Che LIN, Rong-hao ZHENG, Gang-feng YAN, Shi-yuan LU, 2018. Convergence analysis of distributed Kalman filtering for relative sensing networks. *Frontiers of Information Technology & Electronic Engineering*, 19(9):1063-1075.
https://doi.org/10.1631/FITEE.1700066

Convergence analysis of distributed Kalman filtering for relative sensing networks

Key words: Relative sensing network; Distributed Kalman filter; Schur stable; Linear matrix inequality

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Motivations

1. Traditional Kalman filter algorithm is optimal for linear systems with exact system models, but it needs a fusion center to gather all the system information

2. In large-scale WSNs, a centralized estimator for the whole system is both restrictive and infeasible, which motivates us to focus on distributed algorithms

Problem statement

Dynamical model

$$x_i = A_i x_i^- + w_i, \quad w_i \sim N(\mathbf{0}, Q_i)$$

Measurement model

$$y_{ij} = C_{ij} x_{ij} + v_{ij}, j \in N_i, \quad v_{ij} \sim N(\mathbf{0}, R_{ij})$$

Measurement model of an anchor node

$$y_{i0} = C_{i0} x_{i0} + v_{i0}, \quad v_{i0} \sim N(\mathbf{0}, R_{i0})$$

Assumption 1 Suppose that each pair (A, C_{ij}), $i \in V, i \neq 0, j \in N_i$, is observable.
Assumption 2 The undirected graph of the system is connected, and the topology of the graph is fixed.

Problem statement

$$\boldsymbol{x}_i = \boldsymbol{A}_i \boldsymbol{x}_i^- + \boldsymbol{w}_i, \quad \boldsymbol{w}_i \sim N(\boldsymbol{0}, \boldsymbol{Q}_i), \qquad (1)$$

$$\boldsymbol{y}_{ij} = \boldsymbol{C}_{ij} \boldsymbol{x}_{ij} + \boldsymbol{v}_{ij}, j \in N_i, \quad \boldsymbol{v}_{ij} \sim N(\boldsymbol{0}, \boldsymbol{R}_{ij}), \quad (2)$$

$$\boldsymbol{y}_{i0} = \boldsymbol{C}_{i0} \boldsymbol{x}_{i0} + \boldsymbol{v}_{i0}, \quad \boldsymbol{v}_{i0} \sim N(\boldsymbol{0}, \boldsymbol{R}_{i0}), \qquad (3)$$

Problems:

Consider a homogeneous relative sensing network modeled by an undirected graph and suppose that:
(a) Each node is governed by the dynamical model in Eq. (1);
(b) Each node has the measurements of the relative states between itself and its neighbors, modeled in Eq. (2);
(c) Each anchor node also has the measurements of its own absolute state, modeled in Eq. (3).
Find a distributed estimation scheme to reduce the overall estimation error of the large-scale network.

Optimal Kalman estimator

Centralized optimal estimators

$$\begin{cases} \bar{X} = A^* \hat{X}^-, \\ \hat{X} = \bar{X} + K(Y - H\bar{X}), \\ K = \bar{P}H^{\mathrm{T}}(R + H\bar{P}H^{\mathrm{T}})^{-1}, \\ \bar{P} = A^* P^- A^{*\mathrm{T}} + Q, \\ P = (I - KH)\bar{P}. \end{cases}$$

Full state observability

Theorem 1 Under Assumptions 1 and 2, a relative sensing network is full state observable if and only if graph G is connected.

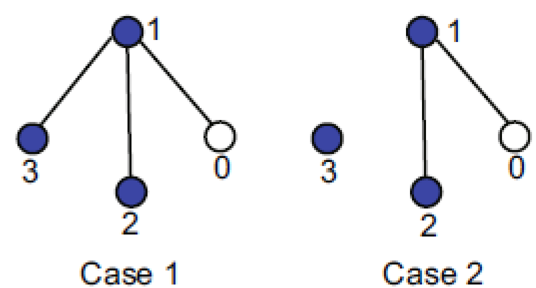

Fig. 3 Intuitive explanation for Theorem 1

Unconnected
(node 3 cannot receive information from anchor node 0)

