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# $H_\infty$ reference tracking control design for a class of nonlinear systems with time-varying delays

**Key words:** $H_\infty$ reference tracking, nonlinear systems, Nonlinear systems, State feedback control, Time-varying delays, Unified model.

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# Introduction

- Tracking control has been widely applied to practical engineering processes, but most of them concentrate on stabilization rather than tracking issues.
- It is not easy to achieve the tracking performances for highly nonlinear systems with time-varying delays and external disturbances. Some methods do exist but no unified approach is proposed in terms of different nonlinear systems.
- A unified method is established based on a unified model to deal with the tracking control issue of different nonlinear systems with time-varying delays.
- $H_\infty$ reference tracking control design for a class of nonlinear systems with time-varying delays has been presented in this work.

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# Method

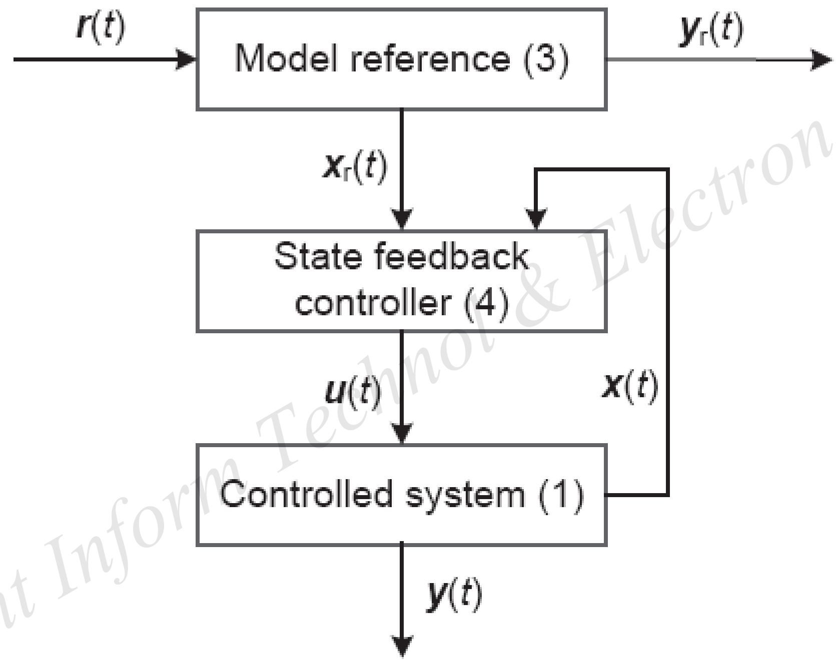


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Fig. 1 Diagram of the closed-loop system

# Main results: stability analysis

**Theorem 1** Given $d \geq 0$, $\mu \geq 0$, and $\gamma > 0$, if there exist symmetric positive definite matrices $\boldsymbol{P}$, $\boldsymbol{Q}_1$, $\boldsymbol{Q}_2$, $\boldsymbol{R}$, a diagonal positive definite matrix $\boldsymbol{\Sigma}$, and a symmetric matrix $\boldsymbol{S}$ that satisfy the following linear matrix inequalities (LMIs):

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13} & \boldsymbol{M}_{14} & \boldsymbol{M}_{15} \\ * & \boldsymbol{M}_{22} & \boldsymbol{M}_{23} & \boldsymbol{M}_{24} & \boldsymbol{M}_{25} \\ * & * & \boldsymbol{M}_{33} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & \boldsymbol{M}_{44} & \boldsymbol{M}_{45} \\ * & * & * & * & \boldsymbol{M}_{55} \end{bmatrix} < 0,$$

$$\begin{bmatrix} \boldsymbol{R} & \boldsymbol{S} \\ * & \boldsymbol{R} \end{bmatrix} > 0,$$

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# Main results: stability analysis

where

$$
\begin{aligned}
M_{11} &= P\widetilde{A} + \widetilde{A}^{\mathrm{T}}P + Q_1 + Q_2 - R \\
&\quad + \widetilde{C}_y^{\mathrm{T}}T\widetilde{C}_y + d^2\widetilde{A}^{\mathrm{T}}R\widetilde{A}, \\
M_{12} &= P\widetilde{A}_d + R - S + d^2\widetilde{A}^{\mathrm{T}}R\widetilde{A}_d, \\
M_{13} &= S,\ M_{14} = P\widetilde{B}_p + \widetilde{C}_q^{\mathrm{T}}\Sigma H + d^2\widetilde{A}^{\mathrm{T}}R\widetilde{B}_p, \\
M_{15} &= \widetilde{C}_y^{\mathrm{T}}T\widetilde{D}_{yw} + P\widetilde{B}_w + d^2\widetilde{A}^{\mathrm{T}}R\widetilde{B}_w, \\
M_{22} &= -(1-\mu)Q_2 - 2R + 2S + d^2\widetilde{A}_d^{\mathrm{T}}R\widetilde{A}_d,
\end{aligned}
$$

