Zhao Wang, Zhong-xuan Luo, Jie-lin Zhang, Emil Saucan, 2016. ARAP++: an extension of the local/global approach to mesh parameterization. *Frontiers of Information Technology & Electronic Engineering*, **17**(6):501-515. http://dx.doi.org/10.1631/FITEE.1500184



# ARAP++: an extension of the local/global approach to mesh parameterization

**Key words:** Mesh parameterization, Convex combination weights, Stretch operator, Jacobian matrix

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# Motivation

- Mesh parameterization is an important research topic in computer graphics (CG), and it has been widely used in digital geometry processing tasks, such as texture mapping, surface fitting, and surface remeshing.
- We propose a novel local/global mesh parameterization approach, ARAP++. Our work is inspired mainly by the as-rigid-as-possible (ARAP) approach and the convex combination approach. ARAP++ adopts the idea of ARAP regarding the approximation of the Jacobian matrix with a fitting matrix, and then achieves the global flattened result by stitching together the local 1-ring patches. To optimize the local spring energy, we introduce the convex combination weights and stretch operator to ARAP++.

# Main idea

- We propose a novel local/global parameterization, ARAP++, for single- and multi-boundary triangular meshes. It is an extension of the as-rigid-as-possible (ARAP) approach, which stitches together 1-ring patches instead of individual triangles. To optimize the spring energy, we introduce a linear iterative scheme which employs convex combination weights and a fitting Jacobian matrix corresponding to a prescribed family of transformations.

- Our algorithm is simple, efficient, and robust. The geometric properties (angle and area) of the original model can also be preserved by appropriately prescribing the singular values of the fitting matrix. To reduce the area and stretch distortions for high-curvature models, a stretch operator is introduced.

# Method

1. We devise ARAP++ based on the optimization of spring energy, and analyze the relation to the ARAP approach which is based on the optimization of the Dirichlet energy. In addition, a broader class of convex combination weights are considered in our method.
2. We give a simple and fast calculation method to obtain an authalic fitting matrix.
3. To deal with high-curvature models, we introduce a stretch operator to ARAP++.

# Major results (1)

- ARAP++ achieves a better result in texture mapping

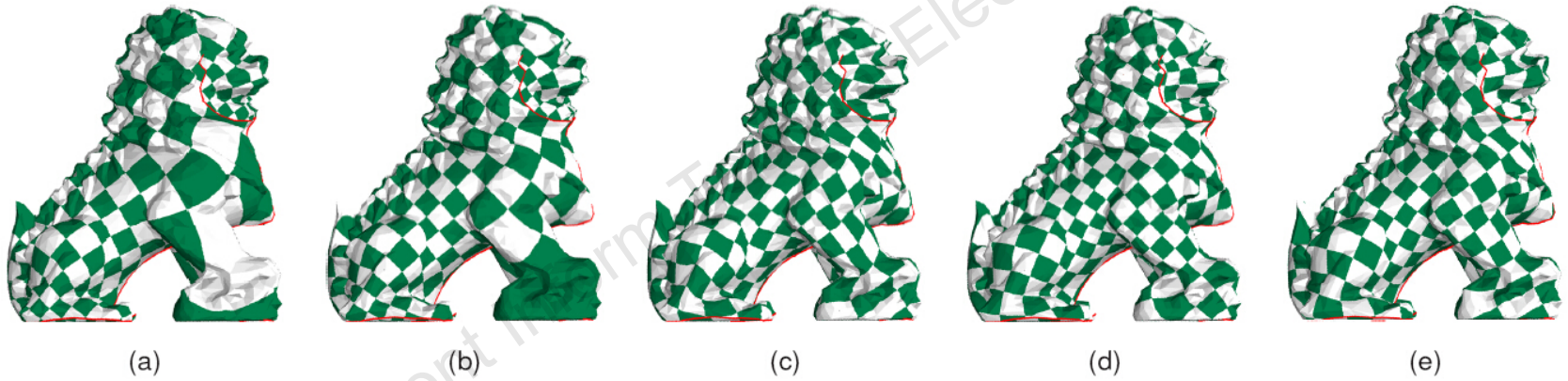




**Fig. 1** Texture of different mesh parameterizations for a Chinese dragon model (red lines represent the seams of the closed mesh when cut to a disk): (a) least squares conformal maps (LSCM) (Lévy *et al.*, 2002); (b) linear angle-based flattening (LABF) (Zayer *et al.*, 2007); (c) as-rigid-as-possible (ARAP) (Liu *et al.*, 2008); (d) bounded distortion as-rigid-as-possible (BD-ARAP) (Lipman, 2012); (e) ARAP++. References to color refer to the online version of this figure

# Major results (2)

- To reduce the area and stretch distortions for high-curvature models, the stretch operator is employed to improve our scheme

