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Optimization of formation for multi-agent systems based on LQR

Key words: Linear quadratic regulator (LQR), Formation control, Algebraic Riccati equation (ARE), Optimal control, Multi-agent systems

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Motivation/Main ideas

➢ **Motivation**

· The formation for agents has its wide range of applications both in civilian and military.

· Optimal formation is to obtain the best performance index due to economic or other reasons, such as the least energy expenditure or the shortest formation time.

➢ **Main ideas**

We study optimal linear formation algorithms for multi-agent systems with single-integrator dynamics in three aspects from the linear quadratic regulator (LQR) perspective, and focus on the situation considering both the collective objective of all agents and the individual objective of each agent.

LQR formation for initially isolated multi-agent systems

System dynamics: $\dot{\boldsymbol{x}}_i = \boldsymbol{u}_i$

Optimal index:

$$J = \int_0^{\infty} \left(\sum_{i=1}^{n} \sum_{j=1}^{i-1} h_{ij} (\boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{\Delta}_{ij})^{\mathrm{T}} (\boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{\Delta}_{ij}) + \sum_{i=1}^{n} e_i (\boldsymbol{x}_i - \boldsymbol{\delta}_i)^{\mathrm{T}} (\boldsymbol{x}_i - \boldsymbol{\delta}_i) + \sum_{i=1}^{n} r_i \boldsymbol{u}_i^{\mathrm{T}} \boldsymbol{u}_i \right) \mathrm{d}t$$

Control algorithm:

$$\boldsymbol{u}_i = -k_i (\boldsymbol{x}_i - \boldsymbol{\delta}_i) - \sum_{i=1}^{n} a_{ij} (\boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{\Delta}_{ij})$$

LQR formation for initially isolated multi-agent systems

Theorem 1 In the LQR formation for single-integrator multi-agent systems with cost function (3), the optimal algorithm is

$$u = -[(R^{-1}(Q+E))^{1/2} \otimes I_m]\tilde{x} = -[(L+K) \otimes I_m]\tilde{x}, \quad (10)$$

where

$$L = (R^{-1}Q + R^{-1}E)^{1/2} - \mathrm{diag}[(R^{-1}Q + R^{-1}E)^{1/2} 1_n] \quad (11)$$

is a Laplacian matrix, and

$$K = \mathrm{diag}[(R^{-1}Q + R^{-1}E)^{1/2} 1_n] \quad (12)$$

is a positive definite diagonal matrix.

Simulation results (Example 1)

