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# HAPE3D—a new constructive algorithm for the 3D irregular packing problem

**Key words:** 3D packing problem, Layout design, Simulation, Optimization, Constructive algorithm, Meta-heuristics

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# HAPE

What is **HAPE**?

↓

**H**euristic **A**lgorithm based on the principle of minimum total **P**otential **E**nergy

↓

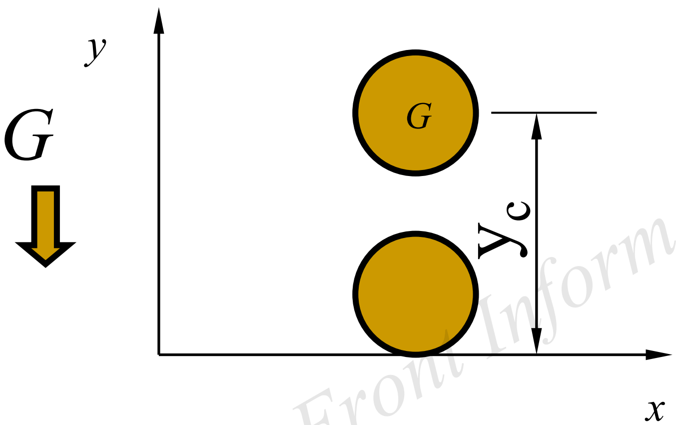
What is **Total Potential Energy**?

## Total potential energy

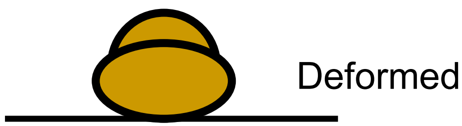
*Π*: Total energy

*V*: Potential energy, of the applied forces

*U*: Elastic strain energy, stored in the deformed body




$V=Gy_c$




*U*: Elastic strain energy

$\Pi=V+U$

# Total potential energy for packing problems

Rubber ball

Steel ball

As the parts are supposed to be rigid for the packing problems, the elastic energy can be ignored. So, $U=0$

We can not change gravity, but we can choose an *optimal* $y_c$.

