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Synchronization of two different chaotic systems using Legendre polynomials with applications in secure communications

Key words: Observer-based synchronization; Chaotic systems; Legendre polynomials; Secure communications

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Motivations

1. According to the orthogonal functions theorem (Kreyszig, 2007), Legendre polynomials can approximate nonlinear functions with arbitrarily small approximation errors; they are universal approximators. Consequently, they may play the role of fuzzy systems and neural networks.

2. Legendre polynomials are simpler than neural networks and fuzzy systems since they have fewer tuning parameters. The tuning parameters of Legendre polynomials are simply their coefficients, while those of fuzzy controllers include the center and width of the Gaussian membership functions and also the weight of each rule.

Main ideas

1. Our proposed controller consists of a linear state feedback, a Legendre estimator, and a robust control term for compensation for the truncation error. The Legendre estimator is responsible for compensation for the lumped uncertainty, including parametric uncertainty and external disturbances.

2. In secure communication systems, only one state of the master chaotic system is sent through the channel, thus observer-based synchronization is required. In this study, observer-based chaos synchronization using Legendre polynomials is proposed. The role of Legendre polynomials in the observer is to overcome external disturbances.

1. Legendre polynomials and function approximation

Legendre polynomials defined as:

$$\varphi_0(x) = 1,$$
$$\varphi_1(x) = x,$$
$$(i+1)\varphi_{i+1}(x) = (2i+1)x\varphi_i(x) - i\varphi_{i-1}(x),$$
$$i = 1, 2, \cdots, m-1,$$

According to Kreyszig (2007), a nonlinear function can be represented as $h_{\mathrm{LP}}(x) = \sum_{i=1}^{m} a_i \varphi_i(x) = \boldsymbol{\alpha}^{\mathrm{T}} \boldsymbol{\varphi}$ in which a_i are Legendre coefficients calculated by

$$a_i = \frac{1}{A_i} \int_{x_1}^{x_2} h(x)\varphi_i(x)\mathrm{d}x, \ i = 0, 1, \cdots, m,$$

$$\int_{x_1}^{x_2} \varphi_i(x)\varphi_j(x)\mathrm{d}x = \begin{cases} 0, & i \neq j, \\ A_i, & i = j. \end{cases}$$

The most important problem in applying orthogonal functions to control systems is that the function $h(x)$ is not available. Thus, the coefficients a_i cannot be calculated according to these equations. In control systems, these coefficients are adjusted online using adaptation laws derived from stability analysis.

2. Synchronization controller

The control law consists of a state feedback and an uncertainty estimator using Legendre polynomials:

$$u = -kE - \hat{f} - u_r,$$

where k is the matrix of feedback gains and is designed using the pole placement algorithm such that all the eigenvalues of matrix $A_c = A - Bk$ are placed at some predefined desired points. The function is our uncertainty estimator using the first m terms of Legendre polynomials, and u_r is considered for compensation for the truncation error.