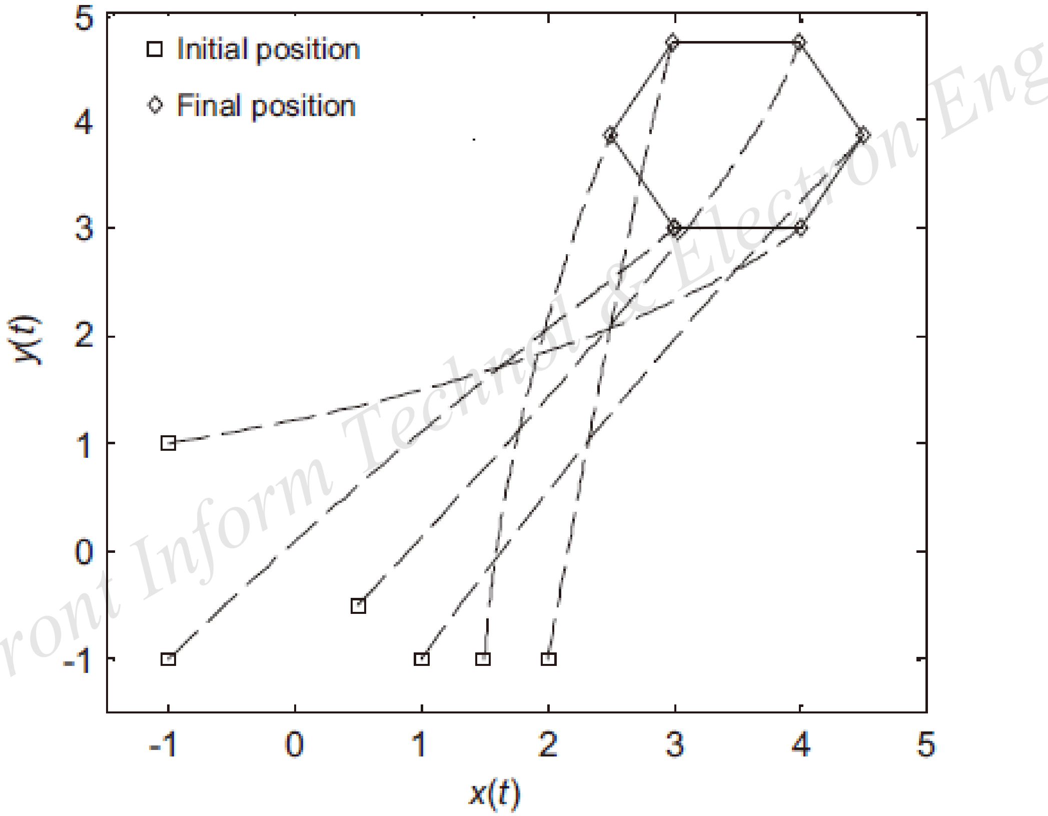

Fig. 1 Formation for the six agents (Example 1)

Simulation results (Example 1)

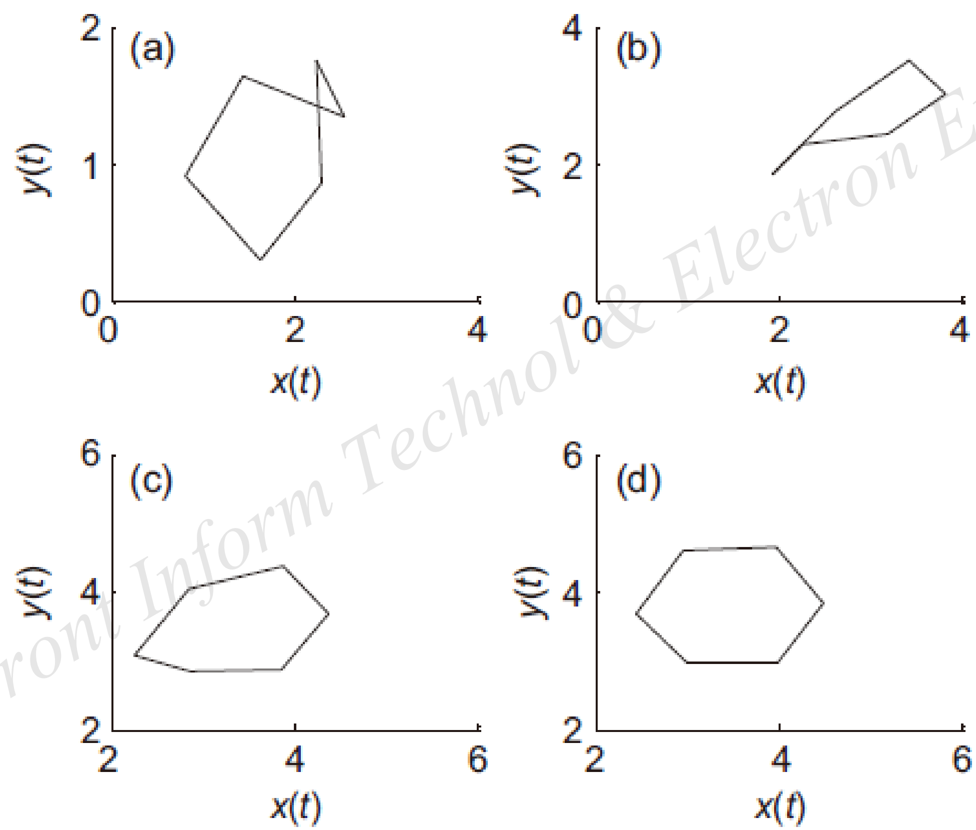

Fig. 2 Snapshots for the six agents (Example 1): (a) $t = 1.4$ s; (b) $t = 2.8$ s; (c) $t = 4.2$ s; (d) $t = 5.6$ s

Simulation results (Example 1)