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# Main results: stability analysis

$$M_{23} = R - S,\ M_{24} = \widetilde{C}_{qd}^{\mathrm{T}} \Sigma H + d^2 \widetilde{A}_d^{\mathrm{T}} R \widetilde{B}_p,$$
$$M_{25} = d^2 \widetilde{A}_d^{\mathrm{T}} R \widetilde{B}_w,\ M_{33} = -Q_1 - R,$$
$$M_{44} = D_p^{\mathrm{T}} \Sigma H + H \Sigma D_p - 2\Sigma + d^2 \widetilde{B}_p^{\mathrm{T}} R \widetilde{B}_p,$$
$$M_{45} = d^2 \widetilde{B}_p^{\mathrm{T}} R \widetilde{B}_w + H \Sigma \widetilde{D}_w,$$
$$M_{55} = -\gamma^2 I + \widetilde{D}_{yw}^{\mathrm{T}} T \widetilde{D}_{yw} + d^2 \widetilde{B}_w^{\mathrm{T}} R \widetilde{B}_w,$$
$$H = \mathrm{diag}(h_1,\ h_2,\ \ldots,\ h_L),$$

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then the controlled system is asymptotically stable without external disturbances , and the prescribed $L_2$ gain is derived.

# Main results: controller design

**Theorem 2** Given $d \geq 0$, $\mu \geq 0$, and $\gamma > 0$, if there exist symmetric positive definite matrices $\boldsymbol{X}$, $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$, $\boldsymbol{Y}_3$, $\boldsymbol{Y}_4$, a diagonal positive definite matrix $\boldsymbol{V}$, and a matrix $\boldsymbol{W}$ that satisfy Eq. (17) (see the next page) and the following LMIs:

$$\begin{bmatrix} \boldsymbol{Y}_3 & \boldsymbol{Y}_4 \\ * & \boldsymbol{Y}_3 \end{bmatrix} > 0,$$

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# Main results: controller design

$$N_1 = \begin{bmatrix} \varXi_3 & \widetilde{A}_d X + Y_3 - Y_4 & Y_4 & \varXi_4 & \widetilde{B}_w & \varXi_6 & \varXi_7 \\ * & -(1-\mu)Y_2 - 2Y_3 + 2Y_4 & Y_3 - Y_4 & X\widetilde{C}_{qd}^{\mathrm{T}} H & 0 & dX\widetilde{A}_d^{\mathrm{T}} & 0 \\ * & * & -Y_1 - Y_3 & 0 & 0 & 0 & 0 \\ * & * & * & \varXi_5 & H\widetilde{D}_w & dV\widetilde{B}_p^{\mathrm{T}} & 0 \\ * & * & * & * & -\gamma^2 I & d\widetilde{B}_w^{\mathrm{T}} & \widetilde{D}_{yw}^{\mathrm{T}} T \\ * & * & * & * & * & -X - X^{\mathrm{T}} + Y_3 & 0 \\ * & * & * & * & * & * & -T \end{bmatrix} < 0, \tag{17}$$

$$\varXi_3 = (\bar{A}X + \bar{B}_u W)^{\mathrm{T}} + \bar{A}X + \bar{B}_u W + Y_1 + Y_2 - Y_3, \ \ \varXi_4 = \widetilde{B}_p V + X\bar{C}_q^{\mathrm{T}} H + W^{\mathrm{T}} D_u^{\mathrm{T}} H,$$

$$\varXi_5 = -2V + VD_p^{\mathrm{T}} H + HD_p V, \ \ \varXi_6 = dX\bar{A}^{\mathrm{T}} + dW^{\mathrm{T}}\bar{B}_u^{\mathrm{T}}, \ \ \varXi_7 = X\bar{C}_y^{\mathrm{T}} T - W^{\mathrm{T}} D_{yu}^{\mathrm{T}} T.$$

then the controlled system is asymptotically stable without external disturbances, and the prescribed $L_2$ gain is derived. Furthermore, the controller gain is as follows:

$$K = WX^{-1}.$$

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# Simulation results: sinusoidal signal