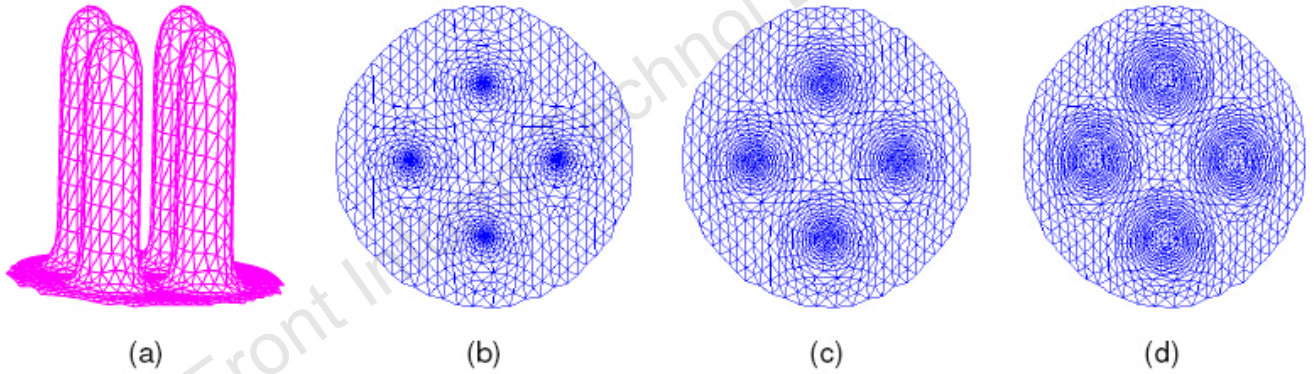




**Fig. 9** The 3D mesh (a) and the stretch operator for $\theta = 0$ (b), $\theta = 1$ (c), and $\theta = 1.6$ (d) for the bump model

# Major results (3)

- ARAP++ performs better than ARAP in area distortion

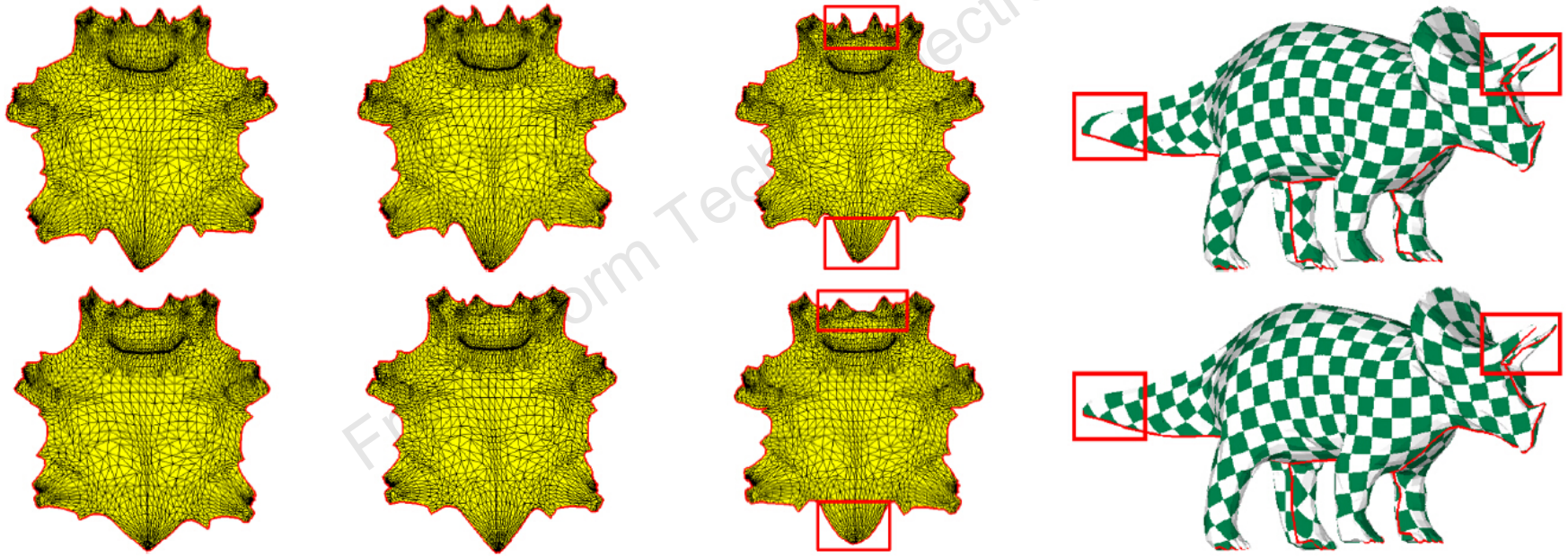

Fig. 11 Parameterizations and textures of ARAP (the first line) and ARAP++ (the second line)

# Major results (4)

- The textured high-curvature models produced by ARAP++ are more uniform and aesthetical than others