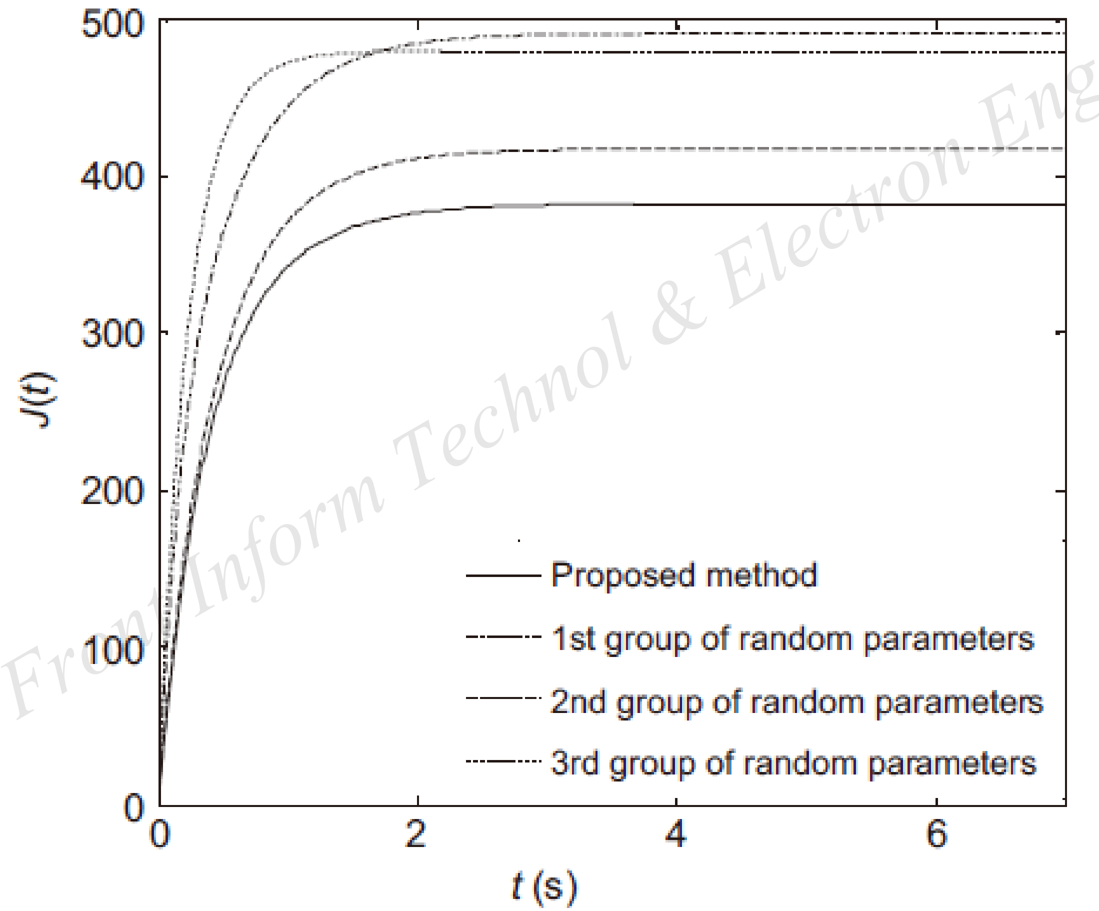

Fig. 3 Trajectories of cost function J (Example 1)

LQR formation for physically couplings multi-agent systems

System dynamics:

$$\dot{x}_i = \sum_{i=1}^{n} a_{ij}(x_j - x_i - \boldsymbol{\Delta}_{ji}) + u_i$$

Optimal index:

$$J = \int_0^{\infty} \left(\sum_{i=1}^{n} \sum_{j=1}^{i-1} a_{ij}(x_i - x_j - \boldsymbol{\Delta}_{ij})^{\mathrm{T}}(x_i - x_j - \boldsymbol{\Delta}_{ij}) + \sum_{i=1}^{n} r_1 (x_i - \delta_i)^{\mathrm{T}}(x_i - \delta_i) + \sum_{i=1}^{n} r_2 u_i^{\mathrm{T}} u_i \right) \mathrm{d}t$$

