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# Controllability analysis of second-order multi-agent systems with directed and weighted interconnection

**Key words:** Multi-agent systems, Controllability, Directed tree

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# Introduction

- We consider the controllability problem of second-order multi-agent systems with directed and weighted interconnection and aim to obtain easy-to-verify criteria for controllability checking in terms of the interaction weights.
- A very simple necessary and sufficient condition regarding the weighting scheme is obtained in this scenario.
- A type of topology is concerned with a more general graph that can be reduced to a directed tree by contracting a cluster of nodes to a component.

# The topology investigated in this work

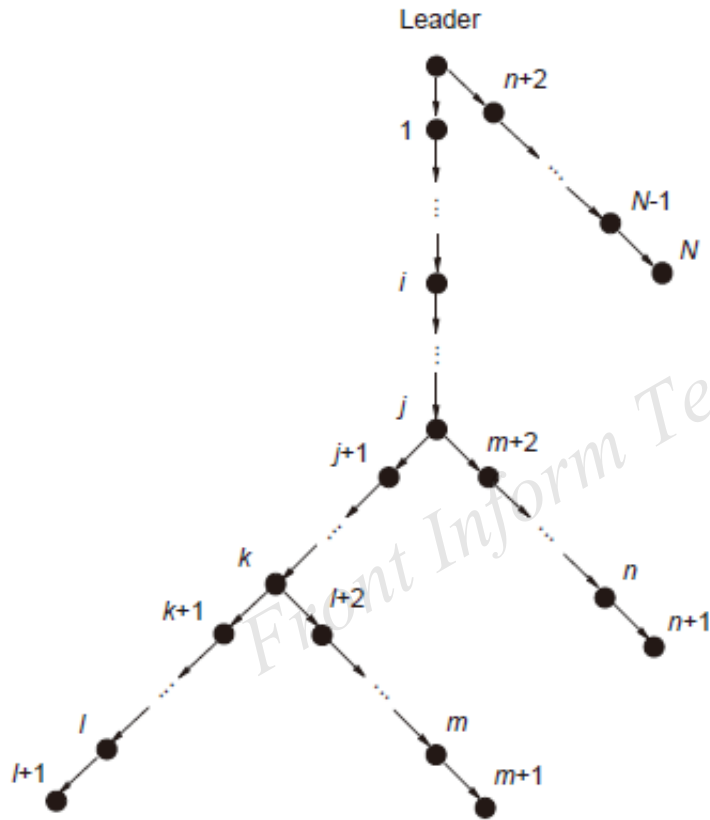


Fig. 1 A directed tree

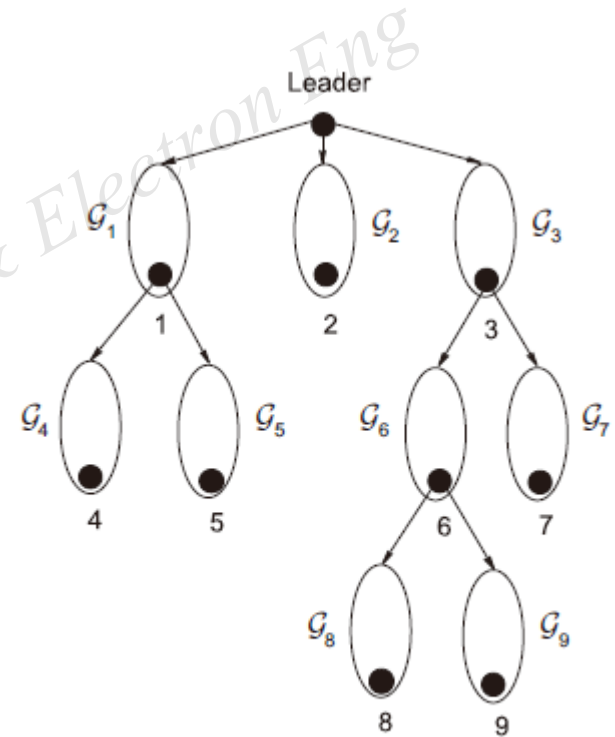
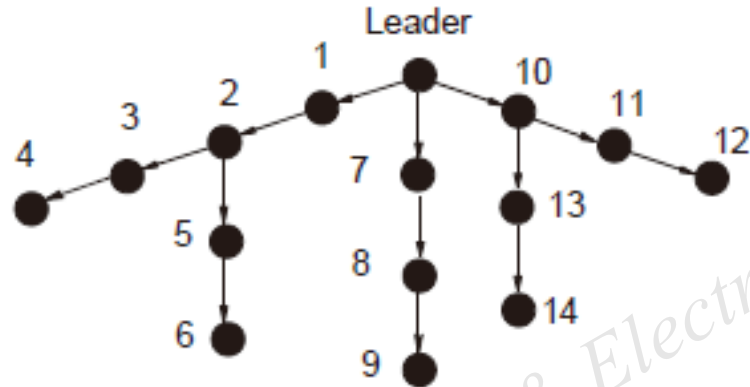


