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Derivation and analysis on the analytical structure of interval type-2 fuzzy controller with two nonlinear fuzzy sets for each input variable

Key words: Interval type-2 fuzzy controller, Analytical structure, Karnik-Mendel type reducer

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Introduction

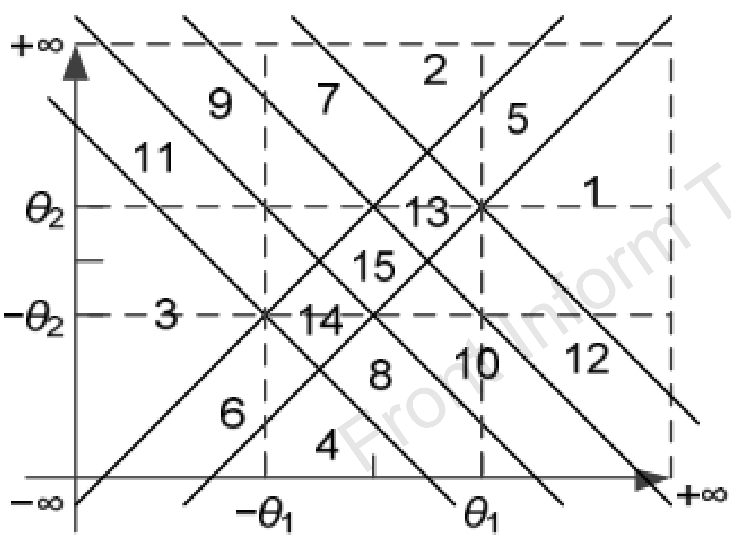
- Although the advantages of the IT2 fuzzy controllers have been demonstrated through case studies, the controllers are still treated as black boxes and cannot be amenable for analysis and interpretation by engineers.

- So far, a very limited number of papers have addressed the analytical structure of an IT2 controller, and when doing so, it is about linear IT2 fuzzy sets for input variables.

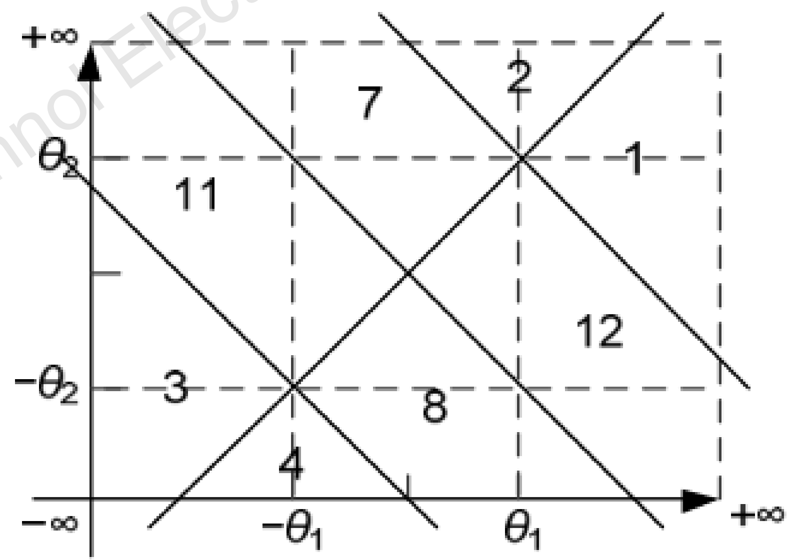
- A derivation method and an analysis of the analytical structure of the IT2 fuzzy controller involving nonlinear IT2 fuzzy sets have been presented in this work.

Derivation of the analytical structure of the IT2 fuzzy controller

1. Dividing the input space of the interval type-2 fuzzy controller



$\theta_1 \neq \theta_2$



$\theta_1 = \theta_2$

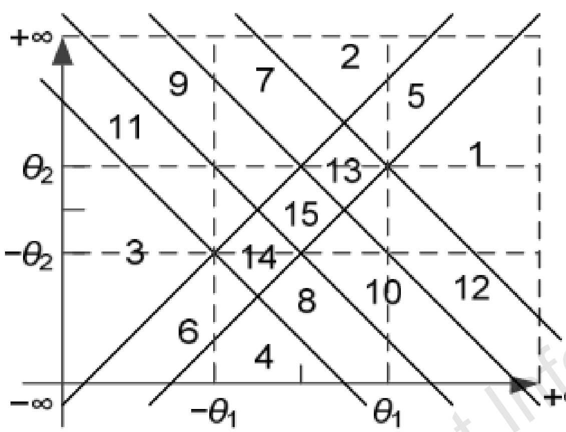
2. Derivation of input-output expression on each partition of the interval type-2 fuzzy controller