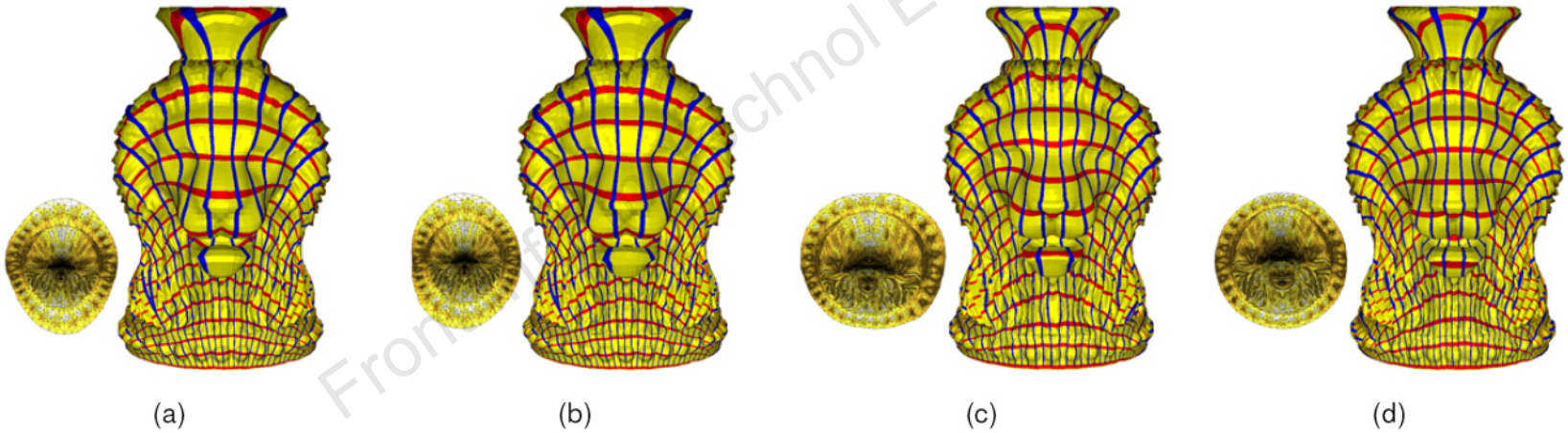


**Fig. 14** Parameterizations and textures of high-curvature models: (a) LSCM; (b) LABF; (c) BD-ARAP; (d) ARAP++

# Major results

- The surface remeshed high-curvature models produced by ARAP++ are more uniform and aesthetical than others

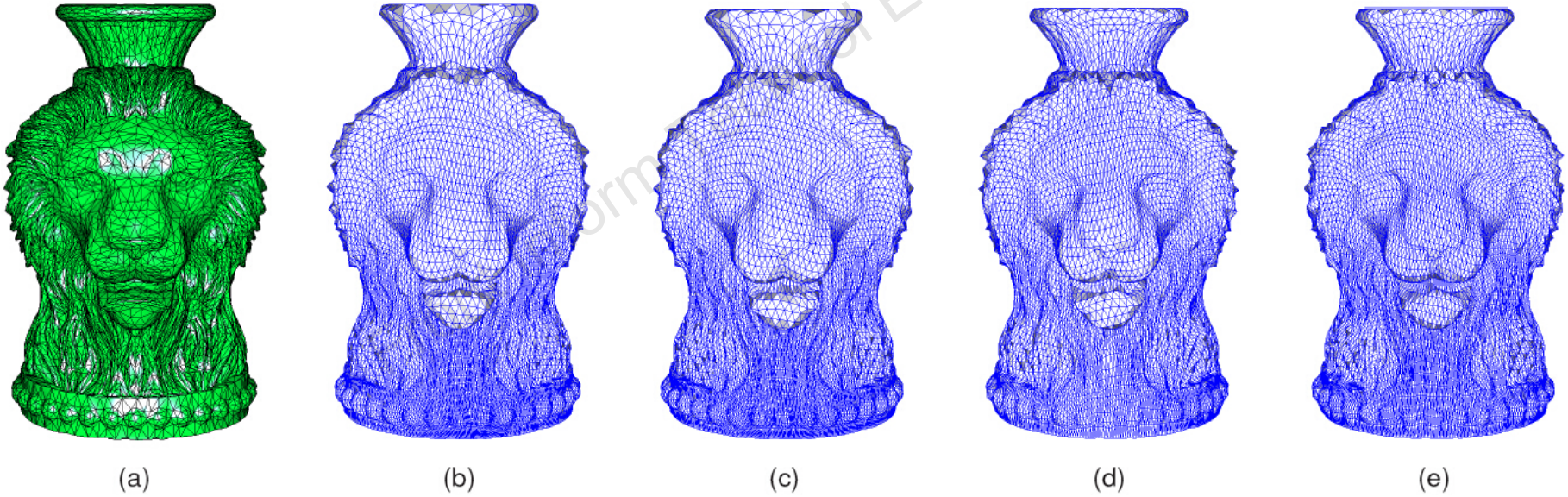


Fig. 15 Surface remeshing: (a) original mesh; (b) LSCM; (c) LABF; (d) BD-ARAP; (e) ARAP++

# Conclusions

- In this paper, we have presented a simple and efficient approach, ARAP++, to flatten a 3D mesh surface. This is an extension of the local/global approach ARAP, which can obtain the flattened results by stitching together the 1-ring patches.

- In addition, to deal with high-curvature models, the stretch operator is introduced, which can better control area and stretch distortions, and has attained better visualization performance in applications, such as texture mapping and surface remeshing.