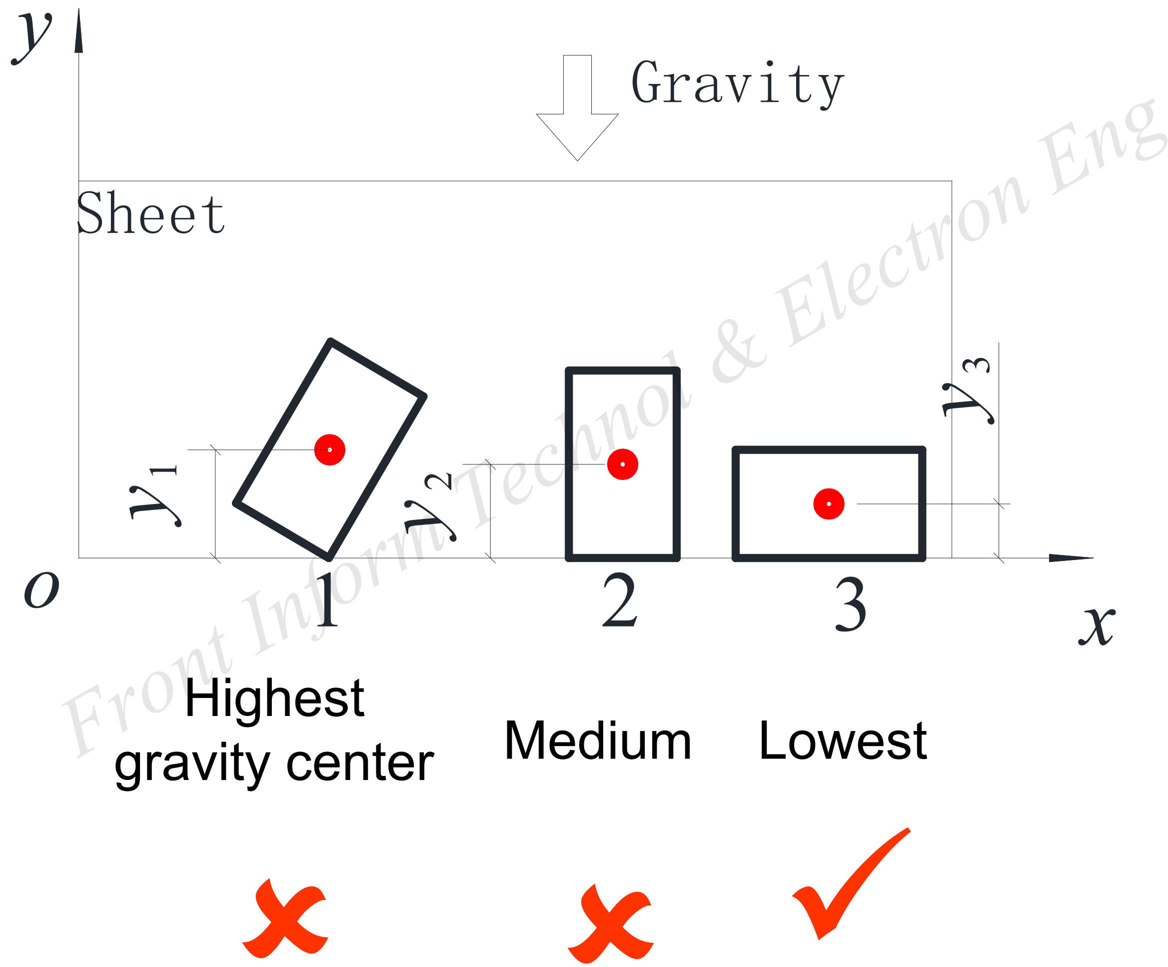
$$\Pi = U + V = V = G y_c$$

$G$: gravity

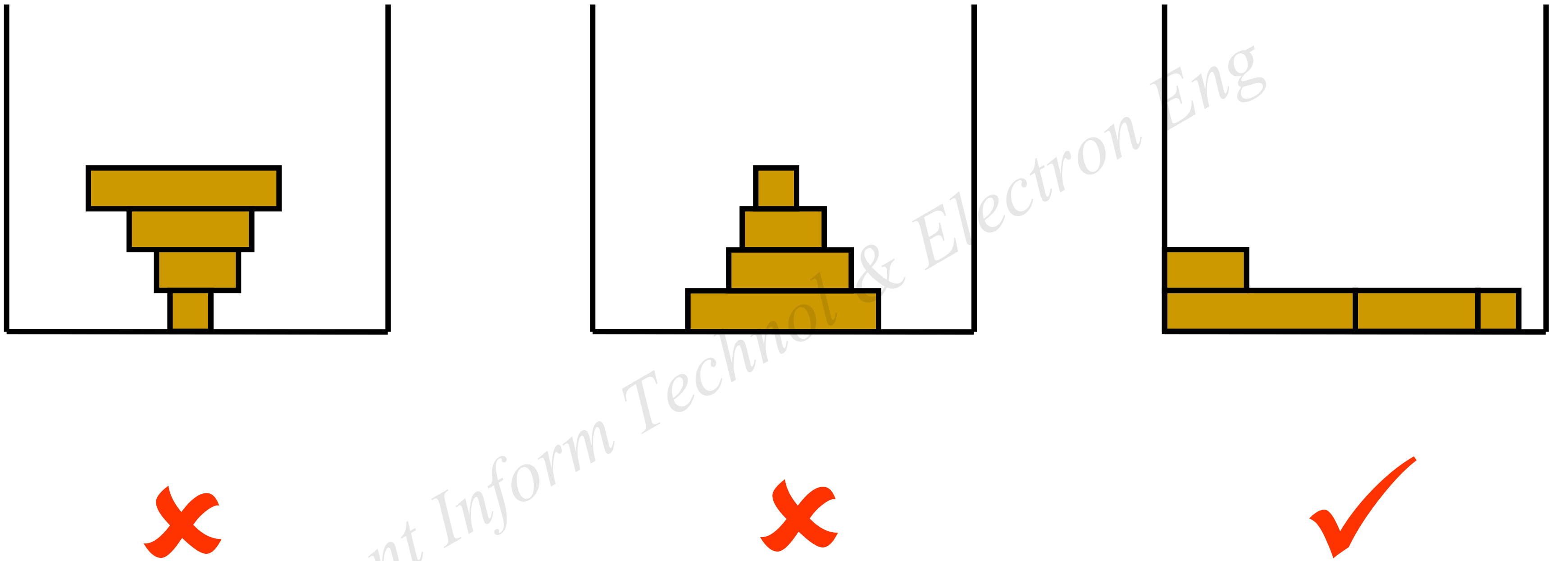
$y_c$: vertical coordinate of the center of $G$

# Principle of minimum total potential energy (3)

The tendency of all weights to lower their position is a basic law of nature.



