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Properties of a general quaternion-valued gradient operator and its applications to signal processing

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Key words: Quaternion, gradient operator, signal processing, LMS algorithm, nonlinear adaptive filtering, adaptive beamforming

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Motivation

- Quaternion calculus has been introduced in signal processing with application areas involving three- or four-dimensional signals, such as color image processing, vector-sensor array systems, three-phase power systems, quaternion-valued wireless communications, and wind profile prediction.

- One important operation in quaternion-valued signal processing is the quaternion-valued gradient operator. There have been studies in this area applicable to real-valued functions of quaternion-valued variables, but a general quaternion-valued gradient operator is missing in literature and we aim to fill this gap in this work and study its properties and applications in detail.

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Contribution (I)

- Firstly, we give a detailed derivation of the relation between the gradients and the increment of a quaternion function, highlighting the difference between the left and the right gradients due to the non-commutativity of quaternion algebra.

- Secondly, we document several properties of the operators that have not been reported before, in particular several different versions of product rules and chain rules.

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Contribution (II)

- Thirdly, we derive a general formula for the restricted HR derivatives of a wide class of regular quaternion-valued nonlinear functions, among which are the exponential, logarithmic, and the hyperbolic tangent functions.

- Finally, we prove that the restricted HR gradients are consistent with the usual definition for the gradient of a real function of a real variable.

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Contribution (III)

- Its application to the derivation of a quaternion-valued least mean squares (QLMS) adaptive algorithm and a nonlinear adaptive algorithm based on the hyperbolic tangent function is also briefly discussed.

- As an example for quaternion-valued signal processing, we consider the reference signal based adaptive beamforming problem for vector sensor arrays consisting of multiple crossed-dipoles and provide some simulation results.

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Conclusions

- We have proposed a restricted HR gradient operator and discussed its properties, in particular several different versions of product rules and chain rules.
- Using the rules that we established, we derive a general formula for the derivative of a large class of nonlinear quaternion-valued functions, and prove that, for a wide class of functions, the restricted HR gradient becomes the usual derivatives for real functions with respect to real variables, when the independent quaternion variable tends to the real axis, thus showing the consistency of the definition.
- Both linear and nonlinear adaptive filtering algorithms are derived to show the applications of the operator.

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