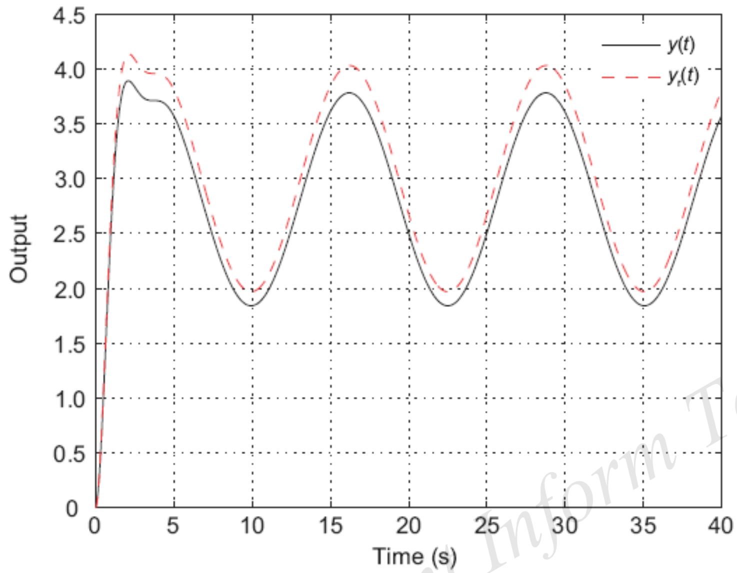


**Fig. 2** The output of system (31) and the reference input (33) when $k = 8$

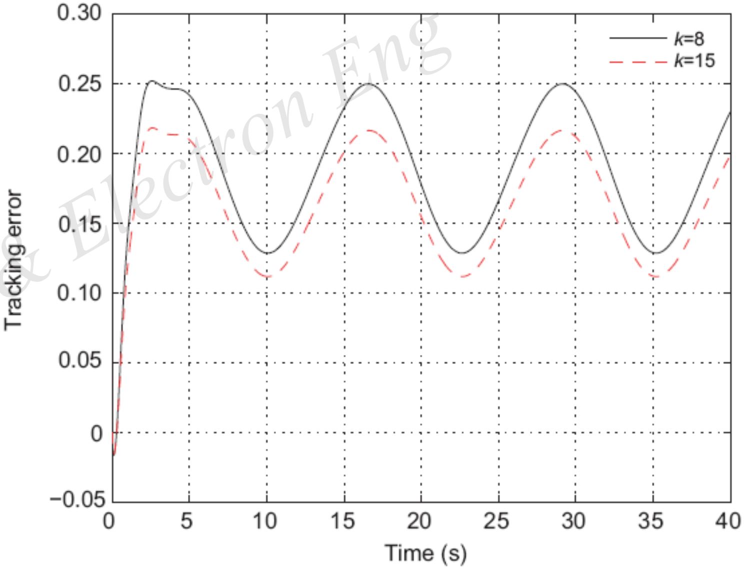


**Fig. 3** Tracking errors in the cases of $k = 8$ and $k = 15$, respectively, with reference input (33)

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# Simulation results: square signal

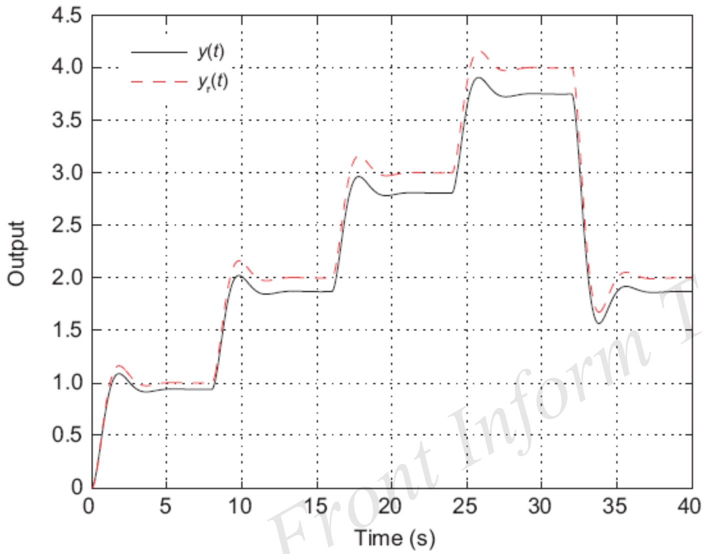


**Fig. 4** The output of system (31) and the reference input (34) when $k = 8$

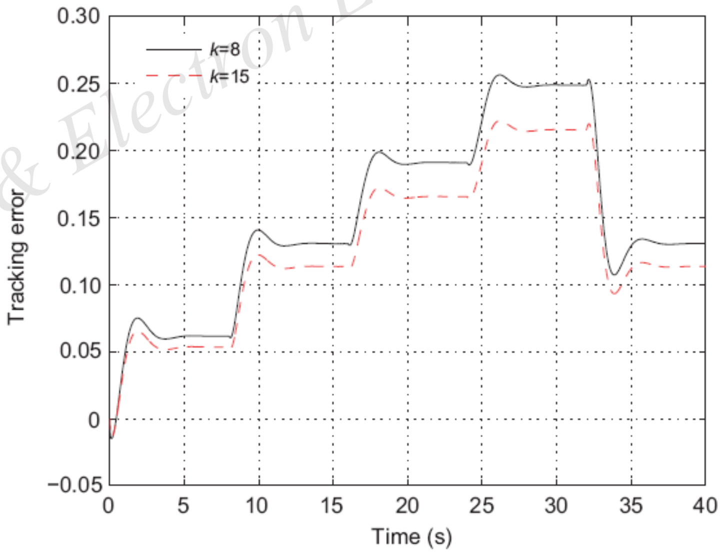


**Fig. 5** Tracking errors in the cases of $k = 8$ and $k = 15$, respectively, with reference input (34)

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# Conclusions

- A state feedback controller is designed which guarantees that the closed-loop system is stable and the output of the system tracks the model reference trajectory in the $H_\infty$ sense even in case of unknown time-varying delays.
- The reference model used in this work is more flexible than previous ones and the scaling factor introduced as a positive integer reduces the tracking error effectively .

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