Binchi WANG, Yitong GUO, Guodong REN, Jun MA, 2026. Energy dynamics and circuit implementation for a neuron with a memcapacitive membrane. *ENGINEERING Information Technology & Electronic Engineering*, 27(1):250024. https://doi.org/10.1631/ENG.ITEE.2025.0024

Energy dynamics and circuit implementation for a neuron with a memcapacitive membrane

Key words: Neural circuit; Neuron model; Hamilton energy; Memcapacitor; Coherence resonance

Corresponding author: Jun MA
E-mail: hyperchaos@163.com
ORCID: https://orcid.org/0000-0002-6127-000X

Background

- Neuromorphic circuits seek low-power neuronal dynamics, where memory devices help merge storage and computation.
- Real neurons have deformable double-layer membranes; effective capacitance and energy vary with polarization and stimuli.
- Memcapacitors provide history-dependent capacitance, enabling circuit-level modeling of membrane adaptation and deformation.

Main content

- Build a FitzHugh–Nagumo (FHN)-type neural circuit with a capacitor (inner membrane) and a memcapacitor (outer membrane) coupled by a resistor.
- Derive a dimensionless neuron model with two capacitive state variables and a memcapacitor internal state under forcing/noise.
- Formulate field energy and convert it to an equivalent Hamilton energy, validated via Helmholtz decomposition criterion.
- Reveal bifurcation and firing modes versus memcap parameters and stimulus amplitude/frequency.
- Link mode transitions to average energy and energy components, and then regulate transitions via an adaptive control strategy.

Circuit overview: physical circuit mapped to a 4D neuron model

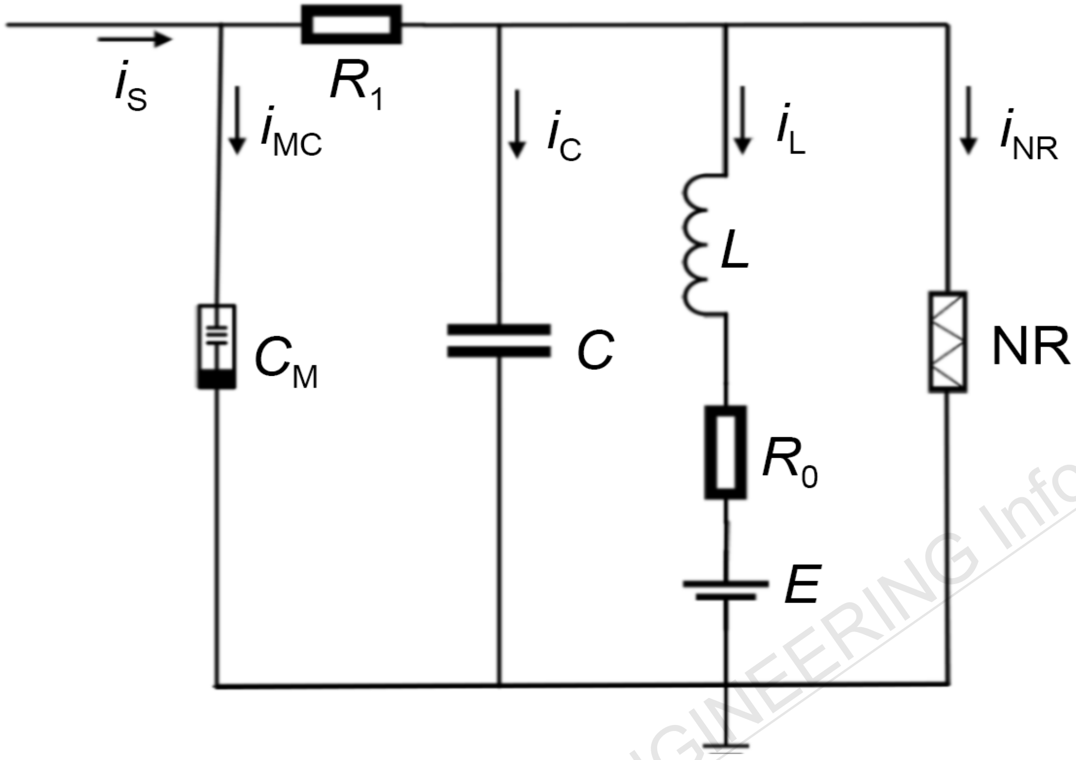

Fig. 1 Neural circuit with a memcapacitive membrane

Circuit equations

$$\begin{cases} C\dfrac{\mathrm{d}V_C}{\mathrm{d}t}=\dfrac{V_{MC}-V_C}{R_1}-i_L-i_{NR}, \\ L\dfrac{\mathrm{d}i_L}{\mathrm{d}t}=V_C-E-R_0 i_L, \\ i_{MC}=-\dfrac{V_{MC}-V_C}{R_1}+i_S. \end{cases}$$

