Li-rong Shen, Xiao-ping Li, Hai-feng Sun, Hai-yan Fang, Meng-fan Xue, 2015. A novel period estimation method for X-ray pulsars based on frequency subdivision. *Frontiers of Information Technology & Electronic Engineering*, **16**(10):858-870. [doi:10.1631/FITEE.1500052]

A novel period estimation method for X-ray pulsars based on frequency subdivision

Key words: Pulsar navigation, Period estimation, Frequency subdivision, Continuous Lomb periodogram

Contact: Li-rong Shen
E-mail: slr_xidian@163.com
ORCID: http://orcid.org/0000-0002-9131-1079

Introduction

- Since an X-ray pulsar signal is extremely weak when it reaches a spacecraft, in X-ray pulsar based navigation, pulsar period with high precision is important for folding the pulse profile and estimating the TOA.

- To the best of our knowledge, frequency estimation precision of existing models is generally below 10^{-5} Hz, and period estimation precision is generally below 10^{-8} s in a short-time span of observation. They are not able to fulfill the requirement for high precision measurements, especially in the case of short-term observation with low SNR.

- In this paper, we propose a new local frequency subdivision based method. This method is able to significantly improve precision of period estimation for X-ray pulsars only at the cost of a slight increase in computational complexity. Experimental results show that the proposed method achieves a high period estimation precision of 10^{-9} s when the observation time is 132 s.

Method

We set up the signal model as follows:

$$y_i = a\cos(\omega t_i) + b\sin(\omega t_i) + n_i \qquad (1)$$

Then we define a discrete Fourier transform for X-ray pulsar signals as follows:

$$X(f) = \sqrt{N_0}\sum_{i=1}^{N_0} y_i\left[A\cos(2\pi f t_i) + \mathrm{j}B\sin(2\pi f t_i)\right] \qquad (2)$$

So the continuous Lomb periodogram (CLP) is

$$\begin{aligned} P(f) &= (1/N_0)\cdot|X(f)|^2 \\ &= \left[A\sum_i y_i\cos(2\pi f t_i)\right]^2 + \left[B\sum_i y_i\sin(2\pi f t_i)\right]^2 \end{aligned} \qquad (3)$$

Method (Con'd)

To obtain a similar analysis with the fast Lomb method, we choose

$$A(f) = \left[\sum_i \cos^2(2\pi f t_i) \right]^{-1/2} \quad (4)$$

$$B(f) = \left[\sum_i \sin^2(2\pi f t_i) \right]^{-1/2} \quad (5)$$

Substituting Eqs. (4) and (5) into Eq. (3), we obtain

$$P(f) = \frac{1}{\sigma^2} \left\{ \frac{\left[\sum_i (y_i - \bar{y}) \cos(2\pi f (t_i - \tau)) \right]^2}{\sum_i \cos^2(2\pi f (t_i - \tau))} + \frac{\left[\sum_i (y_i - \bar{y}) \sin(2\pi f (t_i - \tau)) \right]^2}{\sum_i \sin^2(2\pi f (t_i - \tau))} \right\} \quad (6)$$

In order to make *P* (*f*) completely independent of all the t_i's shifting by any constant for each frequency *f* of interest, compute a time-offset *τ* by

$$\tau(f) = \frac{1}{2 \times 2\pi f} \arctan \left[\frac{\sum_i \sin(2 \times 2\pi f t_i)}{\sum_i \cos(2 \times 2\pi f t_i)} \right] \quad (7)$$

Method (Con'd)

Frequency subdivision values are calculated using

$$f_{\mathrm{mid}\_i}(j)=\begin{cases}\mathrm{floor}(f_{\mathrm{first}})+j\cdot\Delta f, & i=1,\\ \dfrac{\mathrm{floor}(f_{i\_\mathrm{best}}\cdot m^{i-1})}{m^{i-1}}+j\cdot\Delta f, & i\geq 2,\end{cases} \quad (8)$$

$$\Delta f = m^{-(i+1)} \quad (9)$$

Therefore, the best frequency corresponding to the significant peak of the CLP $P_i(f_j)$ will determine the higher precision period for X-ray pulsar:

$$T = 1/f_{i\_\mathrm{best}} = 1\Big/\arg\max_{f_j\in[f_{1i},f_{2i}]} P_i(f_j) \quad (10)$$

The period estimation mean error (PEME) is defined as follows:

$$\mathrm{PEME}(s)=\frac{1}{N}\sum_{i=1}^{N}|s_c - s_e| \quad (11)$$

where s_c stands for the real pulsar period or the real pulsar frequency, s_e stands for the estimated pulsar period or the estimated pulsar frequency, and N is the number of times that period estimation is undertaken.

