Xiao-ming Gou, Zhi-wen Liu, Wei Liu, You-gen Xu, 2016. Filtering and tracking with trinion-valued adaptive algorithms. *Frontiers of Information Technology & Electronic Engineering*, **17**(8):834-840.
http://dx.doi.org/10.1631/FITEE.1601164

Filtering and tracking with trinion-valued adaptive algorithms

Key words: Three-dimensional processes, Trinion, Least mean squares, Kalman filter

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Motivation

- Multidimensional (*m*-D) signal processing has a variety of applications and the modeling of multiple variables is carried out traditionally within the real-valued matrix algebra, while in recent years we have observed the successful exploitation of hyper-complex numbers in areas including color image processing, vector-sensor array processing, and wireless communications.

Motivation (Cont'd)

- The most widely used hyper-complex numbers are quaternions, with rigorous physical interpretation for 3-D and 4-D rotational problems. In particular, for 3-D cases, such as 3-D altitude and 3-D wind speed, they are usually modeled with pure quaternions in the literature. However, pure quaternions do not belong to a mathematical ring, leading to redundant computations. Furthermore, we will see in this paper that pure quaternions cannot be used to model the general 3-D tracking problems.

Major results

- We introduce a new type of hyper-complex number termed trinion for 3-D adaptive filtering and tracking. Trinions form a 3-D ring and are commutative by definition, which implies that the trinion algebra could be a competitive candidate for modeling 3-D processes.
- In our first contribution, a class of trinion-valued least mean squares (LMS) algorithms is developed to show that trinions are computationally more efficient than quaternion algebra for 3-D adaptive filtering applications. The augmented second-order statistics are also considered.

Major results (Cont'd)

- Second, we extend the classic Kalman filter into the trinion domain for efficient and effective 3-D tracking. We can see that, for the most general case, a pure quaternion model will not work while trinion algebra provides a convenient and compact solution.

Conclusions

- A trinion-valued model for filtering and tracking of 3-D signals has been proposed.
- The corresponding algorithms were derived, including two LMS-type algorithms (trinion-valued LMS and its augmented version) for adaptive filtering and a Kalman filtering algorithm for tracking.
- Simulation results show that the trinion model is a competitive candidate for 3-D signal processing with merits of reduced computational complexity (related to its compactness) and effective modeling of more complicated 3-D processes (related to its closure property).