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Galerkin approximation with Legendre polynomials for a continuous-time nonlinear optimal control problem

Key words: Generalized Hamilton-Jacobi-Bellman equation; Nonlinear optimal control; Galerkin approximation; Legendre polynomials

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Motivation

- It is very difficult for the continuous-time nonlinear optimal control problems to obtain optimal control laws because these problems depend on the solution to a partial differential equation, which is called the HJB equation.
- The “curse of dimensionality” occurs in most methods such as the dynamic programming when they find optimal numerical solutions satisfying a given accuracy in a required domain to the HJB equations.
- A new method is used to obtain numerical solutions of the HJB equation for nonlinear optimal control.

Main idea

- A Galerkin approximation with Legendre polynomial (GALP) method for the generalized Hamilton-Jacobi-Bellman (GHJB) equation is proposed. The main advantage of the GALP algorithm is that much fewer integrals need to be computed and the feedback control laws can be implemented in different ways.
- The GAPL algorithm essentially computes the coefficients offline using Legendre base functions. Once the coefficients are calculated, the control laws will be obtained online.

Method

1. The problem of finding the solution to the nonlinear HJB equation has been transformed into the problem of finding the solution to the linear GHJB equation.
2. The Galerkin numerical approximation with a series of Legendre bases was used to approximate the near-optimal solution to the GHJB equation.
3. A value function can be found from a set of algebraic equations.

Major results

- Compared with Beard's method, the computational cost of the approximation optimal control with our method can be decreased when we strengthen the stop criterion.

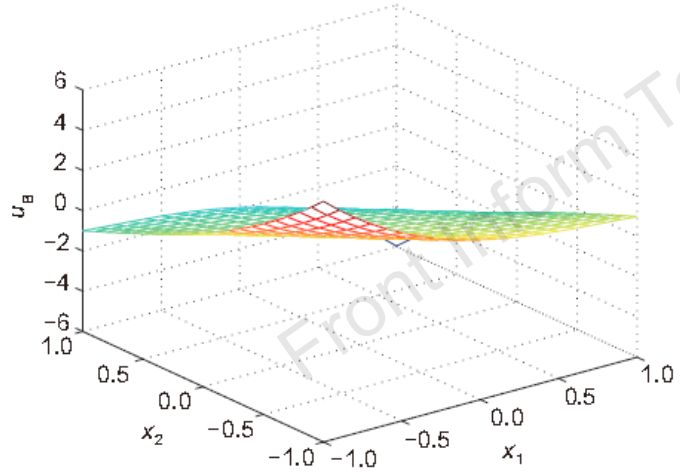

Fig. 1 u_B for the optimal control problem obtained using Beard's control method

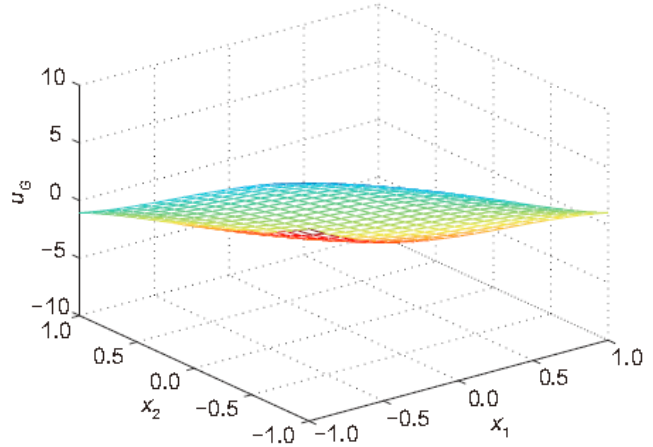

Fig. 2 u_G for the optimal control problem obtained using the GALP control method

Major results

- It is shown that the control obtained by our method performs much better than the one synthesized using exact feedback linearization.

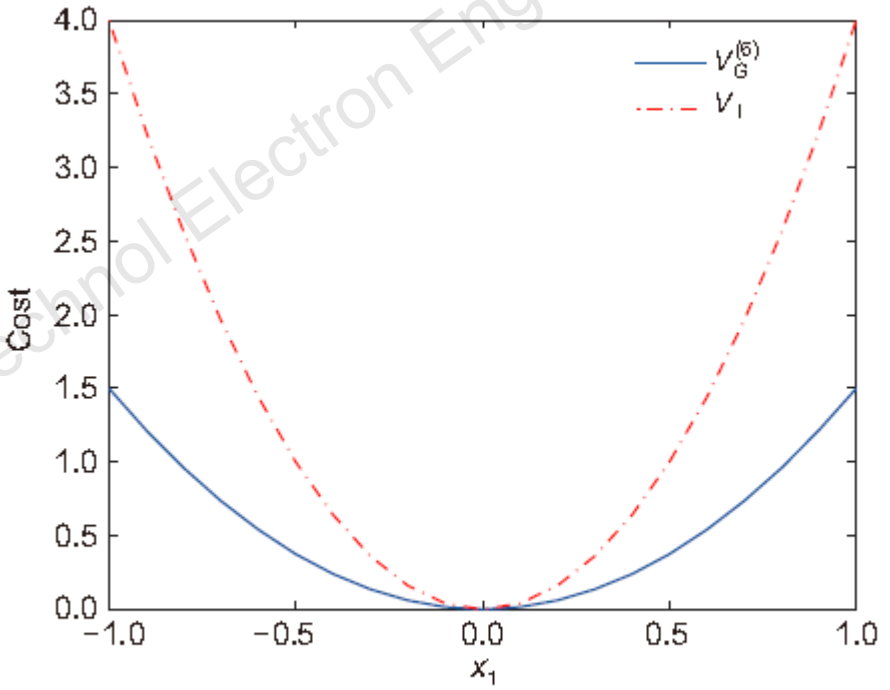

Fig. 3 Comparison with feedback linearization

Conclusions

- It is important to have a numerical algorithm to approximate the solution to the HJB equation in the nonlinear optimal control problem.
- A new GALP method is proposed which provides a suboptimal solution to the GHJB equation for the nonlinear optimal control problem.
- We have proved that the resulting control laws are stable and converge to the optimal control laws.