Experimental results

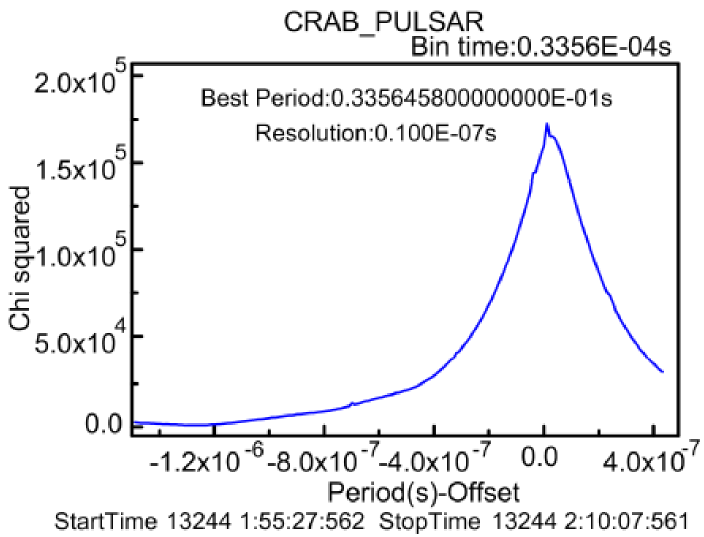

Fig. 1 Period estimation using efsearch in HEAsoft

Table 1 Difference between the raw and preprocessed photon TOAs

Item	Raw	Preprocessed
NUM	9 091 449	7 653 921
T_{obs} (s)	880	738.9009
P_{est} (s)	0.033 564 594 7	0.033 566 924 4
F_{est} (Hz)	29.793 298 75	29.791 230 99

NUM: number of the photons; T_{obs}: observation time; P_{est}: estimated period; F_{est}: estimated frequency

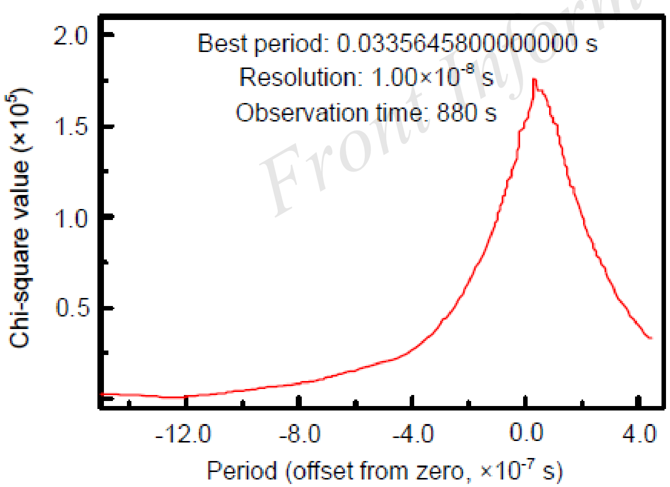

Fig. 2 Period estimation using the extended efsearch in MATLAB (Crab pulsar, bin time: 3.356×10^{-5} s)

Table 2 PEME of the Crab pulsar using four methods

Method	PEME(f) (Hz)	PEME(p) (s)
Fast Lomb	4.02×10^{-4}	4.53×10^{-7}
FFT	1.29×10^{-3}	1.45×10^{-6}
Efsearch	6.70×10^{-5}	7.54×10^{-8}
CLP	2.15×10^{-6}	2.42×10^{-9}

Experimental results (Con'd)

