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Approximate Gaussian conjugacy: parametric recursive filtering under nonlinearity, multimodality, uncertainty, and constraint, and beyond

Key words: Kalman filter; Gaussian filter; time series estimation; Bayesian filtering; nonlinear filtering; constrained filtering; Gaussian mixture; maneuver; unknown inputs.

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Motivation

- Great progress has been seen for parametric recursive filtering theory and algorithms in the last decade, with the rapid development of sensors and ever-increasing proliferation of smartphones, mobile robots and unmanned vehicles, etc.
- Existing reviews/surveys omit or only addressed briefly some important issues : (1) a unifying framework to analyse the common essences of different filters; (2) very informative observation systems; (3) the classification of multimodal systems, intractable uncertainties, and constraints.
- Several interesting, crucial findings in the literature that can be easily overlooked need the high attention of the community.

Main idea

- We base the review on a transparent and concise framework termed 'approximate Gaussian conjugacy (AGC)', which organizes different approaches along the same line.
- To go beyond a pure review, we also include discussions on alternatives to the first-order hidden Markov model (HMM) and on filter evaluation regarding computing speed, with our new thoughts and prospect.
- We highlight several significant issues that can easily be ignored, which summarize the key parts of our review.

Highlights (I)

- CRLB (Cramer-Rao Lower Bound) limits only the variance of unbiased estimators and lower MSE (mean squared error) can be obtained by allowing for a bias in the estimation, while ensuring that the overall estimation error is reduced.
- The KF (Kalman filter) is conditionally biased with a non-zero process noise realization in the given [deterministic] state sequence and is not an efficient estimator in a conditional sense, even in a linear and Gaussian system.
- Among all possible distributions of the observation noise w with a fixed covariance matrix, the CRLB for x attains its maximum when ww is Gaussian, i.e., the Gaussian scenario is the ``worst-case'' for estimating x.

Highlights (II)

- For sufficiently precise measurements, none of the KF variants, including the KF itself, are based on an accurate approximation of the joint density. Conversely, for imprecise measurements all KF variants accurately approximate the joint density, and therefore the posterior density. Differences between the KF variants become evident for moderately precise measurements.
- While the BCRLB (Bayesian Cramer-Rao Lower Bound) sets a best line (in the sense of MMSE) that any unbiased sequential estimator can at maximum achieve, the O2 (Observation-only) inference sets the bottom line that any ``effective'' estimator shall at worst achieve.

Highlights (III)

- Many adaptive-model approaches proposed for MTT (manuevering target tracking) may show superiority when the target indeed maneuvers but perform disappointingly or even significantly worse than those without using an adaptive model, when there is actually no maneuver. We call this over-reaction due to adaptability.
- The theoretically best achievable second order error performance, namely the CRLB, in target state estimation is independent of knowledge (or the lack of it) of the observation noise variance.

Highlights (IV)

- Robust filtering is much more related to robustness with respect to statistical variations than it is to optimality with respect to a specified statistical model. Typically, the worst case estimation error rather than the MSE needs to be minimized in a robust filter. As a result, robustness is usually achieved by sacrificing the performance in terms of other criteria such as MSE and computing efficiency.
- The standard structure of recursive filtering is based on infinite impulse response (IIR), namely all the observations prior to the present time have effect on the state estimate at present time and therefore the filter suffers from legacy errors.

Highlights (V)

- Computing speed matters! Disregarding this key issue may lead to endlessly seeking complicated modeling and/or filtering strategies for a fantastically better result, which may never come true in reality.

Conclusions (I)

- Advances in time series parametric filters have been reviewed in four major categories, including nonlinearity (especially VIO nonlinear systems), multimodality (including GM filtering and MTT), intractable uncertainties (including unknown and non-Gaussian inputs/noise) and constraints. We pointed out that a key concept behind these works is AGC. A few important points have been given in highlights, as well as some of our thoughts on HMM and practical filter evaluation.

Conclusions (II)

- Instead of addressing any applications of these filters, we put our focus on the common and general theory and algorithm design. However, we note that, efficient filter design should be based on the specific problem characteristic and requirement, e.g., estimation in robotics is very different to that in geosciences and the problem of fault diagnosis is very different to that of target tracking. But, one thing is for sure: VIO (very informative observation) plays progressively more important roles in all realms due to the revolutionary development of sensors and their massive deployment.

Conclusions (III)

- The rapid development of sensors and their joint deployment, e.g., large scale sensor networks, provide a foundation for new paradigms to address the challenges that arise in harsh environments. Consequently, the signal processing community starts to manifest increasing eagerness in novel data fusion/mining methods such as clustering, data fitting, and model learning, for incorporating advanced statistical tools and rich sensor data for substantial performance enhancement.