finalized the paper.

Conflict of interest

All the authors declare that they have no conflict of interest.

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Appendix: Proof of Claim 1

The columns of $\boldsymbol{E_s}$ span the signal subspace, which coincides with the column space of $\boldsymbol{C}(\bar{\rho}, \hat{\theta}_d^k)$, and the signal subspace is orthogonal to the noise subspace spanned by the columns of $\boldsymbol{E_n}$. Therefore,

$$\left\|\boldsymbol{E_n^H T_{WB} P C}(\bar{\rho}, \hat{\theta}_d^k)\right\|_2^2 = 0 \tag{31}$$

For the $(k+1)$th iteration, the algorithm begins with the obtained $\hat{\theta}_d^k$ to find the $\hat{\rho}^{k+1}$ to minimize the cost function, which can be represented as follows:

$$\hat{\rho}^{k+1} = \arg\min_{\bar{\rho}} \boldsymbol{C}^H(\bar{\rho}, \hat{\theta}_d^k)\boldsymbol{P}^H\boldsymbol{T}_{WB}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{T}_{WB}\boldsymbol{P}\boldsymbol{C}(\bar{\rho}, \hat{\theta}_d^k) \tag{32}$$

For convenience, we assume that $\boldsymbol{V} = \boldsymbol{T}_{WB}\boldsymbol{P}$, $\boldsymbol{Q} = \left[\boldsymbol{c}(\hat{\theta}_d^k), \boldsymbol{c}(\hat{\theta}_m^k)\right]$, $\boldsymbol{\xi} = [1, \bar{\rho}]^T$. Then, we can get

$$\hat{\bar{\rho}}^{k+1} = \arg\min_{\bar{\rho}} \boldsymbol{\xi}^H\boldsymbol{Q}^H\boldsymbol{V}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{V}\boldsymbol{Q}\boldsymbol{\xi} \tag{33}$$

This is an unconstrained optimization problem. Let $\boldsymbol{\omega} = [1,0]^T$, so the problem becomes an optimization problem under constraints:

$$\begin{cases} \min\limits_{\bar{\rho}} \boldsymbol{\xi}^H\boldsymbol{Q}^H\boldsymbol{V}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{V}\boldsymbol{Q}\boldsymbol{\xi} \\ \text{s.t.} \quad \boldsymbol{\xi}^H\boldsymbol{\omega} = 1 \end{cases} \tag{34}$$

The above problem can now be solved by the Lagrange multipliers, let the objective function be

$$L(\boldsymbol{\xi}) = \boldsymbol{\xi}^H\boldsymbol{Q}^H\boldsymbol{V}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{V}\boldsymbol{Q}\boldsymbol{\xi} + \lambda(\boldsymbol{\xi}^H\boldsymbol{\omega} - 1) \tag{35}$$

Taking partial derivatives of $\boldsymbol{\xi}$ and λ respectively, we obtain

$$\begin{cases} \frac{\partial L}{\partial \xi} = \boldsymbol{Q}^H\boldsymbol{V}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{V}\boldsymbol{Q}\boldsymbol{\xi} + \lambda\boldsymbol{\omega} = \boldsymbol{0} \\ \frac{\partial L}{\partial \lambda} = \boldsymbol{\xi}^H\boldsymbol{\omega} - 1 = \boldsymbol{0} \end{cases} \tag{36}$$

Finally, we obtain

$$\boldsymbol{\xi} = \frac{[\boldsymbol{Q}^H\boldsymbol{V}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{V}\boldsymbol{Q}]^{-1}\boldsymbol{\omega}}{\boldsymbol{\omega}^H[\boldsymbol{Q}^H\boldsymbol{V}^H\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{V}\boldsymbol{Q}]^{-1}\boldsymbol{\omega}} \tag{37}$$

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