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Parameter estimation in exponential models by linear and nonlinear fitting methods

Key words: Exponential model, Parameter estimation, Linear least squares, Nonlinear fitting

Corresponding author: Hui Huang
E-mail: huangh@zju.edu.cn
ORCID: http://orcid.org/0000-0002-1000-4764

Motivation

- There are many physical models based on the exponential function. To determine the unknown parameters of the exponential model, it's common to fit the parameterized model to the measurement data.
- The linear fitting method has obvious advantages, but the estimates of parameters achieved by the linear method are not optimal when noise exists in the measurement data. The discrepancy between the true values of the parameters and the estimates can be further minimized by the nonlinear fitting method.

Main idea

- We investigated both the linear and nonlinear fitting methods for estimating the parameters of unknown exponential model and provided theoretical analysis.
- Numerical simulation of two methods have been made to evaluate the effects on separate parameters estimation and the fitting results.
- An experimental comparison is performed on fitting water spectral attenuation coefficients into an exponential model.

Method

1. The inaccuracy brought by the linear fitting method is theoretically analyzed.

2. Numerical simulation is performed to compare the performance of two methods.

The numerical model is

$$y_i = a_0 \, \mathrm{e}^{b_0 x_i} + p \cdot n_i$$

where n_i represents noise with the standard Gaussian distribution, and p represents noise magnitude. The unknown parameters a_0 and b_0 are estimated from $\{x_i, y_i\}_{i=1}^{m}$.

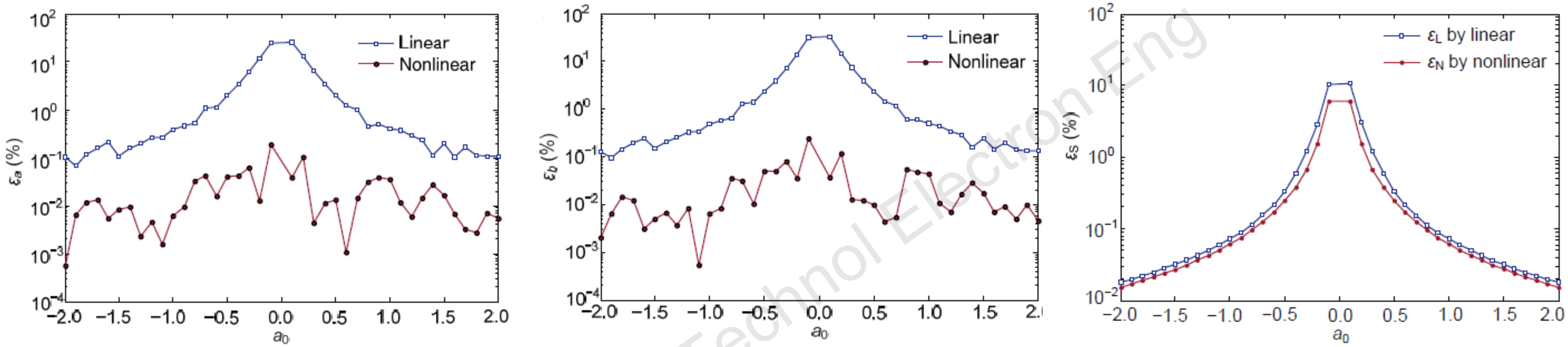
ε_a and ε_b are the relative errors of estimated parameters a_0 and b_0; ε_S is the relative sum of squared errors.