# Which is more stable and compact?

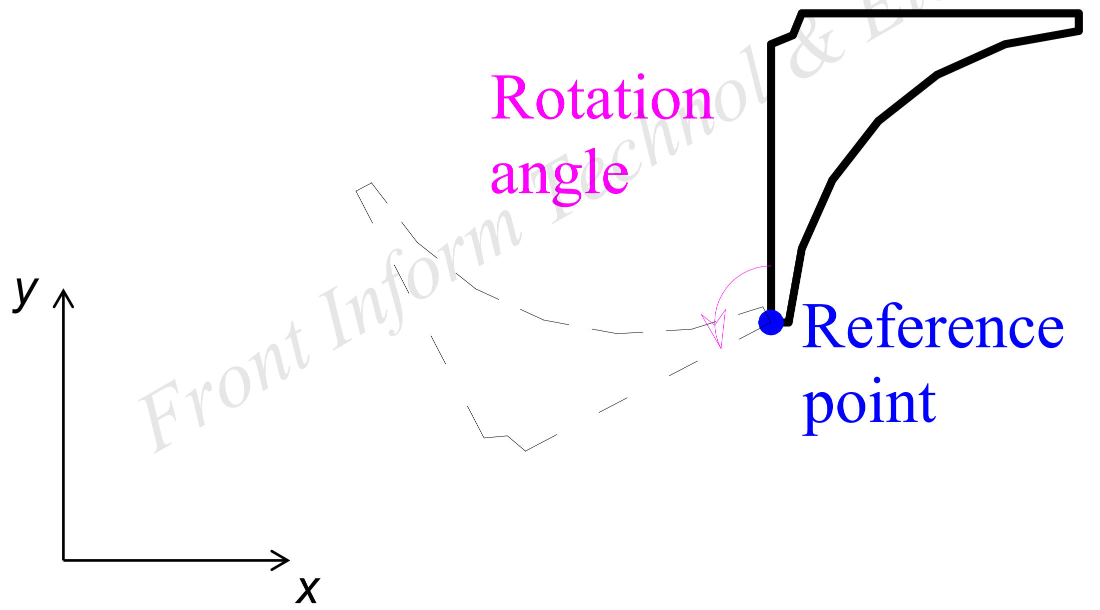


Highest gravity center

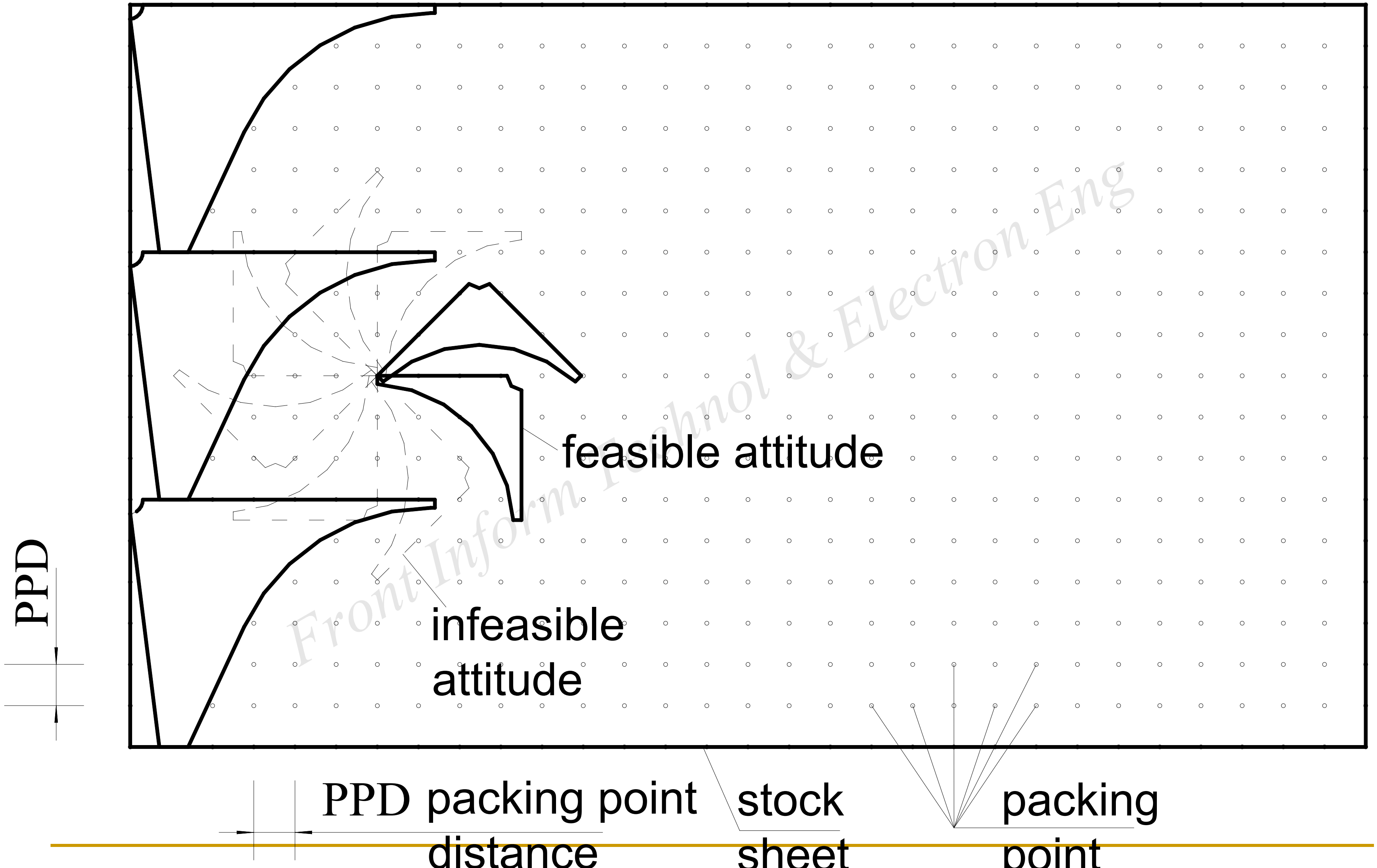
Medium

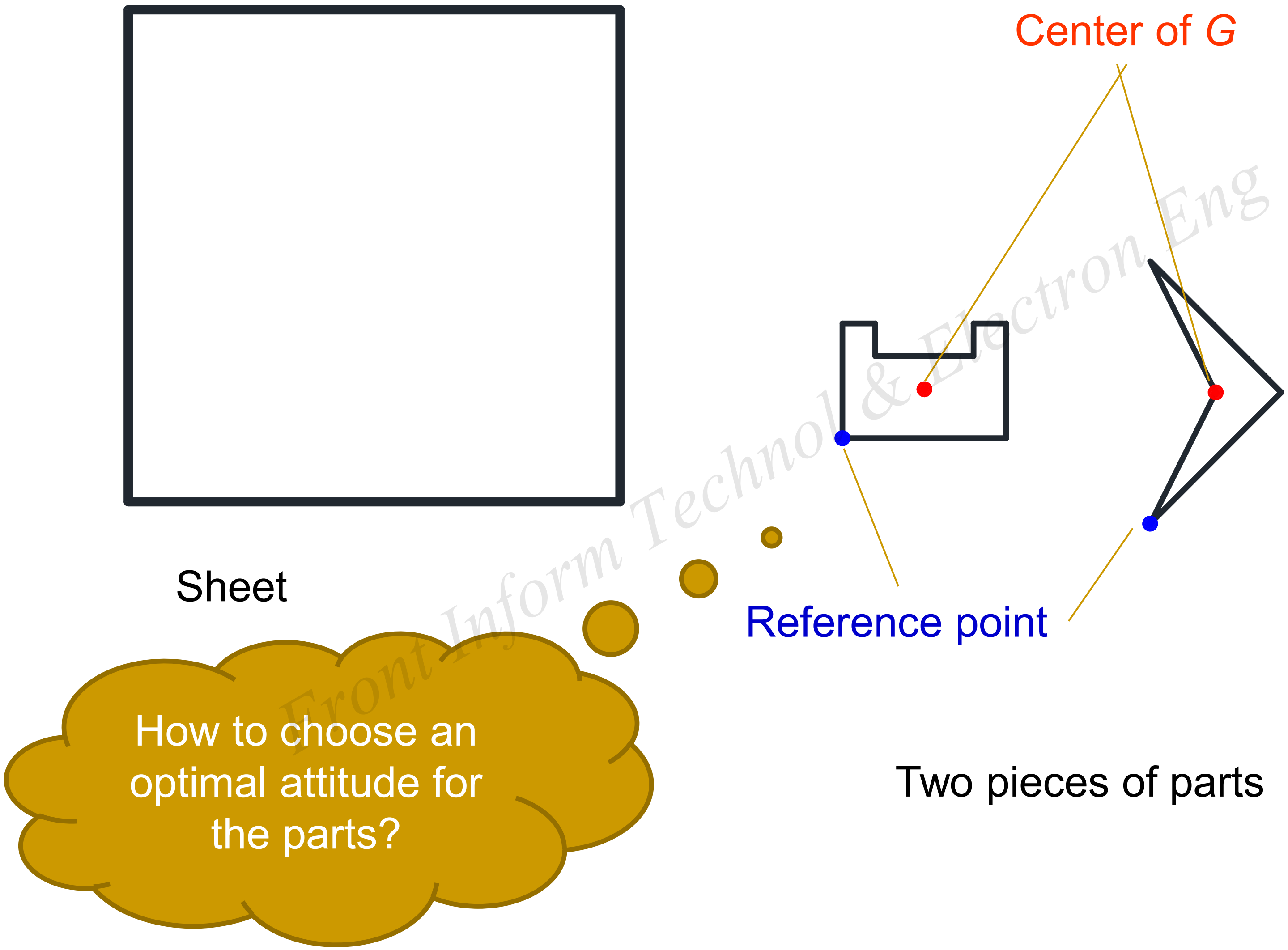
Lowest

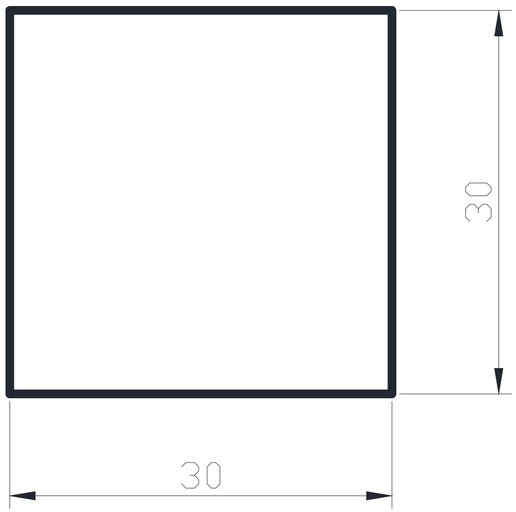