Distributed suboptimal estimators

Design of estimators

$$\begin{cases} \bar{x}_i = A\hat{x}_i^-, \\ \hat{x}_i = \bar{x}_i + K_i(Y_i - H_i\bar{X}), \\ K_i = \bar{P}_i H_{ii}^{\mathrm{T}}(R_i + \sum_j H_{ij}\bar{P}_j H_{ij}^{\mathrm{T}})^{-1}, \\ \bar{P}_i = AP_i^- A^{\mathrm{T}} + Q_i, \\ P_i = (I - K_i H_{ii})\bar{P}_i, \end{cases}$$

Convergence analysis

Theorem 2 P_i in the distributed algorithm is convergent if there exist gains K_i and positive definite matrices $P_i^*, i = 1, 2, \ldots, n$, such that

$$P_i^* > (I - K_i H_{ii}) A P_i^* A^{\mathrm{T}} (I - K_i H_{ii})^{\mathrm{T}} + \sum_{j, j \neq i} K_i H_{ij} A P_j^* A^{\mathrm{T}} H_{ij}^{\mathrm{T}} K_i^{\mathrm{T}}. \quad (18)$$

Also, K_i is convergent.

Theorem 3 Suppose Eq. (18) is satisfied. Then the real estimation error covariance matrix P_{dis} will finally converge if and only if S_e^* is Schur stable.

Simulation

System models

$$\begin{cases} x_i = Ax_i^- + w_i, \\ y_{ij} = C_{ij}x_{ij} + v_{ij}, \end{cases}$$

where $A = C = R = I_2$, $Q = 0.01I_2$, $C_i = C_{ij} = C$, $R_{ij} = R_i$, $Q_1 = Q_2 = Q_3 = 5Q$, $Q_4 = Q_6 = 2Q$, $Q_5 = 12Q$, $R_1 = R_5 = R$, $R_2 = R_4 = 2R$, and $R_3 = R_6 = 4R$.

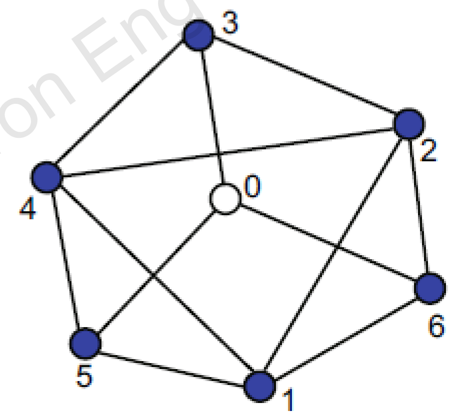

Fig. 4 Network topology of simulation 1

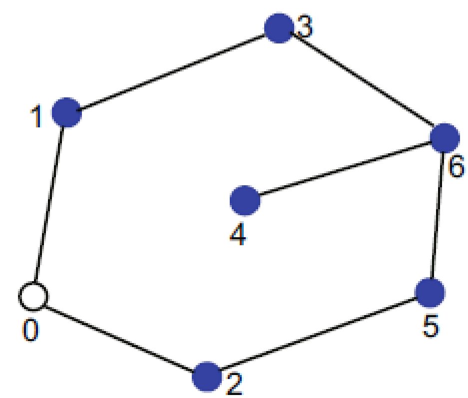

Fig. 5 Network topology of simulation 2

Simulation

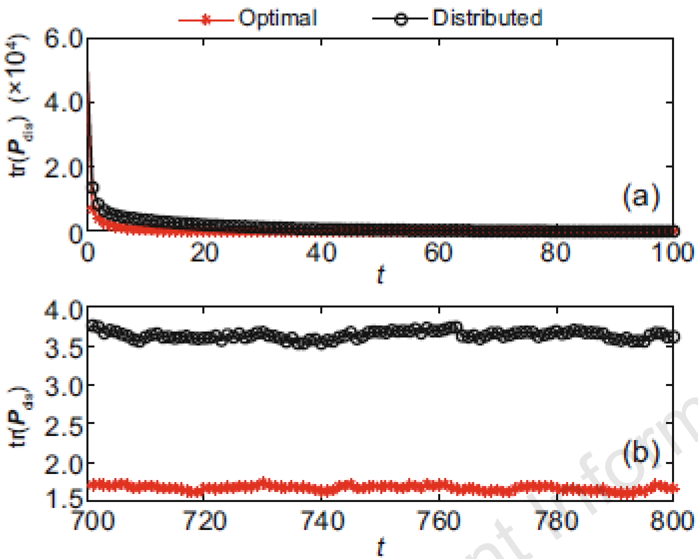

Fig. 6 Results of tr(P_{dis}) at each time step in simulation 1: (a) from time step 0 to time step 100; (b) from time step 700 to time step 800