$$
\begin{aligned}
\Delta u_{\mathrm{IC}_1} &= \frac{y_1 + y_{\mathrm{r}}}{2} \\
&= \left[\frac{P_2 P_4 H \beta_1(E,R)}{2(3+\exp(E-\theta_1))(1+\exp(R-\theta_2))} \right. \\
&\qquad \left. + \frac{P_1 P_3 H \beta_1(E,R)}{2(3+\exp(E+\theta_1))(1+\exp(R+\theta_2))} \right] R \\
&\quad + \left[\frac{P_2 P_4 H \beta_1(E,R)}{2(3+\exp(E-\theta_1))(1+\exp(R-\theta_2))} \right. \\
&\qquad \left. + \frac{P_1 P_3 H \beta_1(E,R)}{2(3+\exp(E+\theta_1))(1+\exp(R+\theta_2))} \right] E \\
&\quad + \left[\frac{P_2 P_4 - 1}{(3+\exp(E-\theta_1))(1+\exp(R-\theta_2))} \right. \\
&\qquad \left. + \frac{P_1 P_3 - 1}{(3+\exp(E+\theta_1))(1+\exp(R+\theta_2))} \right] H \\
&= K_{\mathrm{p}}^1 R + K_{\mathrm{i}}^1 E + \delta^1 .
\end{aligned}
$$

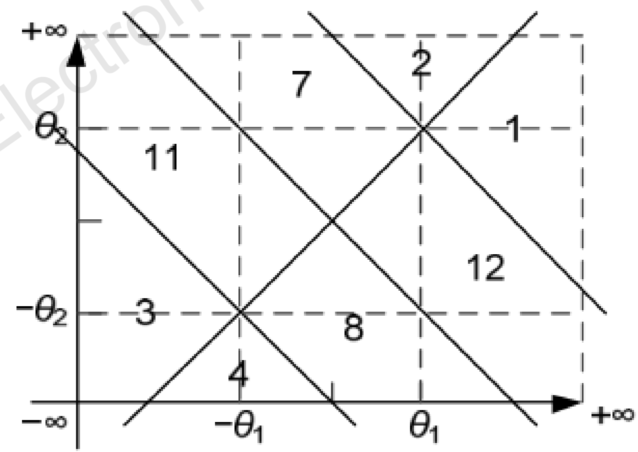
The proposed IT2 fuzzy controller has been proved to be approximately equivalent to a nonlinear PI or PD controller with variable gains.

Analysis of the analytical structure of the IT2 fuzzy controller

1. The relationship between design parameters and regions



$$\theta_1 \neq \theta_2$$



$$\theta_1 = \theta_2$$

By comparing these two figures, some regions exist only when the parameters are unequal, such as IC5–IC6, IC9–IC10, IC13–IC15. The closer the values of θ_1 and θ_2, the smaller the sizes of these regions.