# Definition 1: attitude=reference point+rotation angle



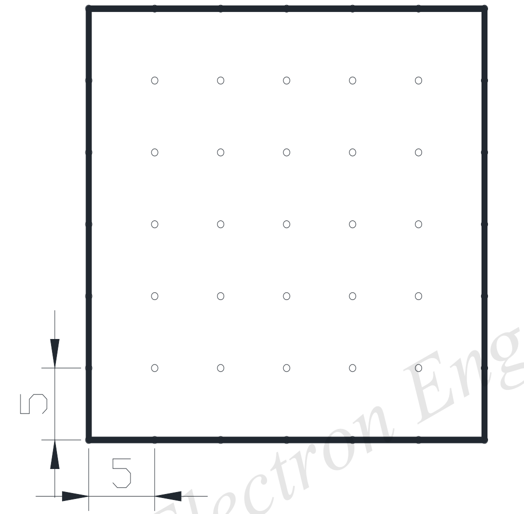
# Definition 2: feasible attitude



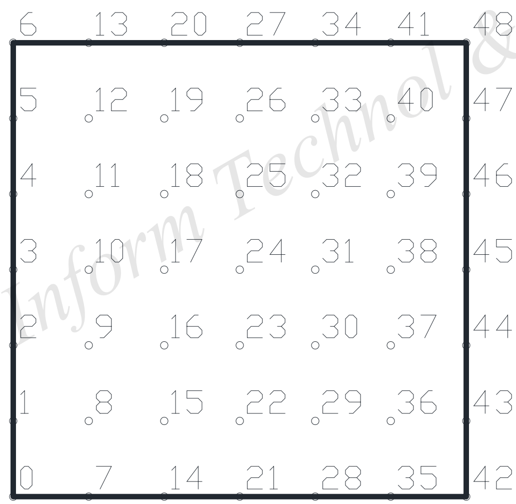

Center of G
Sheet
Reference point
Two pieces of parts
How to choose an optimal attitude for the parts?