Control algorithm:

$$u_i = -\sum_{i=1}^{n} b_{ij}(x_i - x_j - \boldsymbol{\Delta}_{ij}) - k_i(x_i - \delta_i)$$

$$u = -(L_2 \otimes I_m)\tilde{x} - (K \otimes I_m)\tilde{x}$$

LQR formation for physically couplings multi-agent systems

Theorem 2 In the optimal formation under undirected couplings for the cost function (14) subject to Eq. (13), the optimal control parameter matrices are $\boldsymbol{L}_2 = r_2^{-1}\bar{\boldsymbol{P}} - (r_1/r_2)^{1/2}\boldsymbol{I}_n$ and $\boldsymbol{K} = (r_1/r_2)^{1/2}\boldsymbol{I}_n$, where $\bar{\boldsymbol{P}}$ can be constructed as $\bar{\boldsymbol{P}} = \boldsymbol{M}^{\mathrm{T}}\mathrm{diag}(p_1, p_2, \ldots, p_n)\boldsymbol{M}$ with $\boldsymbol{M}$ being an orthogonal matrix such that $\boldsymbol{L}_1 = \boldsymbol{M}^{\mathrm{T}}\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)\boldsymbol{M}$; moreover, we have

$$p_i = -\lambda_i r_2 + [\lambda_i^2 r_2^2 + r(\lambda_i + r_1)]^{1/2}, \quad i = 1, 2, \ldots, n.$$

Simulation results (Example 2)

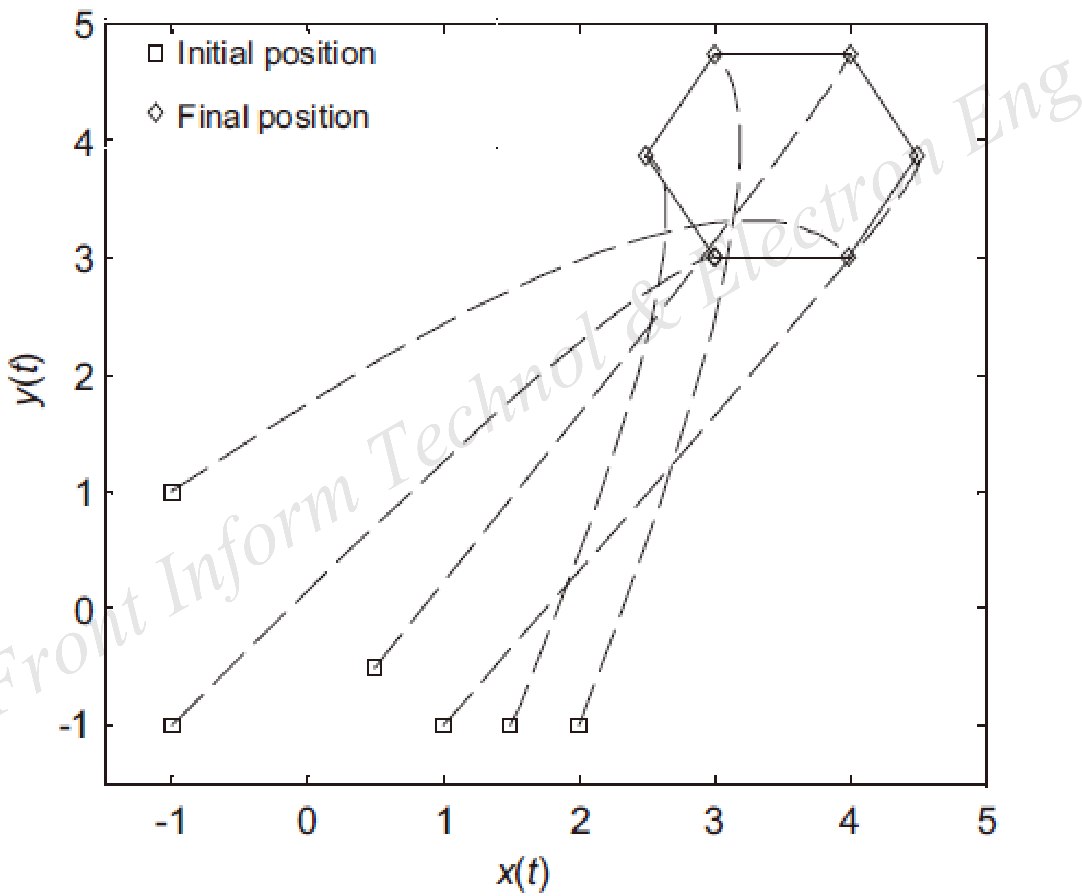

Fig. 4 Formation for the six agents (Example 2)

Simulation results (Example 2)

