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Neuro-heuristic computational intelligence for solving nonlinear pantograph systems

Key words: Neural networks; Functional differential equations; Unsupervised learning; Genetic algorithms; interior-point technique

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Motivation

- The strength of the universal function approximation capability of artificial neural net-works (ANNs), optimized with local and global search techniques, has been exploited immensely to solve constrained and unconstrained optimization problems.
- The accuracy and convergence should be analyzed based on a large number of numerical experimentations by changing the number of neurons in neural networks modeling..
- The design and development of stochastic numerical solvers for nonlinear Pantograph system based on differential equations looks to be a promising area of research.

Main idea

A novel computational intelligence algorithm for approximating the solution of IVP for the functional differential equations of pantograph type using feed-forward artificial neural networks, evolutionary computing technique synchronized on GAs, the interior-point technique (IPT), and their hybrid combinations. The validity of the obtained results is analyzed with available standard solutions, i.e., the exact, numerical, and analytical solutions.

Method

1. ***Initialization***: The initial population is randomly generated with real values represented chromosomes or individuals with same number of elements equal to number of unknown weights in the ANN models. Initialize the values GAs parameters as given in Table 1.
2. ***Calculate Fitness***: The fitness value for each individual of population is evaluated first by using Eq. (3) for the first ANN model and then Eq. (8) for the second ANN model.
3. ***Ranking***: Each individual of the populations is ranked on the basis of the minimum value of the respective fitness functions of the models.

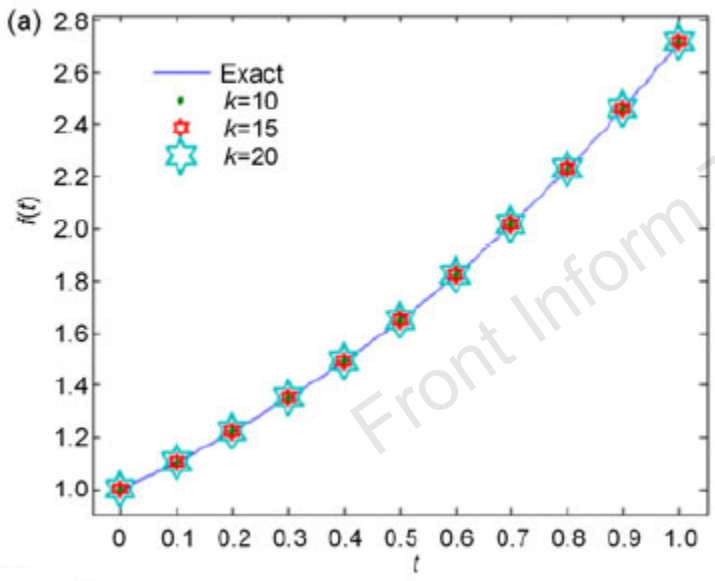
Method (Cont'd)

4. ***Termination:*** The algorithm is terminated when any of following criteria is matched:
   - The predefined fitness value is achieved
   - The number of generations is completed
   - The stoppage criteria given in Table 1.is fulfilled

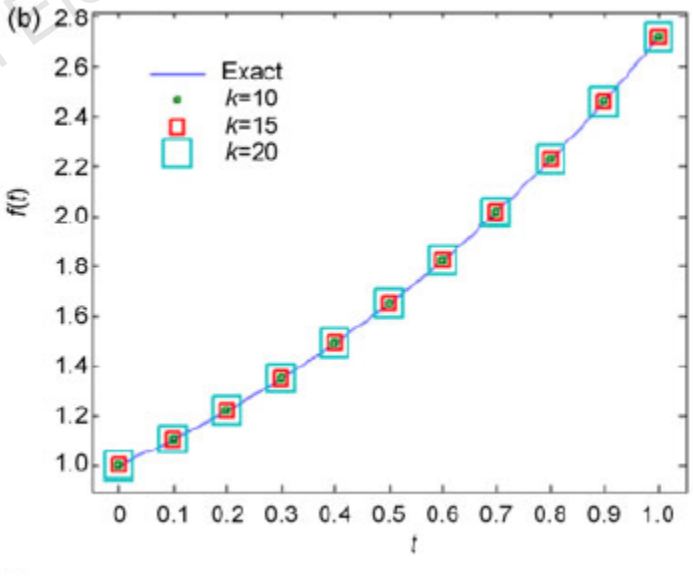
   If any of the above-mentioned termination criteria is met, then go to 6
5. ***Reproduction***: Create next generation at each cycle by using Crossover: Call for heuristic function, Mutation: Call for Gaussian function, Selection: Call for Stochastic Uniform function and Elitism count 05, etc. Repeat the process from steps 2 to 5 with new population.
6. ***Hybridization***: The IPT is used for the refinement of results by using the best individual of GAs as initial weights. The parameter settings used for IPT are given in Table 2.
7. ***Store:*** The final weight vector and fitness values achieved are stored and results are calculated with these weights for finding the solution of nonlinear Pantograph System