2. Analysis of the derived gains of the IT2 fuzzy controller

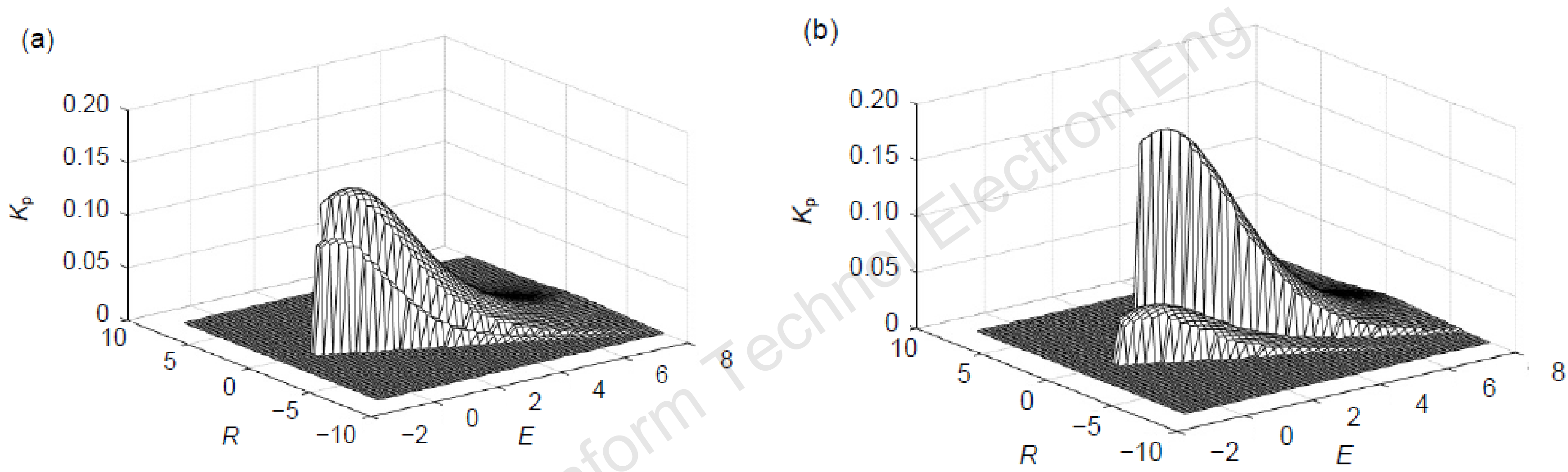


Fig. 10 Proportional gain of the IT2 fuzzy controller on IC1 and IC12: (a) θ_1=θ_2=0.3; (b) θ_1=θ_2=1.0

According to the analytical structure of the proposed IT2 fuzzy controller, increasing the design parameters appropriately is helpful to improve the control performance.

3. Comparison between the outputs of the IT2 fuzzy controller and its T1 counterpart

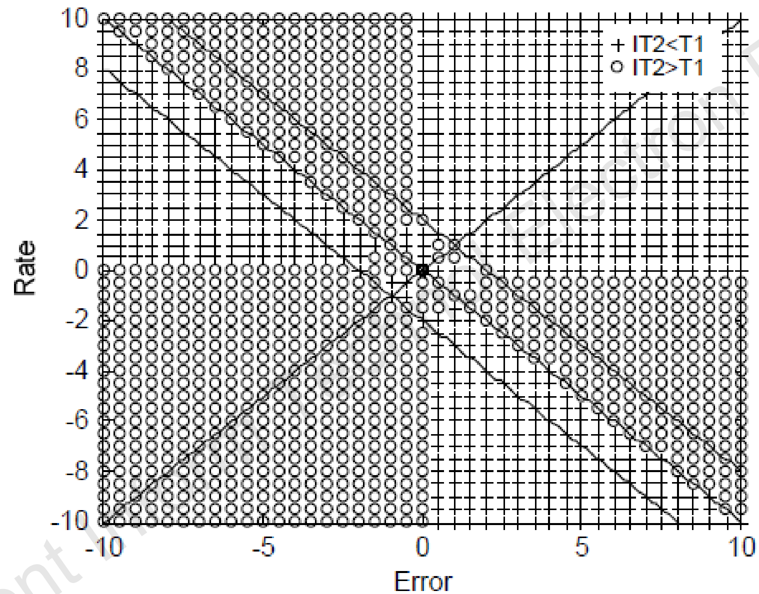


Fig. 11 Comparison between the T1 and IT2 controllers when θ_1=θ_2

Compared with the T1 fuzzy controller, the IT2 fuzzy PI controller can reduce the overshoot and decrease the rising time at the same time.

Simulation and experiment

Simulation results

Table 2 Comparison of the properties of the step response for the T1 and IT2 fuzzy systems

Controller	Rising time (s)	Overshoot (%)	Setting time (s)	ITAE
T1	30	4.6289	93	6747.6
No. 1 IT2	30	2.4333	73	5927.9
No. 2 IT2	30	2.8329	88	6099.3