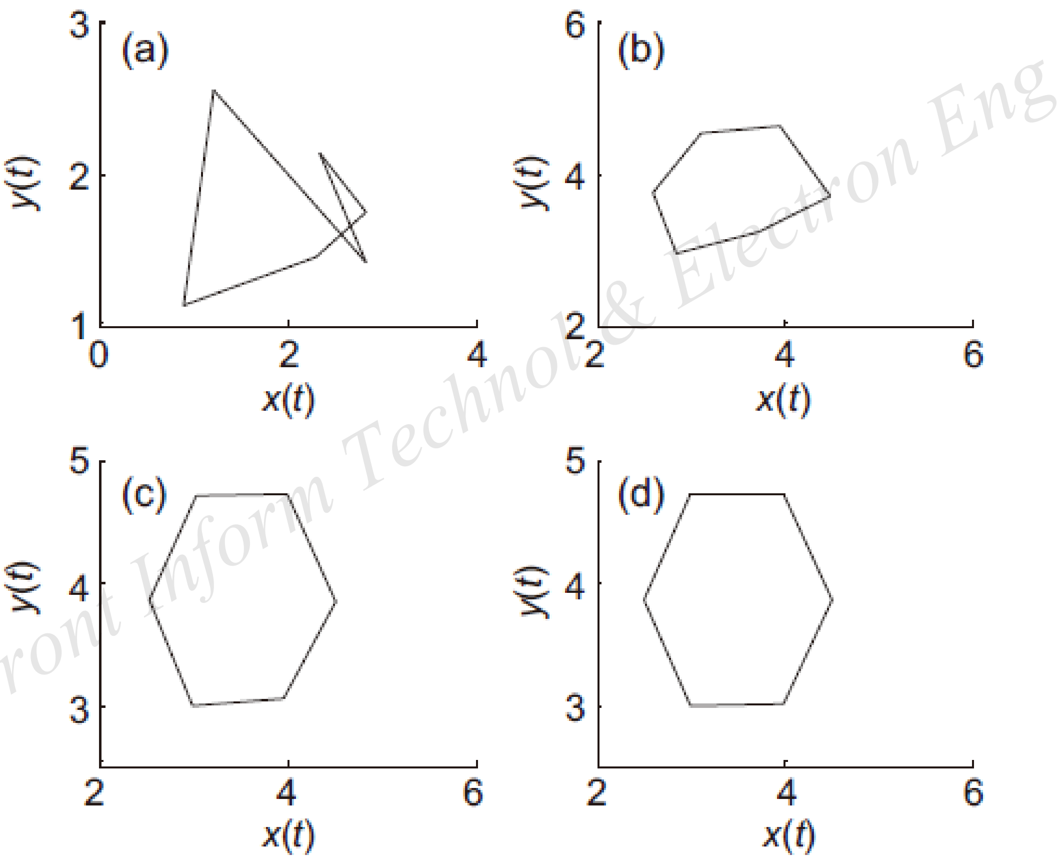

Fig. 5 Snapshots for the six agents (Example 2): (a) $t = 2$ s; (b) $t = 4$ s; (c) $t = 6$ s; (d) $t = 8$ s

Simulation results (Example 2)