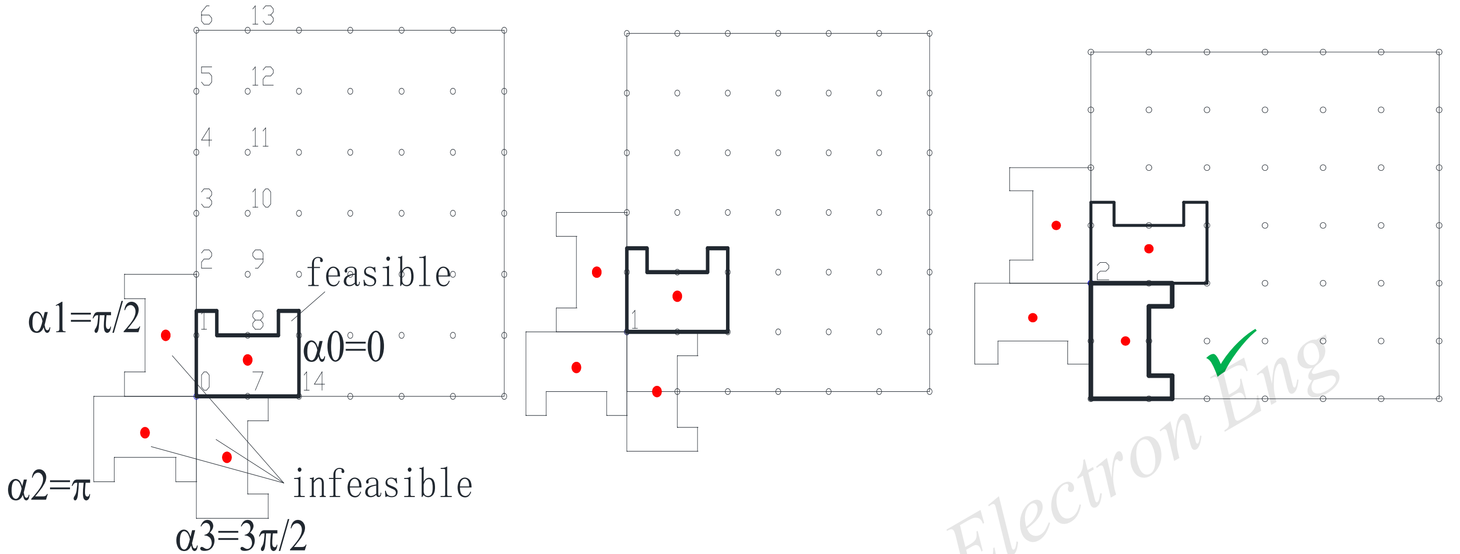

Sheet

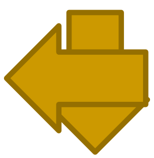



Packing points

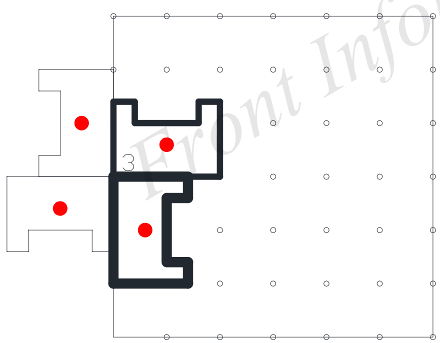


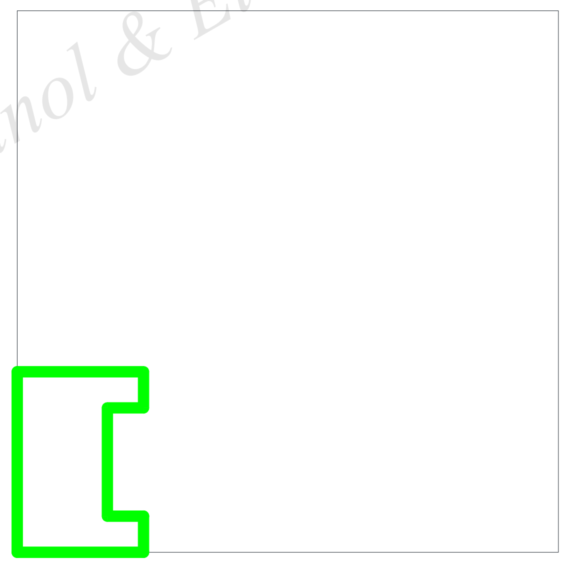

Number of packing points



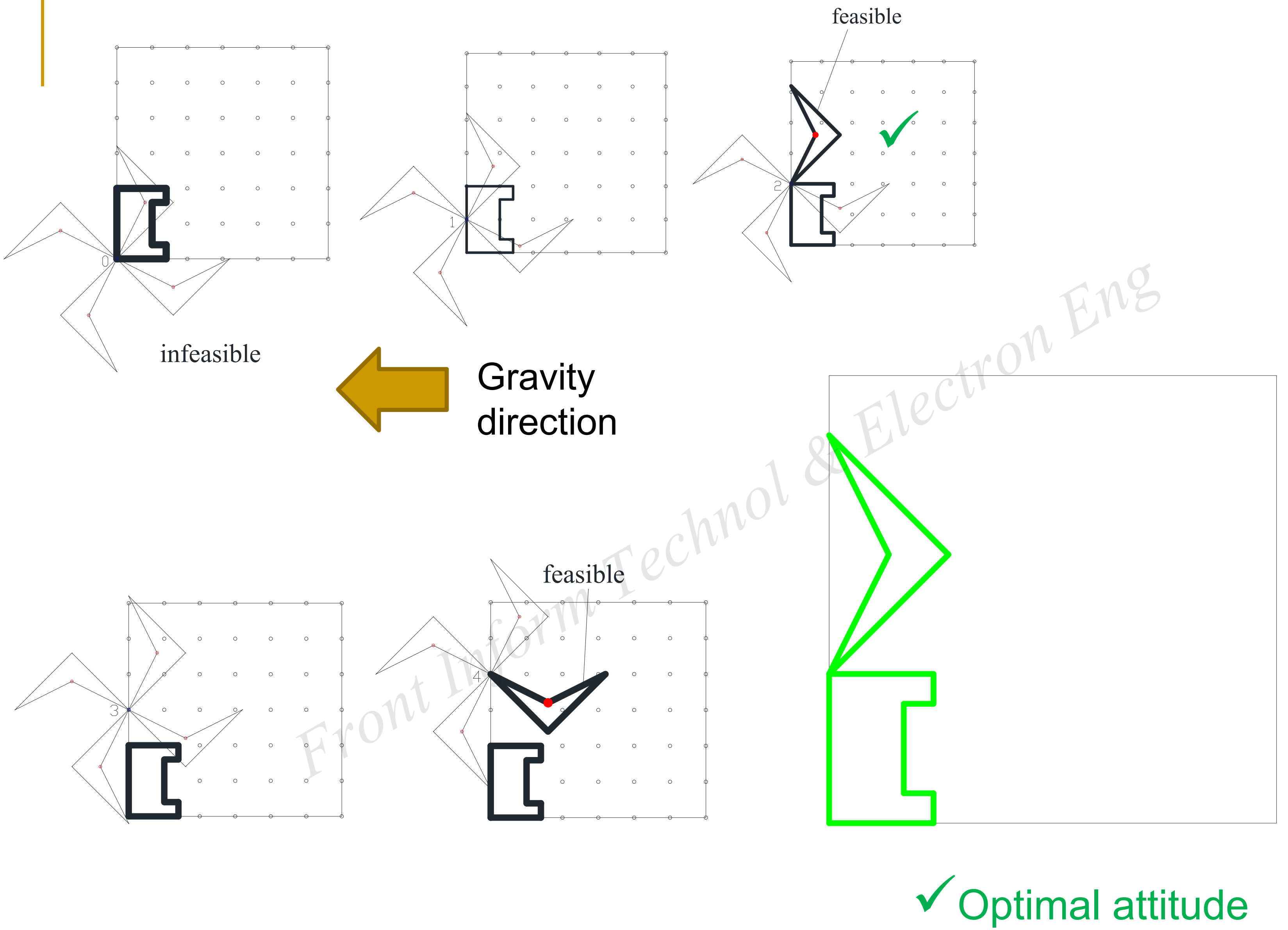



Gravity direction

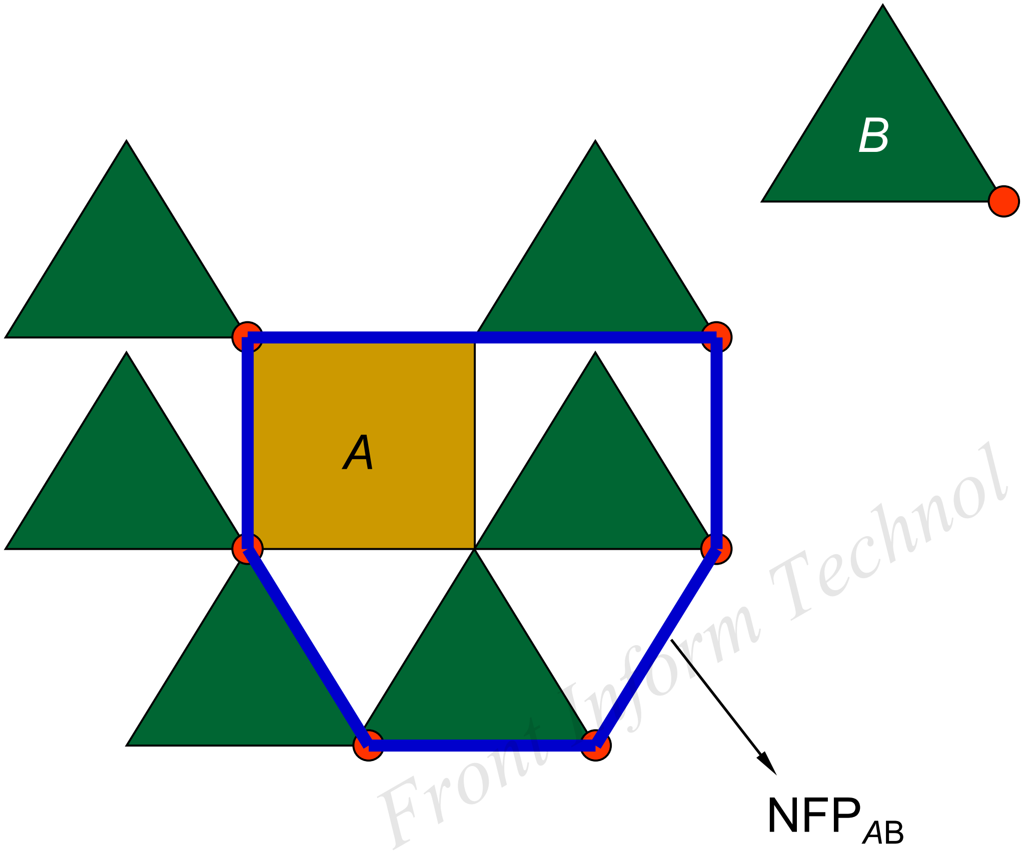


Optimal attitude


feasible
infeasible
Gravity direction
feasible
0
1
2
3
4
Optimal attitude

# NFP$_{AB}$



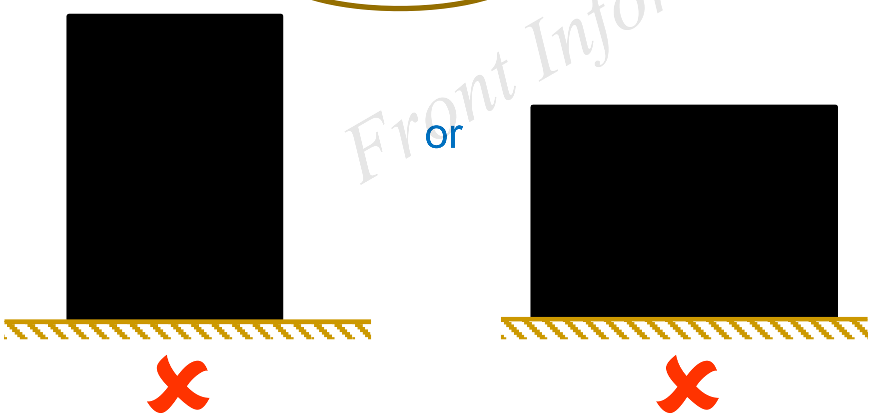

Calculation of NFP is difficult for concaved shapes.

HAPE need NOT to calculate NFP any more!

# A question

A black box on a table

Which attitude is stable/optimal according to the principle of minimum total potential energy?




This example tells us two things:

(1) The principle of minimum total potential energy is still correct in 3D world. So, we can expand HAPE to HAPE3D.

(2) The HAPE3D is much more complicate than HAPE.