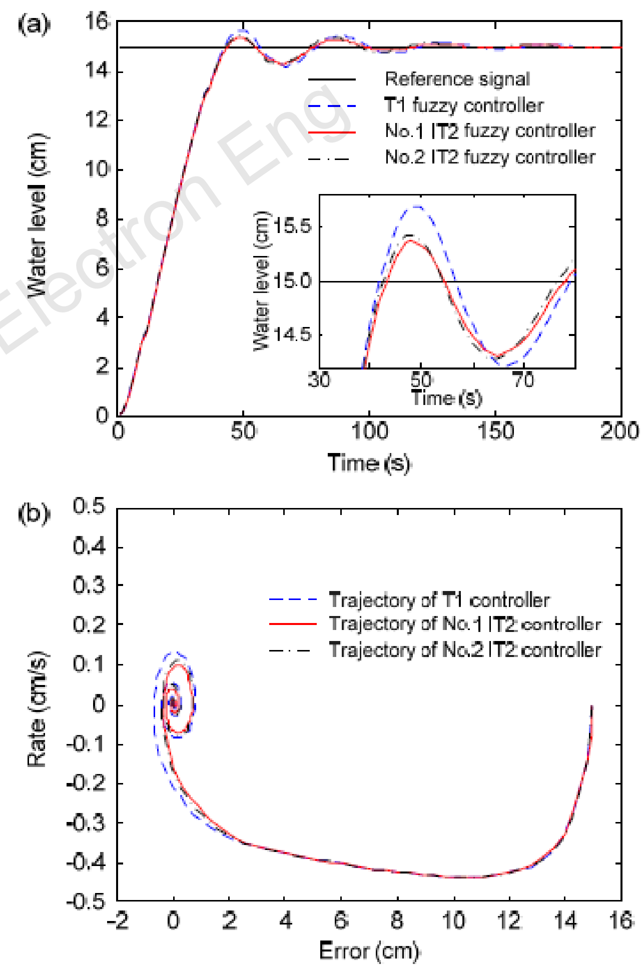


Fig. 12 Results of the coupled-tank water level control simulation: (a) step response of the fuzzy controllers; (b) trajectories of error and rate

Simulation and experiment (Cont'd)

Experiment results

Fig. 13 Industrial mechatronics drives unit

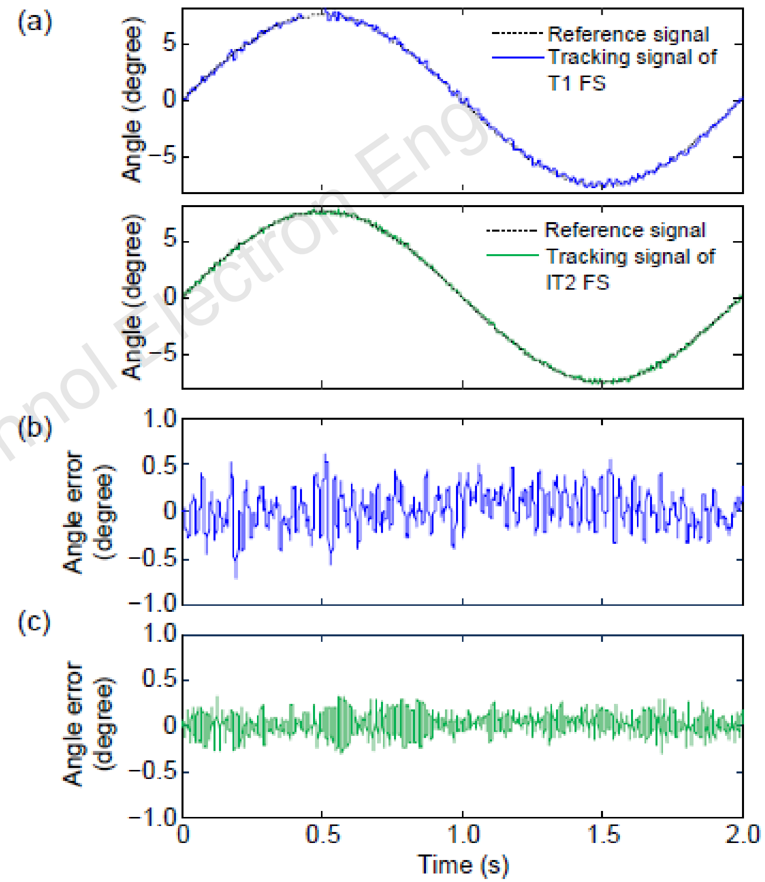


Fig. 14 Results of the sinusoidal tracking experiment: (a) sinusoidal tracking curve of the IT2 fuzzy system and its T1 counterpart; (b) sinusoidal tracking error of the IT2 fuzzy system; (c) sinusoidal tracking error of the T1 fuzzy system

Conclusions

- The analytical structure derivation of the IT2 fuzzy controller is extended to the IT2 fuzzy controller with nonlinear IT2 fuzzy sets and a Zadeh AND operator.
- Through dividing the input space of the IT2 fuzzy controller into 15 partitions, the controller is demonstrated to be approximately equivalent to a nonlinear PI or PD controller with variable gains.
- The analytical structure deepens the understanding of the internal input-output relationships of the IT2 fuzzy controller. A comparative analysis of the derived analytical structure and its T1 counterpart revealed the potential ad-vantages of the IT2 fuzzy controller.