Memcapacitor state variable

$$C_M^{-1}(\sigma)=\frac{R_{12}}{R_{11}}\sigma+\frac{R_{12}}{R_{10}}\sigma^2.$$

Field energy function

$$W=\frac{1}{2}CV^2+\frac{1}{2}Li_L^2+\frac{1}{2}C_M V_{MC}^2$$

Theoretical model and control law

Memcapacitive neuron model

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}\tau}=(\lambda_1-1)x-\dfrac{1}{3}\lambda_1 x^3-y+\lambda_2 zw+\lambda_3 zw^2, \\ \dfrac{\mathrm{d}y}{\mathrm{d}\tau}=\lambda_4 x-\lambda_5 y-\lambda_6, \\ \dfrac{\mathrm{d}z}{\mathrm{d}\tau}=x-\lambda_2 zw-\lambda_3 zw^2+I_{\mathrm{S}}, \\ \dfrac{\mathrm{d}w}{\mathrm{d}\tau}=z. \end{cases}$$

Hamilton energy function

$$H=\frac{W}{CV_0^2}=\frac{1}{2}x^2+\frac{1}{2\lambda_4}y^2+\frac{1}{2}\lambda_2 wz^2+\lambda_3 w^2 z^2,$$

$$H_{\mathrm{C}}=\frac{1}{2}x^2, H_{\mathrm{L}}=\frac{1}{2\lambda_4}y^2, H_{\mathrm{MC}}=\frac{1}{2}\lambda_2 wz^2+\lambda_3 w^2 z^2.$$

Mapping

- *x*: inner-membrane capacitive voltage
- *z*, *w*: memcapacitor charge & memory state
- *y*: inductive-channel variable

Statistical analysis

$$CV=\frac{\sqrt{<T^2>-<T>^2}}{<T>}, \quad <H>=\frac{1}{\tau}\int_0^{\tau} H(\tau)\mathrm{d}\tau \approx \frac{1}{\mathrm{N}}\sum_{i=1}^{N} H_i.$$

Adaptive law

$$\frac{\mathrm{d}\lambda_3}{\mathrm{d}\tau}=\sigma\lambda_3\Theta(H-\lambda), \Theta(P)=\begin{cases}1, P\geq 0, \\ 0, P<0.\end{cases}$$

Scientific contribution and conclusions

- Propose a physically grounded memcapacitive neuron capturing outer-membrane deformation and dual-membrane energy interaction.
- Show memcapacitive energy, dominates total energy, and serves as an indicator of firing regularity and resonance strength.
- Demonstrate channel-dependent noise thresholds; outer-membrane (memcap) perturbation yields stronger coherence resonance.
- Verify numerics with an analog circuit implementation, reproducing periodic/chaotic patterns and supporting hardware feasibility.

Author biographies

Binchi WANG

First author

Lanzhou University of Technology

iD 0009-0004-6919-2503

Binchi WANG received the B.E. degree from Lanzhou University of Technology. He was recommended for postgraduate study without entrance examination and is currently pursuing the M.E. degree in physical electronics at Lanzhou University of Technology. His research supervisor is Professor Jun Ma. His research interests include dynamical control, computational neurodynamics, and functional neural circuit.

E-mail: 13919823182@163.com
https://webofscience.clarivate.cn/wos/author/record/GXM-9955-2022

Jun MA

Corresponding author

Department of Physics, Lanzhou University of Technology

iD 0000-0002-6127-000X

Professor Jun MA received the Master's degree in theoretical physics from Guangxi Normal University in 2003 and the doctorate degree in theoretical physics from the Central China Normal University in 2010. He is an active associate editor for *Nonlinear Dynamics* (*NODY*) and now serves *NODY* as a receiving editor. His research interests include memristor, neural circuit, neurodynamics, synchronization control, neural networks, and electromechanical arm.

E-mail: hyperchaos@163.com
https://webofscience.clarivate.cn/wos/author/record/1609312