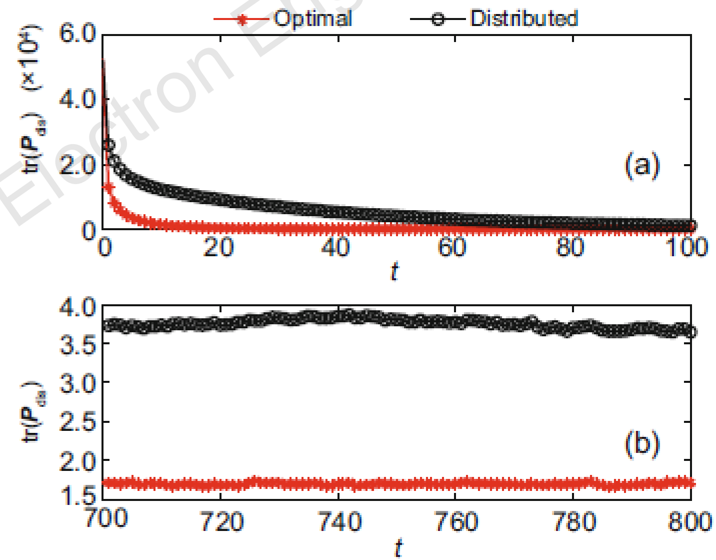

Fig. 7 Results of tr(P_{dis}) at each time-step in simulation 2: (a) from time step 0 to time step 100; (b) from time step 700 to time step 800

1. Errors in all estimators converge to steady states.
2. The estimation errors of the suboptimal estimator are larger than those of the optimal estimator.