A set of selected values of a_0, b_0, p VS 1000 repeated numerical trials

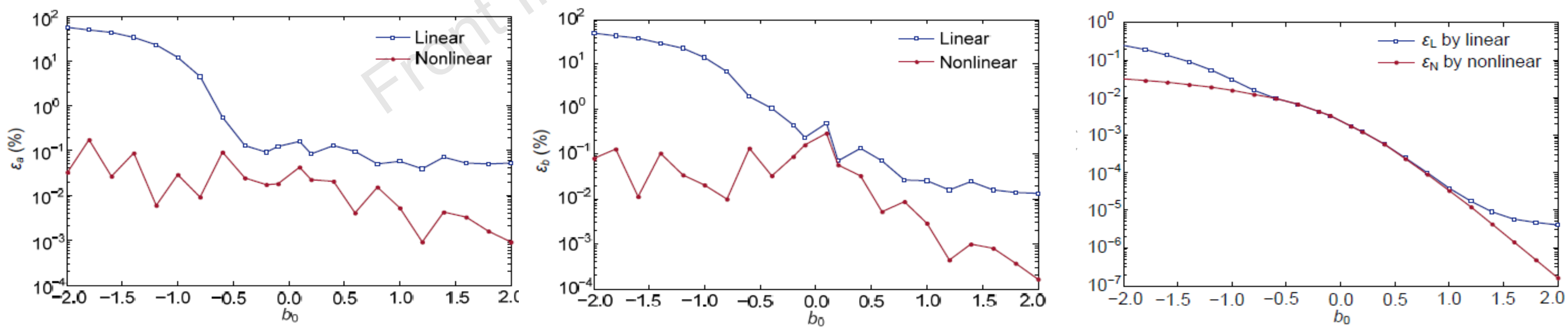
3. The spectral attenuation coefficient of water is estimated directly based on underwater images.