# 3D irregular packing problem

The 3D irregular packing problem belongs to a general class of combinatorial optimization problems which are concerned with packing a set of irregular pieces into one or more containers to minimize the waste or maximize profit. The 3D irregular packing problem occurs in many applications, namely, container loading, human occupied vehicle (HOV) design (Fig. 1), satellite module layout design (Huo *et al.*, 2006) (Fig. 2), building layout, and 3D laser cutting.

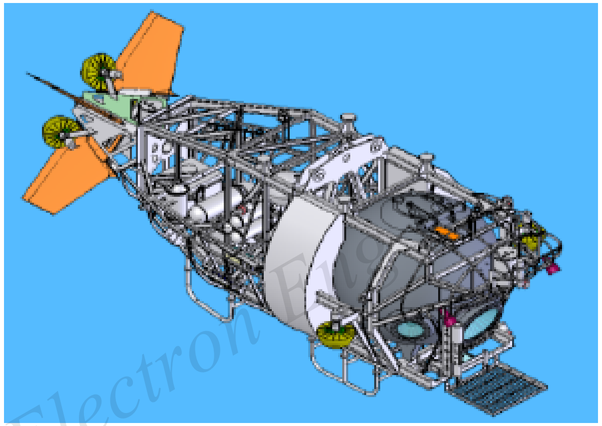


**Fig. 1 A human occupied vehicle (HOV)**

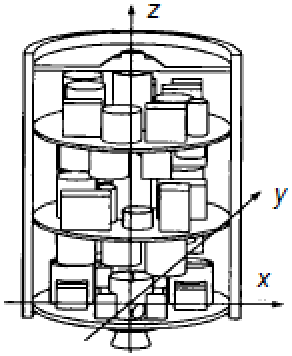
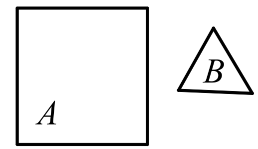



**Fig. 2 A satellite module (Huo *et al.*, 2006)**

# Polygon separation test

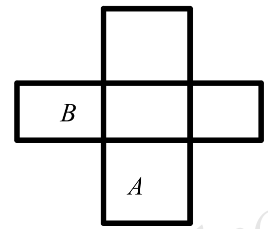
suppose there are two polygons: *A* and *B*




How to judge if *A* and *B* are separated with each other?