Simulation

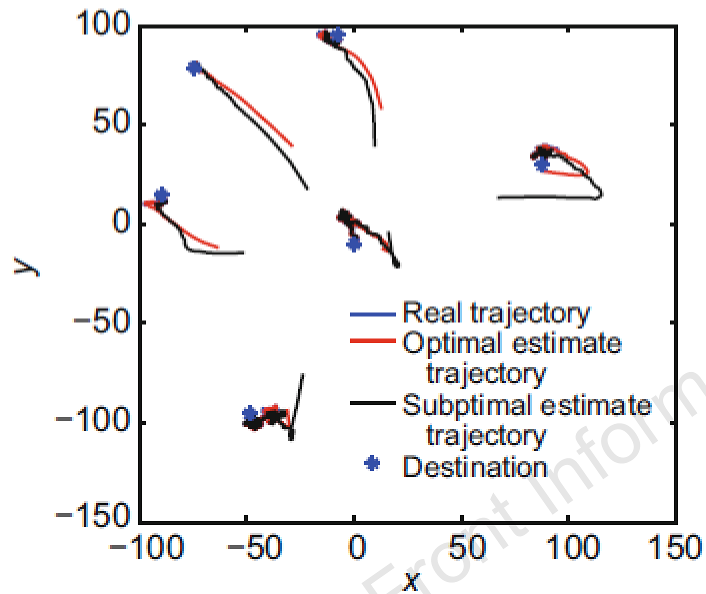

Fig. 8 Trajectory evolution of simulation 1

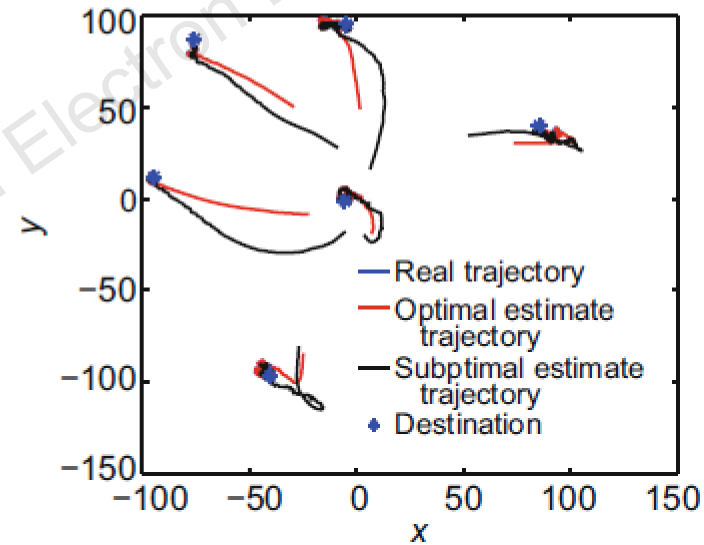

Fig. 9 Trajectory evolution of simulation 2

Both centralized and distributed estimates finally catch up with the real state.

Simulation

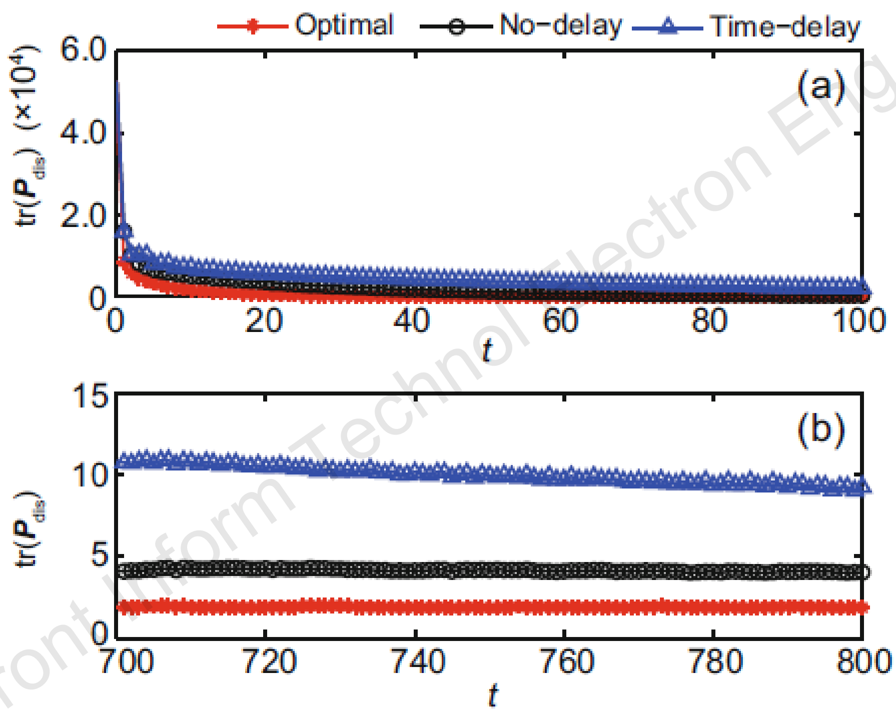

Fig. 10 Comparisons between a no-delay algorithm and a time-delay algorithm: (a) from time step 0 to time step 100; (b) from time step 700 to time step 800

With time delay, the convergence would be slower but can still be achieved with a larger trace of the estimation error

Conclusions

1. The sufficient and necessary condition for observability of relative sensing networks is presented with a detailed analysis.

2. A distributed suboptimal estimator is designed by ignoring the correlated edge covariance of the centralized optimal estimator. The solutions of n LMIs are prepared as the condition for convergence, and the network topology and parameters also influence the stability of the estimation error.

3. Simulations with different topologies are carried out to verify the performance of the proposed estimators.

Future work

In future work, the emergent task is to determine the equivalent condition for the stability of $\boldsymbol{S}_e^*$. In addition, some other distributed algorithms will be discussed, and analyzing the performance differences between them would be interesting and necessary. The effect of time-delay transmission has been simulated in this paper, and further theoretical analysis would be interesting and important. Relative sensing networks modeled by directed graphs and switching topology will also be considered.