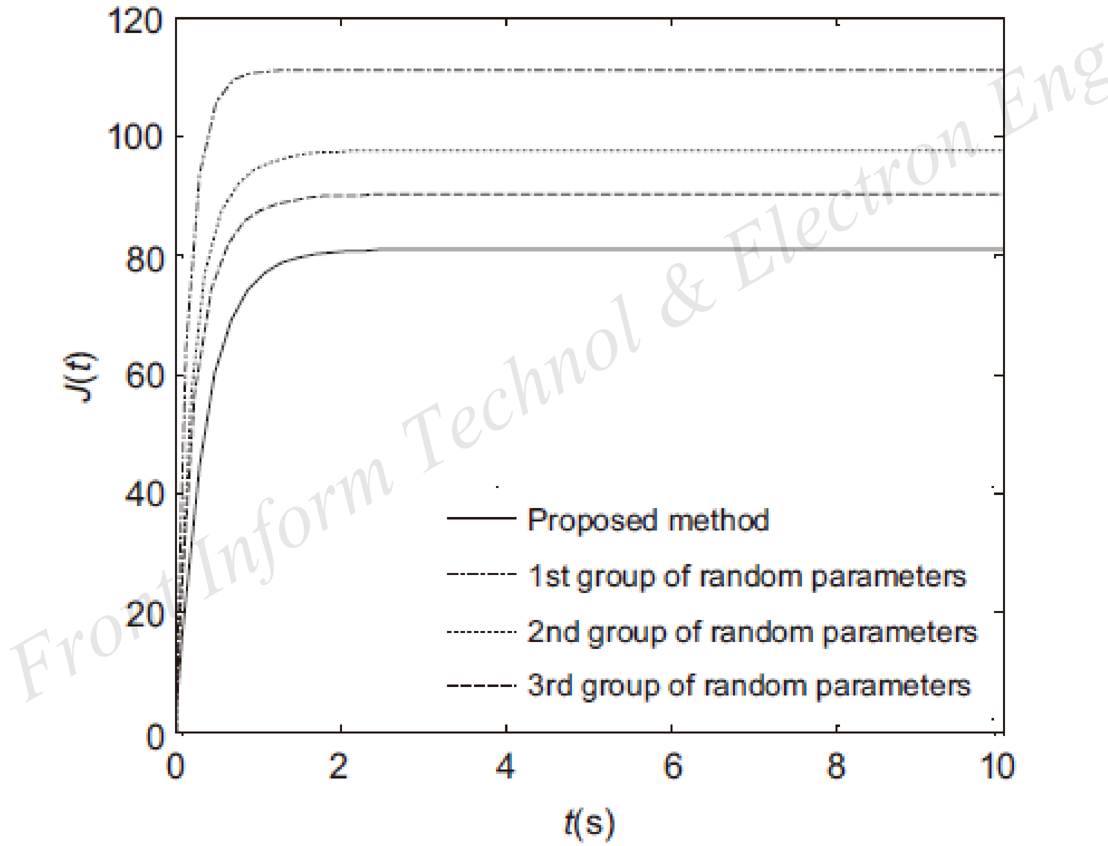

Fig. 6 Trajectories of cost function J (Example 2)

LQR formation for multi-agent systems under fixed topology

System dynamics:

$$\begin{cases} \dot{x} = u_i, \\ u_i = -\sum_{i=1}^{n} a_{ij}(x_i - x_j - \Delta_{ij}) - k(x_i - \delta_i) \end{cases}$$

Optimal index:

$$J = \int_0^{\infty} \left(\sum_{i=1}^{n} \sum_{j=1}^{i-1} a_{ij}(x_i - x_j - \Delta_{ij})^2 + \sum_{i=1}^{n} r_1 (x_i - \delta_i)^2 + \sum_{i=1}^{n} r_2 u_i^{\mathrm{T}} u_i \right) \mathrm{d}t$$

LQR formation for multi-agent systems under fixed topology

Theorem 3 In the LQR formation control problem with the cost function J proposed in Eq. (20) subject to Eq. (19), if the communication topology is fixed and undirected, the optimal control parameter k is given by

$$\begin{aligned} k = &-\frac{\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)(\boldsymbol{L} \otimes \boldsymbol{I}_m)\tilde{\boldsymbol{x}}(0)}{\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)\tilde{\boldsymbol{x}}(0)} \\ &+ \frac{1}{r_2\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)\tilde{\boldsymbol{x}}(0)}\{r_2^2[\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)(\boldsymbol{L} \otimes \boldsymbol{I}_m)\tilde{\boldsymbol{x}}(0)]^2 \\ &- r_2\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)\tilde{\boldsymbol{x}}(0)\tilde{\boldsymbol{x}}^{\mathrm{T}}(0)[r_2(\boldsymbol{L} \otimes \boldsymbol{I}_m)^2 \\ &- \boldsymbol{L} \otimes \boldsymbol{I}_m - r_1\boldsymbol{I}_{mn}]\tilde{\boldsymbol{x}}(0)\}^{1/2}, \end{aligned}$$

where $r_2 \leq 1/(2d_{\max})$, and $d_{\max} = \max_i d_i$, $i = 1, 2, \ldots, n$.