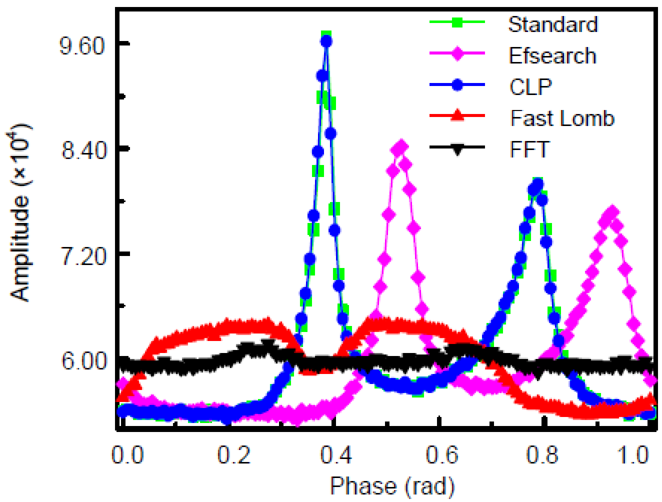

Fig. 3 Comparison between the standard profile and integrated pulse profiles

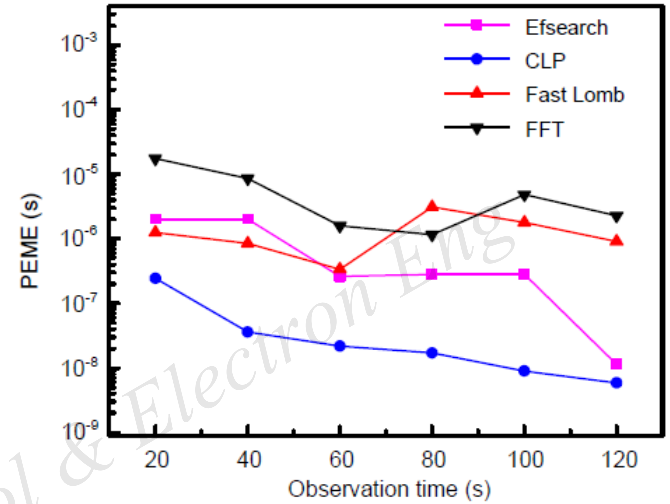

Fig. 5 PEME of the four methods using the data of packet 90802-02-06-00

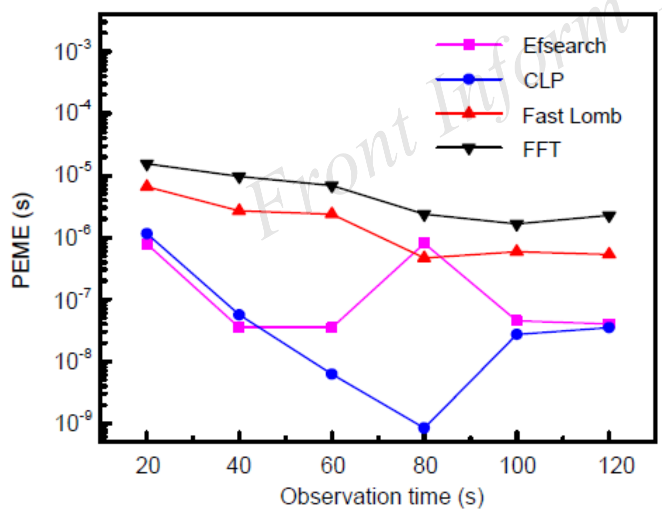

Fig. 4 PEME of the four methods using the data of packet 90802-02-02-00

Table 3 PEME(p) of the four methods

Observation time (s)	PEME(p) (s)			
	FFT	Fast Lomb	Efsearch	CLP
20	1.54×10^{-5}	1.45×10^{-5}	7.72×10^{-6}	8.88×10^{-7}
40	6.66×10^{-6}	6.71×10^{-6}	3.58×10^{-6}	2.91×10^{-7}
60	4.49×10^{-6}	4.53×10^{-6}	9.07×10^{-7}	1.33×10^{-7}
80	4.01×10^{-6}	3.98×10^{-6}	7.28×10^{-7}	1.29×10^{-7}
100	2.88×10^{-6}	2.85×10^{-6}	1.28×10^{-7}	5.36×10^{-8}
120	1.94×10^{-6}	1.93×10^{-6}	6.38×10^{-8}	4.40×10^{-8}

Experimental results (Con’d)

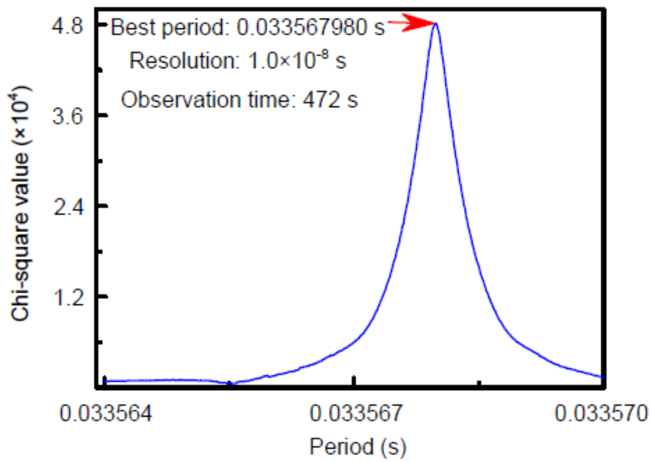

Fig. 6 High precision period estimated using the long-time observation data of packet 90802-02-06-00 (Crab pulsar, bin time: 3.356×10^{-5} s)