Major results

Solution of proposed ANN-GA-IPT algorithm for nonlinear Pantograph Equation



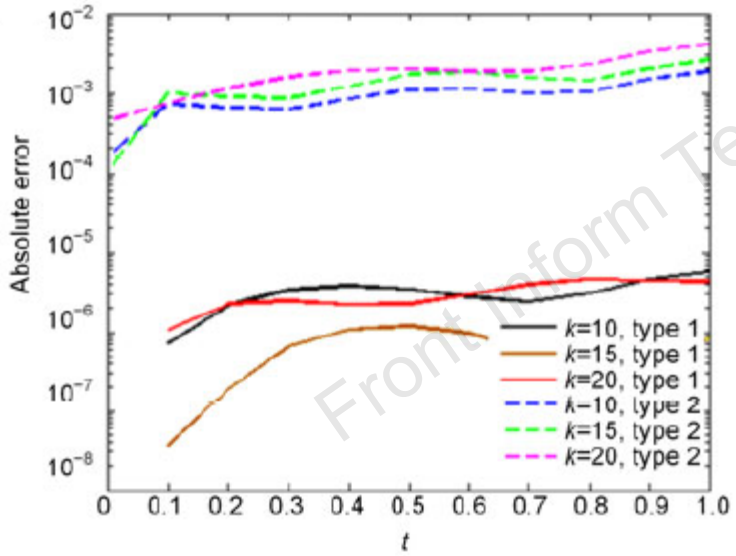
ANN model of type 1



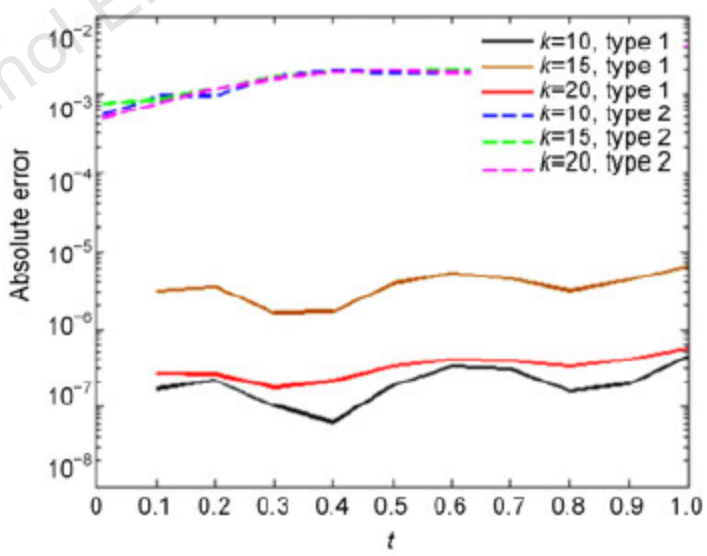
ANN model of type 2

Major results (Cont'd)

Absolute error of proposed ANN-GA-IPT algorithm for solving nonlinear Pantograph Equation



ANN model of type 1



ANN model of type 2

Conclusions

- A new heuristic computational intelligence technique is developed for solving initial value problems of functional differential equations of pantograph type effectively by exploiting the strength of ANN modeling, GA, IPT, and their hybrid approach GA–IPT.
- The behavior of the proposed scheme, based on neural network models with different number of neurons, shows that a slight gain is seen in the precision of the results with increasing number of neurons in the modeling; however, no drastic change in accuracy has been observed. On the other hand, the time of computations has increased exponentially by increasing the number of neurons in the modeling.
- Besides the consistent accuracy and convergence of the proposed scheme, other valuable advantages are the simplicity of the concept, ease in implementations, provision of results on entire continuous grid of inputs, and easily extendable methodology for different applications. All the features established the intrinsic worth of the scheme as a good alternate, accurate, reliable, and robust computing platform for stiff nonlinear Pantograph systems