Simulation results (Example 3)

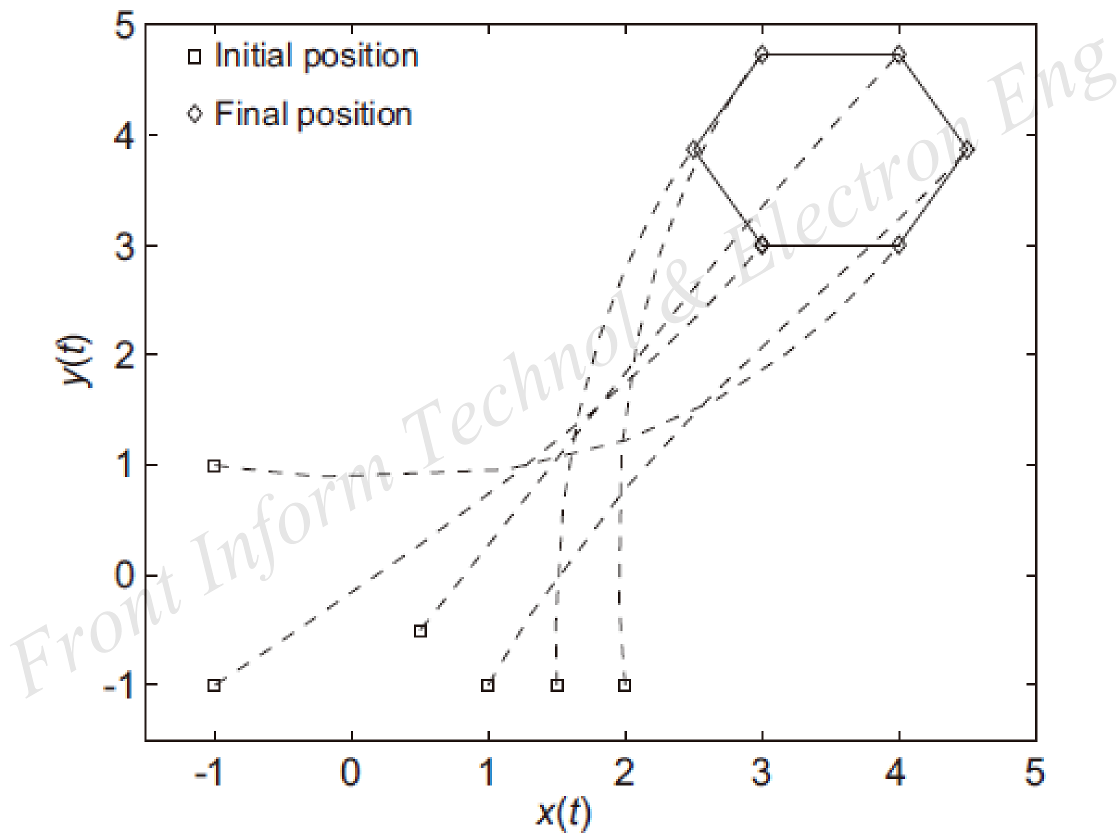

Fig. 7 Formation for the six agents (Example 3)

Simulation results (Example 3)

