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# An enhanced mixed modulated Lagrange explicit time delay estimator with noisy input

**Key words:** Time delay estimation, Adaptive filter, Noisy input, Modulated Lagrange, Unbiased impulse response estimation

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# Motivation

- Time delay estimation (TDE) is one of the most important problems in many applications.
- The explicit time delay estimation (ETDE) algorithm (So *et al.*, 1994) is unbiased only for broadband white noise-like signal. It does not explicitly consider the ubiquitous noise and may deteriorate in low signal-to-noise (SNR) scenarios.
- The mixed modulated Lagrange ETDE (MMLETDE) algorithm (Cheng and Tjhung, 2003) behaves well for narrowband or sinusoidal signals, and is easy to implement due to small filter order demand. However, it does not explicitly consider the noise effect.

# Main idea

- We are inspired by the unbiased impulse response estimation approach when explicitly considering the noise effect at both input and output.
- The proposed algorithm enhances the performance of MMLETDE mainly in low SNR scenarios, and keeps most of the merits of the original MMLETDE

# Method

1. Consider noisy input and output measurements.

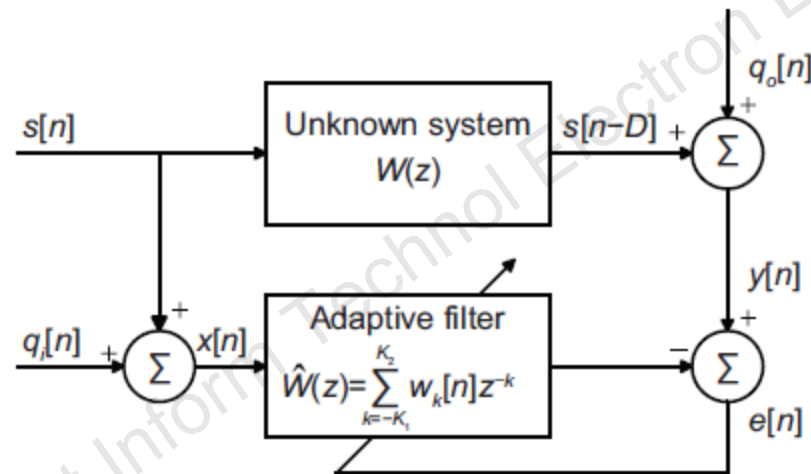
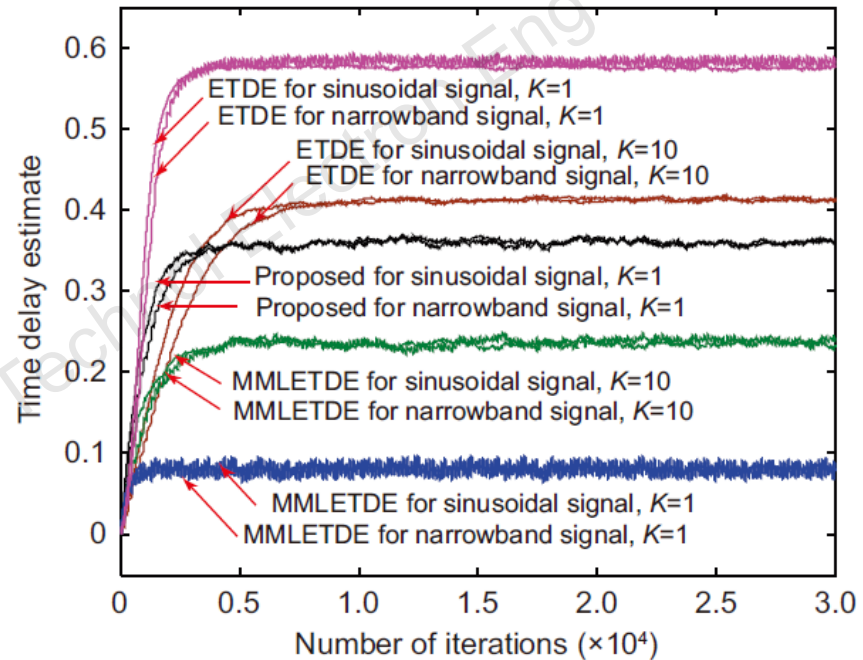


Fig. 1 Adaptive time delay estimator considering noise at both input and output

2. Apply the unbiased impulse response estimation approach (So, 2001) based on the MMLETDE algorithm.

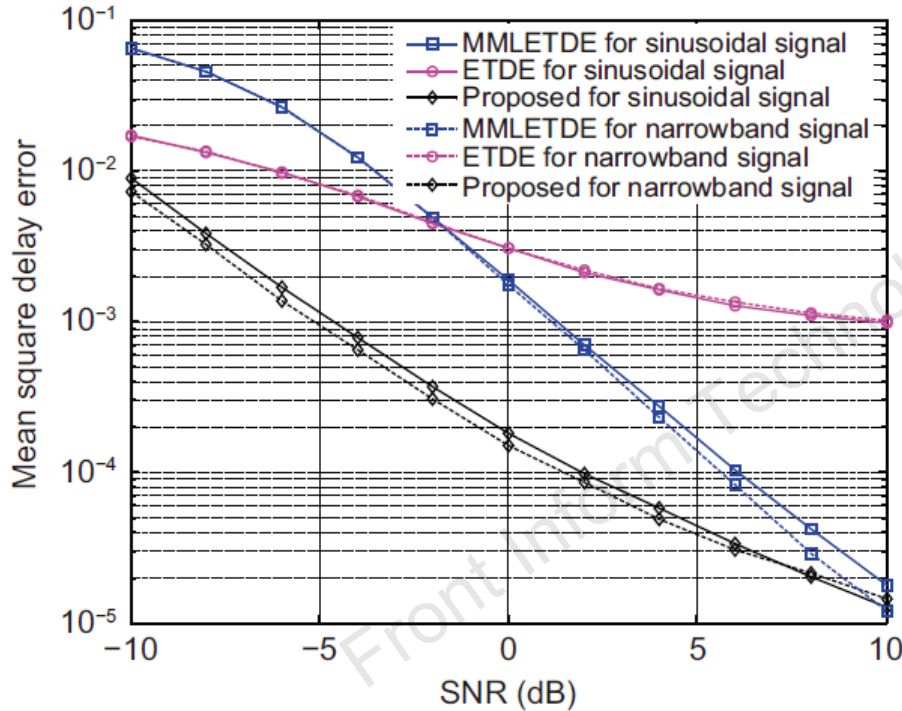
# Major results (I)

- Our proposed algorithm can directly obtain an unbiased explicit time delay estimation with a filter order  $K$  under a low SNR.



**Fig. 4** Time delay estimate of the three algorithms for sinusoidal and narrowband signals (SNR= $-10$  dB)

# Major results (II)



Our proposed algorithm can achieve good estimation performance under low SNR scenarios.

Fig. 5 Steady-state performance for sinusoidal and narrowband signals at different SNRs ( $K = 2$ )

# Conclusions

- Inspired by unbiased impulse response estimation, an enhanced MMLETDE algorithm is proposed for noisy input.
- Under low SNR scenarios, the proposed algorithm can obtain an unbiased ETDE for narrowband and sinusoidal signals with a smaller steady-state estimate.
- For sinusoids, the proposed algorithm obtains time delay estimation with a small filter order, over a wide frequency range with improved accuracy.