3. Simulation results

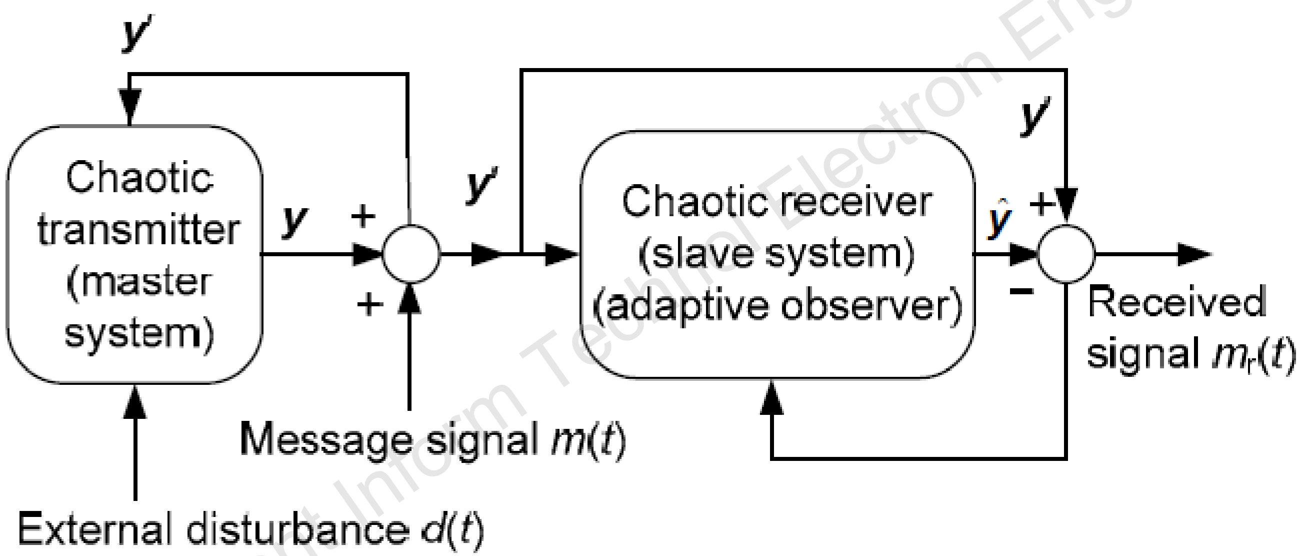

Fig. 5 Block diagram of secure communications using chaotic systems (adapted from Liao and Tsai (2000))

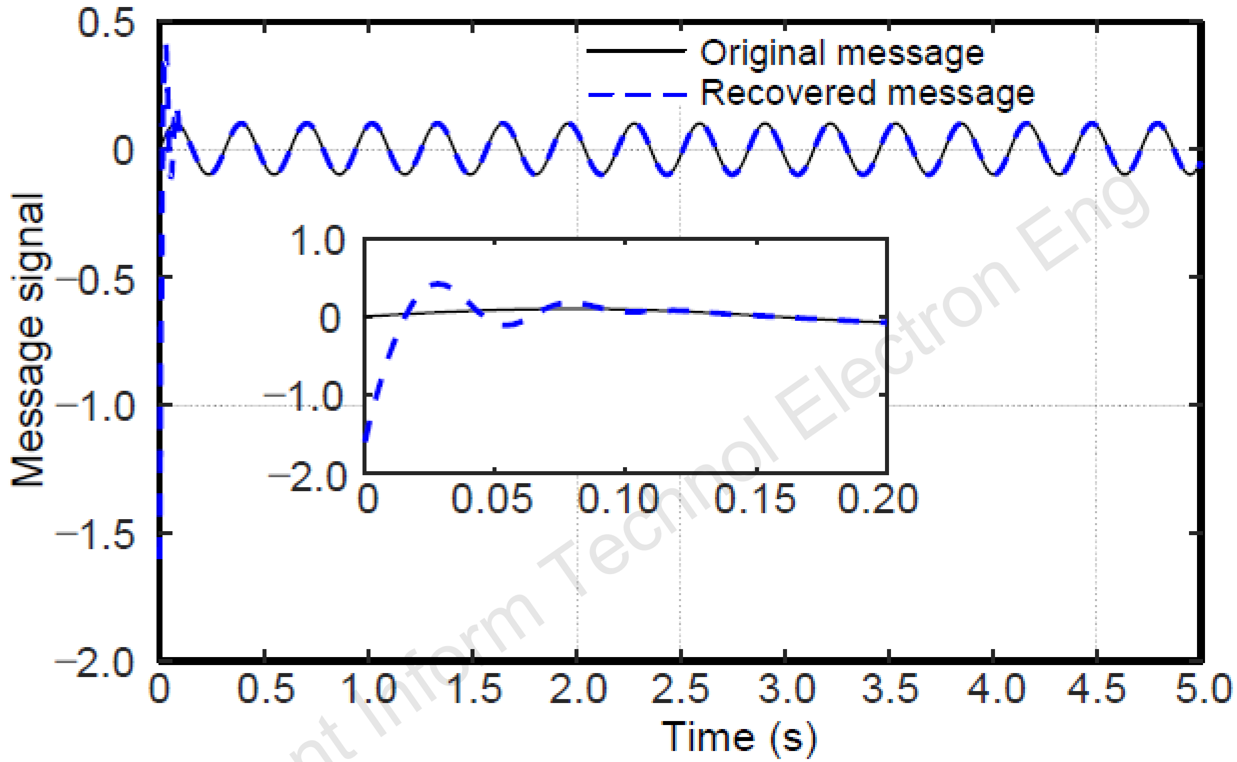

Fig. 6 Original and recovered message signals (the transient state is magnified)

Conclusions

1. Due to the orthogonal functions theorem, Legendre polynomials can approximate nonlinear functions with arbitrarily small approximation errors.

2. Observer-based chaos synchronization using Legendre polynomials has been presented and applied to secure communications.

3. A comparison between Legendre polynomials and a fuzzy sliding mode controller showed that the proposed algorithm can reduce the synchronization error faster.