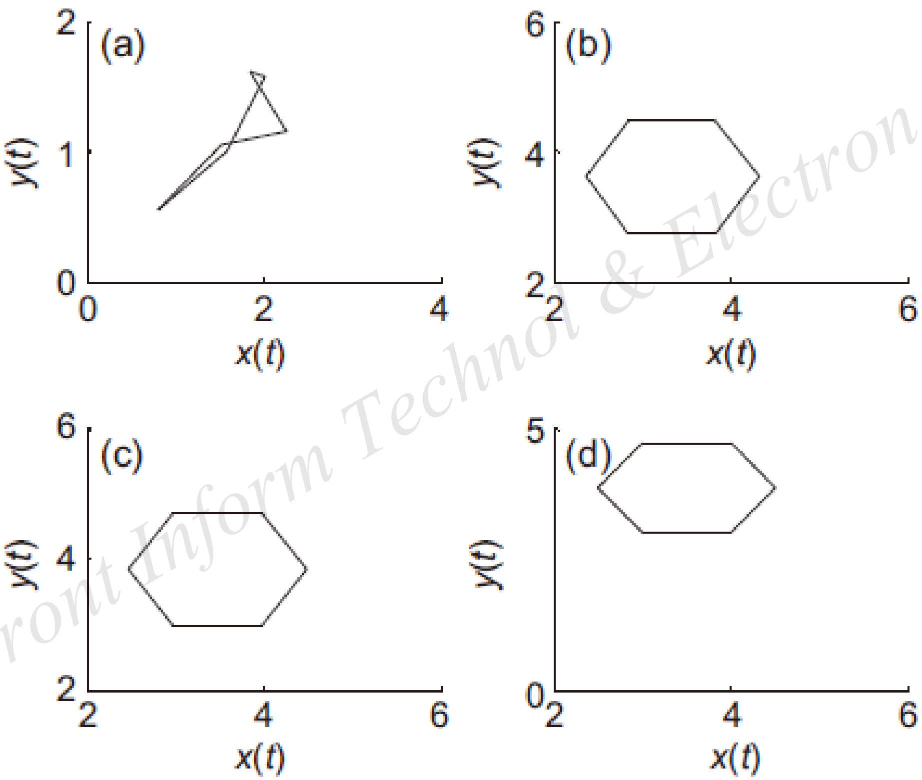

Fig. 8 Snapshots for the six agents (Example 3): (a) $t = 0.8$ s; (b) $t = 1.6$ s; (c) $t = 2.4$ s; (d) $t = 3.2$ s

Simulation results (Example 3)

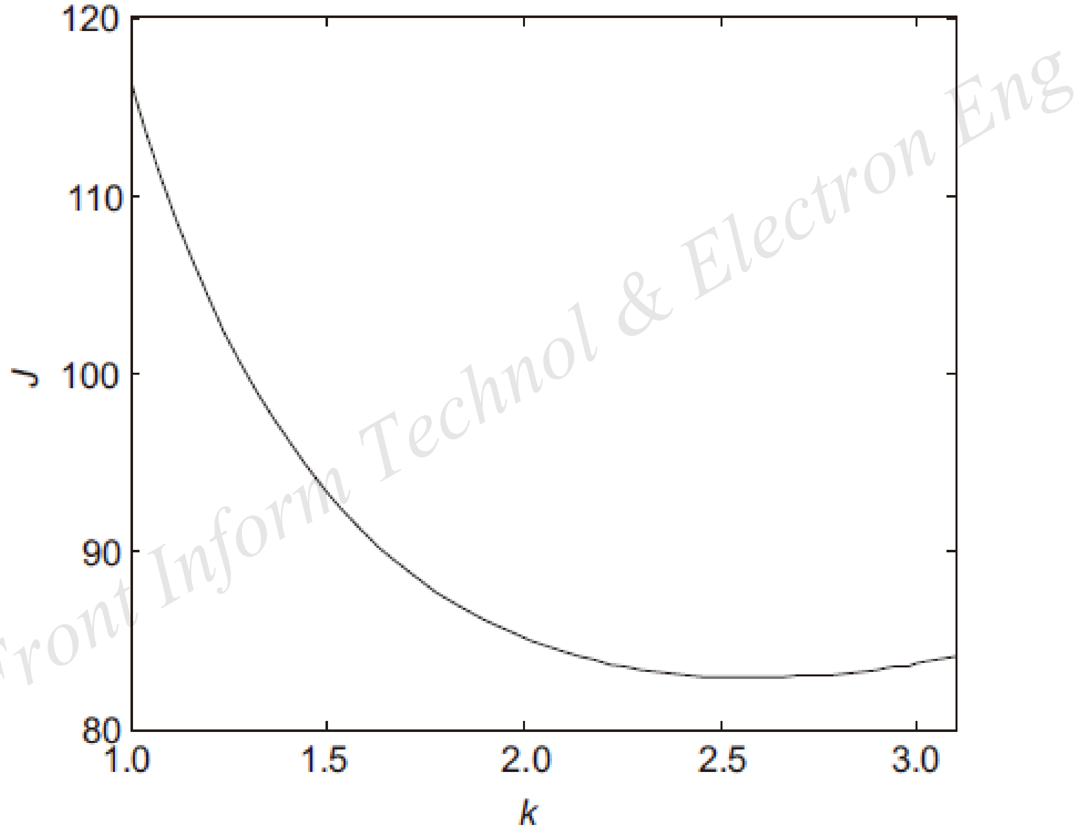

Fig. 9 Trajectory of cost function J as a function of k (Example 3)

Conclusions

- Designed the optimal communication topology and the optimal feedback matrix for no initially coupled multi-agent systems from the solution of an algebraic Riccati equation (ARE).
- Proposed the optimal formation algorithm for physically coupled multi-agent systems from a team of quadratic equations with one unknown.
- Designed the optimal local feed-back gain from a quadratic equation with one unknown for the multi-agent systems under the fixed communication topology.