Major results

- Effect of parameter a_0

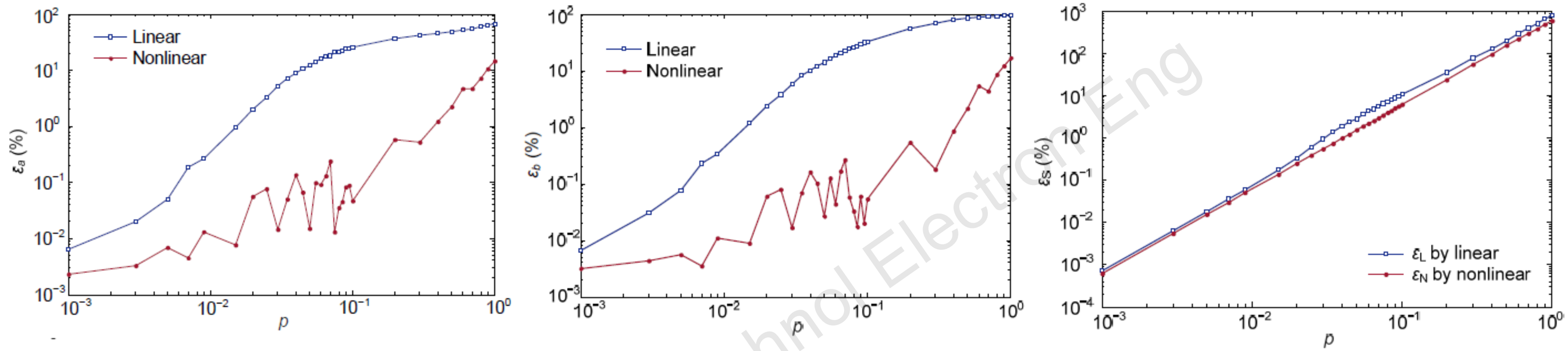


- Effect of parameter b_0



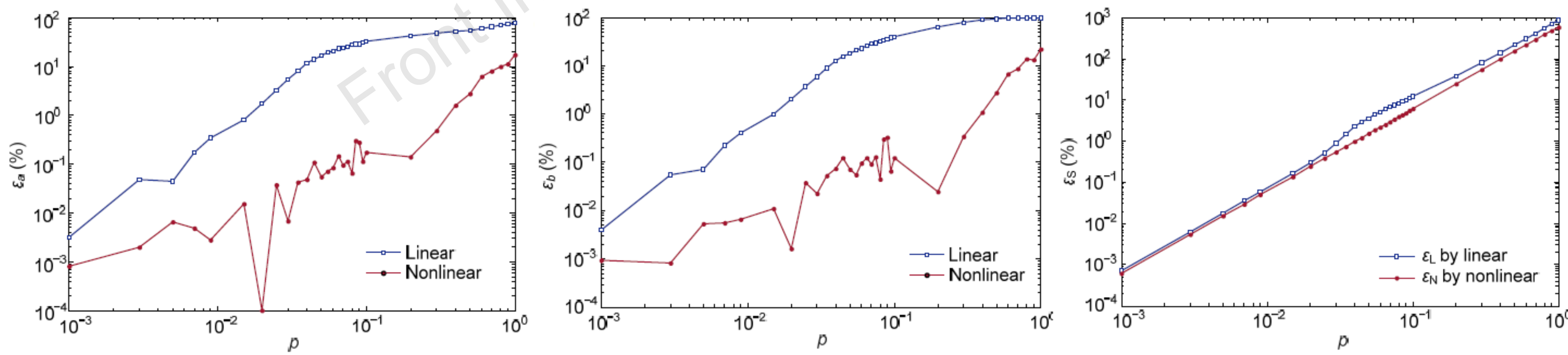
Major results

- Effect of the noise magnitude *p*



- Effect of noise distribution

In this case, the noise conforms to the uniform distribution with zero mean and variance of 1.



Major results

- Experimental comparison

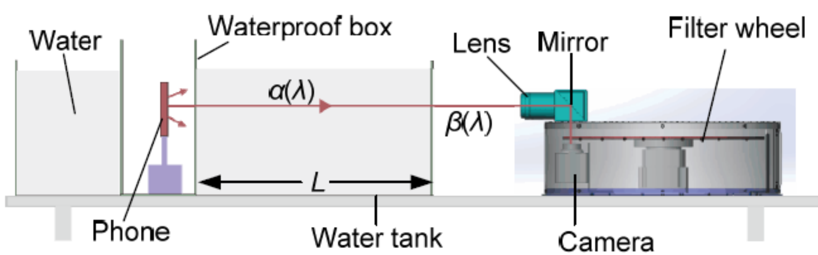


Fig. 10 Experimental setup for underwater imaging with narrowband color filters

$$k(\lambda, L) = \frac{I(\lambda, L)}{I_0(\lambda)} = \beta(\lambda) \mathrm{e}^{-\alpha(\lambda) L}$$

$k(\lambda, L)$: spectral transmittance of the water

λ : wavelength of light

L : underwater imaging distance

$\beta(\lambda)$: transmittance of the glass

$\alpha(\lambda)$: spectral attenuation coefficient of water

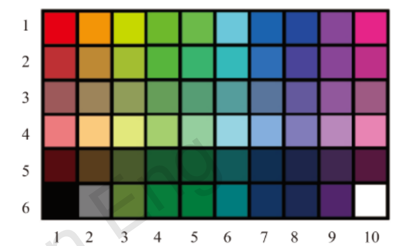


Fig. 11 Image on the mobile phone screen (References to color refer to the online version of this figure)

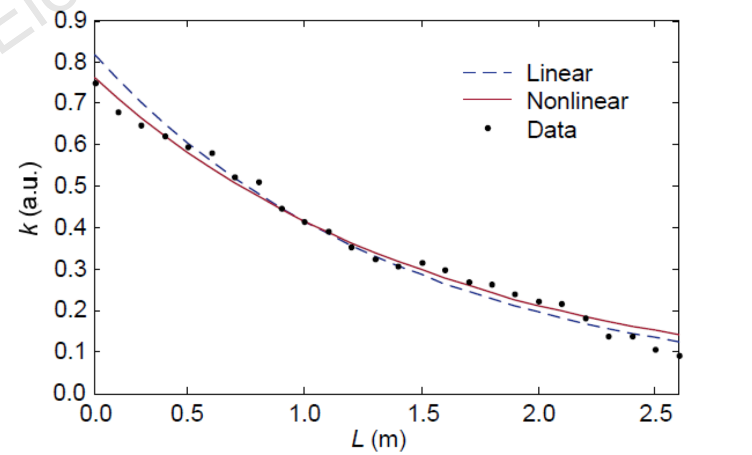


Fig. 12 Measurement data and fitting curves obtained by two methods

Method	Estimate of β(λ)	Estimate of α(λ) (m^{-1})	Sum of squared errors
Linear	0.881	0.751	0.025
Nonlinear	0.815	0.673	0.013

Conclusions

- The unknown parameters estimated by the linear fitting method alone cannot minimize the sum of the squared residual errors when measurement noise is involved in the data.

- Numerical simulation results show that the linear method can only obtain a suboptimal estimate of the unknown parameters but the nonlinear method gives more accurate results.

- Application of the fitting methods supports the improvement in the accuracy of parameter estimation by the nonlinear fitting method.