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Theoretical foundation of a decision network for urban development

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Motivation

- In real-world situations, most planning problems are complex and challenging, not only because the problems themselves are ambiguous and difficult to define, but also because they involve multiple stakeholders with multi-attribute preferences.
- Traditional techniques developed in decision analysis, such as the decision tree, focus on a single decision-maker with unidimensional attributes, such as utility, evaluating a given set of alternatives to select the best.
- In real situations, the complex nature of planning problems renders such techniques less useful than expected in helping decision-makers figure out what to do.

Main idea

- As an alternative to traditional techniques, we propose here a theoretical foundation for a technique called ‘decision network’ for making multiple and linked decisions that involve multiple stakeholders with multi-attribute preferences.
- A detailed description of the conceptual framework of decision network was provided by Han and Lai (2011), together with a description of its application to the management of urban growth boundaries (Han and Lai, 2012).
- Here, we provide a formal and general formulation of that framework, and describe a solution algorithm for that formulation.

Main idea (Cont'd)

- The task is then to make a 'plan' by assigning the given *m* decision-makers, *p* problems, and *q* solutions to *n* decision situations to yield the highest overall expected utility under the structural constraints.
- Mathematically, this assignment task can be easily formulated as a 0-1 integer program, shown as

$$\max \sum_{j=1}^{n} p_j \left(\sum_{i=1}^{m} x_{ij} u_i + \sum_{k=1}^{p} y_{kj} v_i + \sum_{l=1}^{q} z_{lj} w_l \right), \qquad (1)$$

Solution algorithm

- For a small or medium-sized problem, solving Eq. (1) is straightforward using a commercial package, such as LINDO. When the problem size becomes large enough to involve thousands of decision-makers, problems, solutions, and decision situations, it would be cumbersome to construct the model and solve it through LINDO.
- An algorithm for solving large models involving sequential decisions under uncertainty proposed by Kirkwood (1993) could be applied. However, that would require the modeler to reconstruct the decision network problem into a decision tree. With thousands of variables and parameters, this reconstruction would render the solution algorithm inextricable.
- Alternatively, we develop here a solution algorithm that is specific for solving large-scale decision network problems.

Numerical examples

- Following Han and Lai (2011), we use the same numerical example here to show how the algorithm works. Assume that *D*, *A*, and *S* are given as follows:

$$D=\begin{pmatrix}0&1&1&0&1\\1&0&0&1&0\end{pmatrix}, A=\begin{pmatrix}0&1&0&0&1\\0&0&0&1&0\\0&0&1&0&0\end{pmatrix},$$

$$S=\begin{pmatrix}0&1&1&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&1&0&1\end{pmatrix}.$$

Numerical examples (Cont'd)

- Furthermore, suppose that *e* and e_k are the unit matrix and the k-th unit matrix, respectively, where the k-th unit vector in the k-th unit matrix is equal to one or zero. Note that *e′* and e'_k are the transpose of *e* and e_k respectively.

$$e = \begin{pmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{pmatrix}, e' = \left[1 \cdots 1 \cdots 1\right], e_k = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}, e'_k = \left[0 \cdots 1 \cdots 0\right].$$

Numerical examples (Cont'd)

- Assume further that the probabilities associated with the five decision (chance) nodes are 1.0, 1.0, 1.0, 0.7, and 0.5, respectively, that the utilities associated with the two decision makers are 0.7 and 0.3, respectively, that the disutilities associated with the three problems are –0.6, –0.5, and –0.7, respectively, and that the utilities associated with the four solutions are 0.6, 0.3, 0.7, and 0.5, respectively.
- We first show how the access structure is decomposed according to Steps 1 to 3 and finally demonstrate how the solution is obtained.

Numerical examples (Cont'd)

- Step 1: Retrieve the row vectors for the access structure.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

- Step 2: Identify the number of elements for each row where the values of the elements are equal to 1 to obtain decomposed matrices, and we have:

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Numerical examples (Cont'd)

- Step 3: Decompose the solution structure following Steps 1 and 2 to obtain decomposed matrices.

$$S_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, S_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, S_4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Numerical examples (Cont'd)

- Step 4: For each combination of the decomposed matrices of the access and solution structures, compute the overall expected utility and select the combination that yields the highest overall expected utility as the solution.

$$U(n_1) = 1.0 \times \begin{pmatrix} 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} + 1.0 \times \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} -0.6 \\ -0.5 \\ -0.7 \end{pmatrix} + 1.0 \times \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \end{pmatrix} = 0.6;$$

$$U(n_2) = 1.0 \times \begin{pmatrix} 1 & 0 \end{pmatrix} \times \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} + 1.0 \cdot \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} -0.6 \\ -0.5 \\ -0.7 \end{pmatrix} + 1.0 \times \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \end{pmatrix} = 0.1;$$

Numerical examples (Cont'd)

$$U(n_3) = 1.0\times\begin{pmatrix}1 & 0\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix} + 1.0\times\begin{pmatrix}0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix} + 1.0\times\begin{pmatrix}1 & 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix} = 1.1;$$

$$U(n_4) = 0.7\times\begin{pmatrix}0 & 1\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix} + 0.7\times\begin{pmatrix}0 & 1 & 0\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix} + 0.7\times\begin{pmatrix}0 & 0 & 1 & 0\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix} = 0.35;$$

$$U(n_5) = 0.5\times\begin{pmatrix}1 & 0\end{pmatrix}\times\begin{pmatrix}0.7\\0.3\end{pmatrix} + 0.5\times\begin{pmatrix}0 & 0 & 0\end{pmatrix}\times\begin{pmatrix}-0.6\\-0.5\\-0.7\end{pmatrix} + 0.5\times\begin{pmatrix}0 & 0 & 0 & 0\end{pmatrix}\times\begin{pmatrix}0.6\\0.3\\0.7\\0.5\end{pmatrix} = 0.35.$$

Numerical examples (Cont'd)

- Therefore, for the combination of A_1 and S_3, the overall expected utility across the five decision nodes is 0.6 + 0.1 + 1.1 + 0.35 + 0.35 = 2.5.
- The overall expected utilities for other combinations of the access and solution structures can be computed in a similar way.
- The best "plan," or combination, of the access and solution structures is the one that yields the highest overall expected utility, which in this example is A_2S_1 or A_2S_3, which yield an overall expected utility of 2.8.

Conclusions

- Planning problems are characterized by difficulty and complexity. Traditional decision analysis techniques that are commonly used by planners are overwhelmingly focused on making independent decisions for single decision makers.
- Effective planning tools must address multiple stakeholders and multi-attribute preferences at the same time. Here, we provide a theoretical basis for Decision Network, which attempts to make multiple, linked decisions with multiple stakeholders and multi-attribute preferences.
- With sufficient and persistent effort, the theoretical foundation introduced here could serve as a starting point for development into a fully-fledged technology that helps planners deal confidently with challenging planning problems.