(1) All vertices of polygon *A* are exterior points of the polygon *B*, and vice versa.

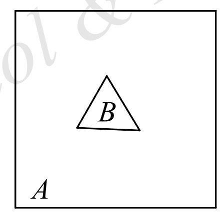
Is this enough?




When the first subset of the separation tests is satisfied, polygons *A* and *B* may still be overlapped

Both of them are equally important!

(2) All line segments of polygon *A* do not cross those of *B*, and vice versa.




When only the second subset is satisfied, vertices of *A* are outside of *B*, *A* may contain *B* completely

Another special condition.

## Polyhedron separation test

(1) All vertices of polyhedron *A* are exterior points of polyhedron *B*, and vice versa;
(2) All line segments of polyhedron *A* do not cross any face of polyhedron *B*, and vice versa.

Note the following two points:
(1) When vertices of *A* are outside B, *A* may contain *B* completely (Fig. 6a);
(2) If only the first subset of separation tests is satisfied, polyhedrons *A* and *B* may still be overlapping (Fig. 6b).

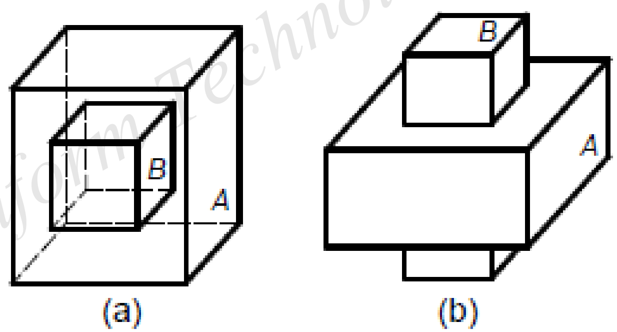
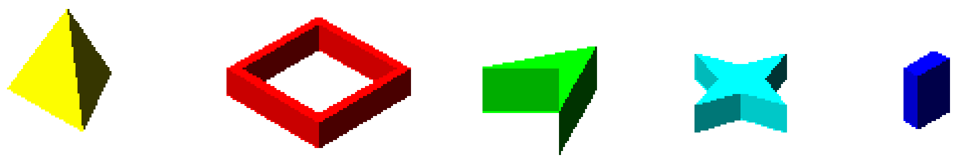


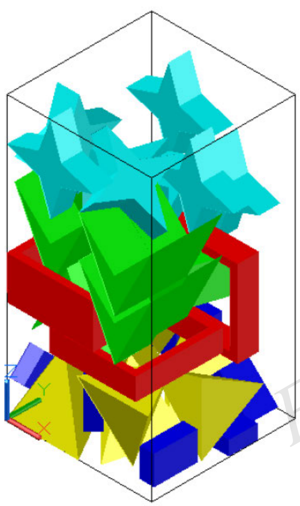

**Fig. 6 Overlapped polyhedrons satisfying only one subset of separation test: (a) *A* contains *B*; (b) *A* and *B* are overlapping**

# Example 1 for HAPE3D

Thirty-six polyhedrons of five types have to be allocated in a box-shaped container . Every polyhedron is allowed to have eight orientations (RN=8) around *x*, *y* and *z* coordinate axes, corresponding to rotation angles of 0° , 45° , 90° , …, 315° .



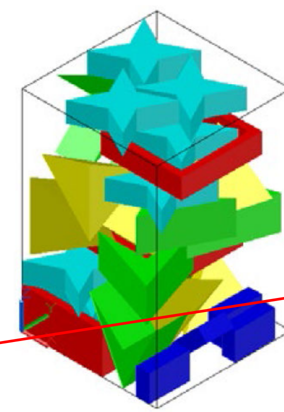
Tetrahedron Ring Arrow Star Cube



(a)
HAPE3D
*h*=39.0 mm
*t*=16.1 s

HAPE3D can also be hybridized with other heuristics, SA for example, and produce better result.

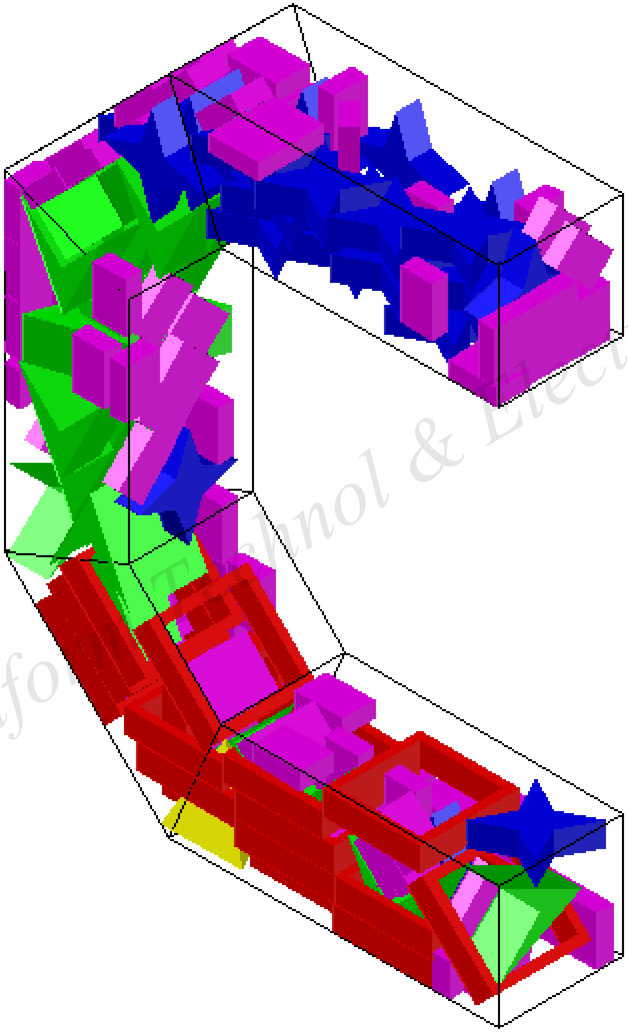
20.0% decreased!