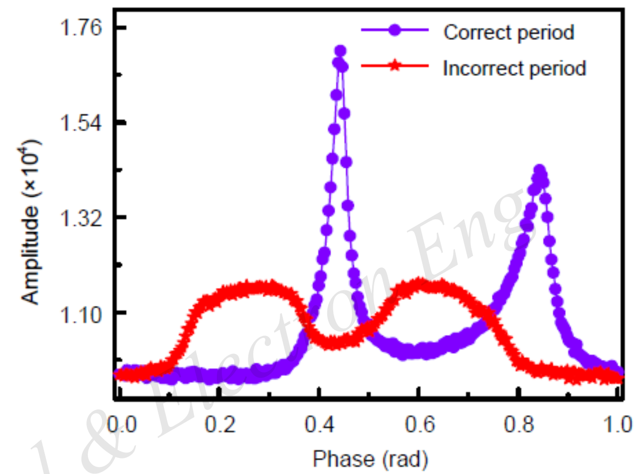

Fig. 8 Integrated profiles folded using correct and incorrect periods

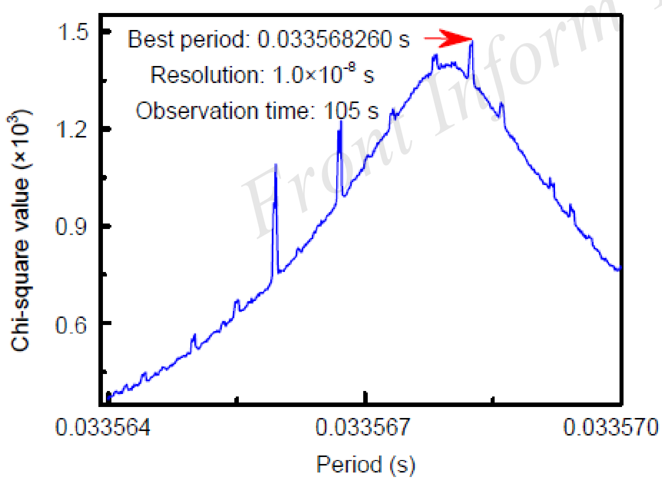

Fig. 7 Low precision period estimated using the short-time observation data of packet 90802-02-06-00 (Crab pulsar, bin time: 3.356×10^{-5} s)

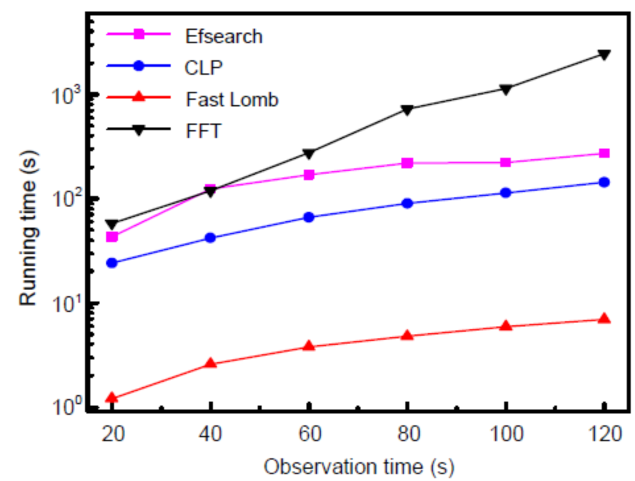

Fig. 9 Mean running time of the four methods

Conclusions

- The period estimation precision of the proposed method is 1–3 orders of magnitude higher than that of the fast Lomb method and FFT method, and that the proposed method is more effective than efsearch when using the same amount of data.
- The proposed method also indicates that the observed data within 120–130 s can be used to estimate and update the period for XPNAV. This period will significantly improve the precision of the folding profile and time delay estimation as well as position and velocity measurement in deep space exploration based on X-ray pulsar navigation.
- The method is also applicable when seeking to measure the rotation period of other variable stars and celestial bodies.