Fig. 2 A contracted tree

# Main results

- For a directed tree, choosing different weights for agents at different branches, the controllability can be ensured.
- A contracted tree is controllable if and only if
  - Every subgraph is controllable.
  - The components in different branches have different eigenvalues.



**Fig. 8 Example 1: a directed tree**

- Choosing different weights for agents at different branches, the system is controllable.
- On the other hand, choose  $w_{12} = w_4$  the system is not controllable.

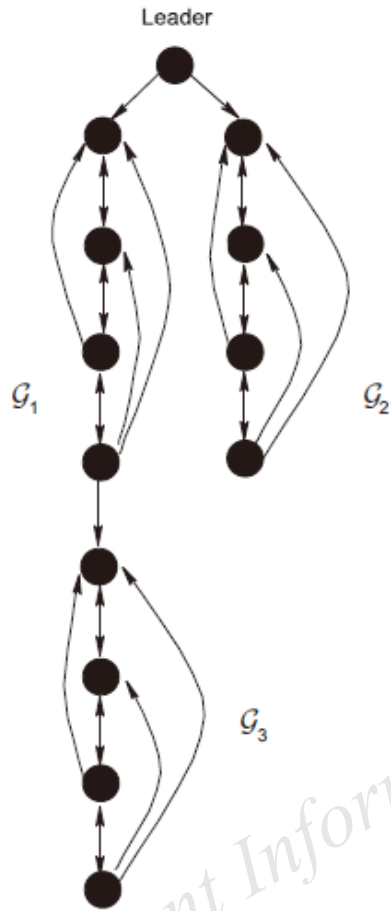


Fig. 9 Example 2: a contracted tree with feedback arcs

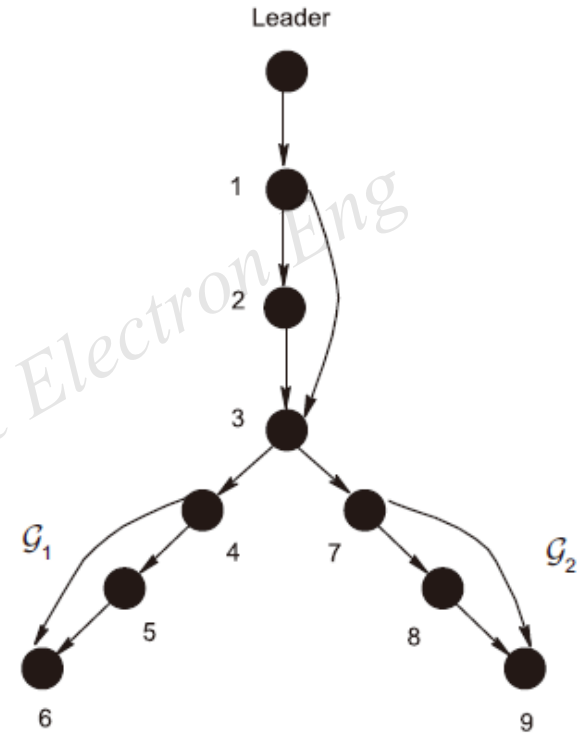


Fig. 10 Example 3: a contracted tree with feedforward arcs

The above two examples also show that the results of this work are correct.

# Conclusions

- This article investigates the controllability problem of second-order multi-agent systems over directed trees and contracted trees.
- Necessary and sufficient controllable conditions are obtained, which are easy to be verified.