(b) HAPE3D+SA
(500 iterations)
*h*= 31.2 mm
*t*=9637.5 s

However, it is highly time consuming!

# Example 1 for HAPE3D (Con'd)



# Example 2 for HAPE3D (comparison)

**Table 2 Comparison of HAPE3D and the method of Stoyan *et al.* (2004)**

| $n$ | Packing height (mm) | | Calculation time (s) | |
|---|---|---|---|---|
| | HAPE3D | Stoyan *et al.* (2004) | HAPE3D | Stoyan *et al.* (2004) |
| 20 | 44.7 | 43.7 | 10.0 | 0.6 |
| 30 | 62.0 | 59.0 | 21.1 | 1.2 |
| 40 | 79.9 | 81.4 | 24.5 | 2.9 |
| 50 | 92.0 | 94.6 | 42.8 | 4.3 |

Note: $n$ is the number of polyhedrons

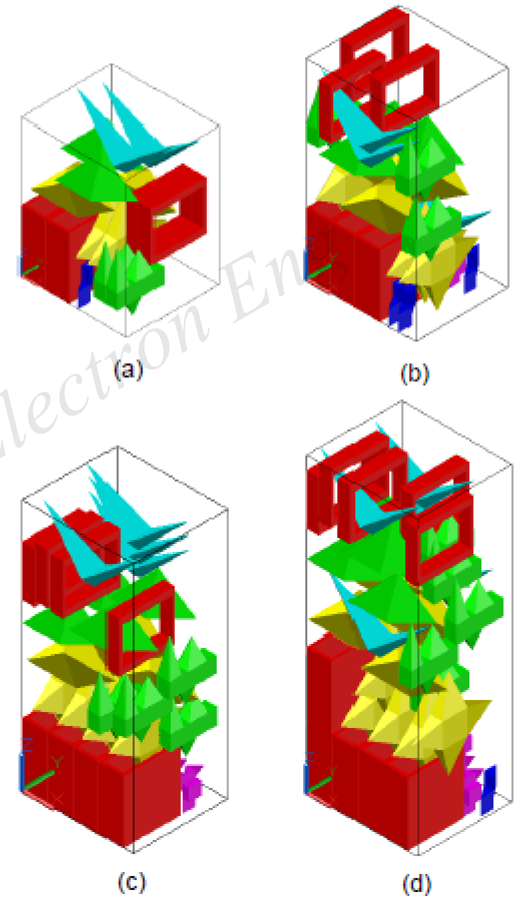



**Fig. 13 Layout generated using HAPE3D when rotation is forbidden (RN=1)**
(a) $n$=20, $h$=44.7 mm; (b) $n$=30, $h$=62 mm; (c) $n$=40, $h$=79.9 mm; (d) $n$=50, $h$=92.0 mm

# Example 2 for HAPE3D (layout by HAPE3D+SA)

**Table 3 Packing height and execution time with different RNs (HAPE3D+SA)**

| *n* | Packing height (mm) | | | Calculation time (s) | | |
|---|---|---|---|---|---|---|
| | RN=1 | RN=2 | RN=4 | RN=1 | RN=2 | RN=4 |
| 20 | 36.9 | 34.0 | 31.0 | 5921.5 | 14100.1 | 26202.1 |
| 30 | 55.6 | 48.0 | 46.0 | 10398.6 | 27745.6 | 53741.5 |
| 40 | 71.8 | 65.0 | 59.0 | 18383.9 | 48950.9 | 99952.0 |
| 50 | 92.0 | 80.1 | 73.6 | 23113.1 | 64463.0 | 125210.6 |

Note: *n* is the number of polyhedrons and RN=4 means each polyhedron is allowed to rotate around each coordinate axis at four angles (0°, 90°, 180°, and 270°)



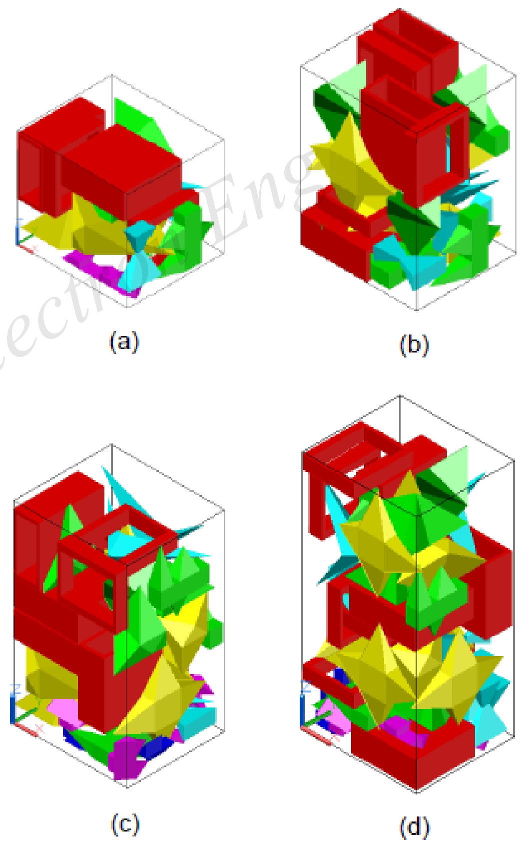



**Fig. 15 Layout by HAPE3D+SA (500 iterations, RN=4)**
(a) $n$=20, $h$=31.0 mm; (b) $n$=30, $h$=46.0 mm; (c) $n$=40, $h$=59.0 mm; (d) $n$=50, $h$=73.6 mm

# Conclusions

- HAPE3D does not need to calculate NFP anymore
- HAPE3D does not forbid the rotation freedom of the piece.
- HAPE3D can be hybridized very well with other meta-heuristic algorithms to further improve the packing efficiency.


Thank you for your attention!
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08:52:59