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Hohmann transfer via constrained optimization

Key words: Hohmann transfer; Nonlinear programming; Constrained optimization; Calculus of variations

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Motivations

1. In 1925, Hohmann first described the well-known optimal orbital transfer between two circular coplanar space orbits by numerical examples. It is found that a strict analytic proof for the global minimum of the Hohmann transfer has not been available.

2. We consider the Hohmann transfer problem as a dynamic optimization problem with equality constraints to validate an indirect optimization method for the orbital transfer and interception problems of spacecraft.

Main ideas

1. Inspired by the geometric method (Marec, 1979), we transform the Hohmann transfer problem into a static nonlinear programming problem with an inequality constraint.

2. Two dynamic optimization problems for the Hohmann transfer problem are formulated, one of which has an unspecified final time and the other has interior point constraints with a known final time.

Methods

1. We analytically prove the global minimum of the Hohmann transfer using the results in nonlinear programming such as the well-known Kuhn-Tucker theorem and a second-order sufficient condition for minima.

2. By variational methods, the related dynamic optimization problems are converted into two- and multi-point boundary-value problems. Matlab boundary value problem solvers are used to solve them.

Major results

1. The global minimum of the Hohmann transfer analytically appears. Two sets of feasible solutions are found: one corresponding to the Hohmann transfer is the global minimum, and the other is a local minimum.

2. Boundary conditions are derived. The numerical solutions show that the variational method as an indirect optimization method provides highly accurate solutions by using Matlab boundary value problem solvers once the solutions are found by trial and error.

Major results (Cont'd)

The analytic proof of the Hohmann transfer is as follows:

Theorem 1 For the constrained optimization problem

$$\min_{x_0, y_0} \Delta v(x_0, y_0) \tag{7}$$

$$\text{s.t. } x_0^2 + (1+y_0)^2(1-\bar{r}_{\mathrm{f}}^{-2}) - 2(1-\bar{r}_{\mathrm{f}}^{-1}) \geq 0,$$

there are two sets of feasible solutions, one of which corresponds to the Hohmann transfer and is the global minimum, and the other is a local minimum.

Numerical examples

Example 1 Static optimization

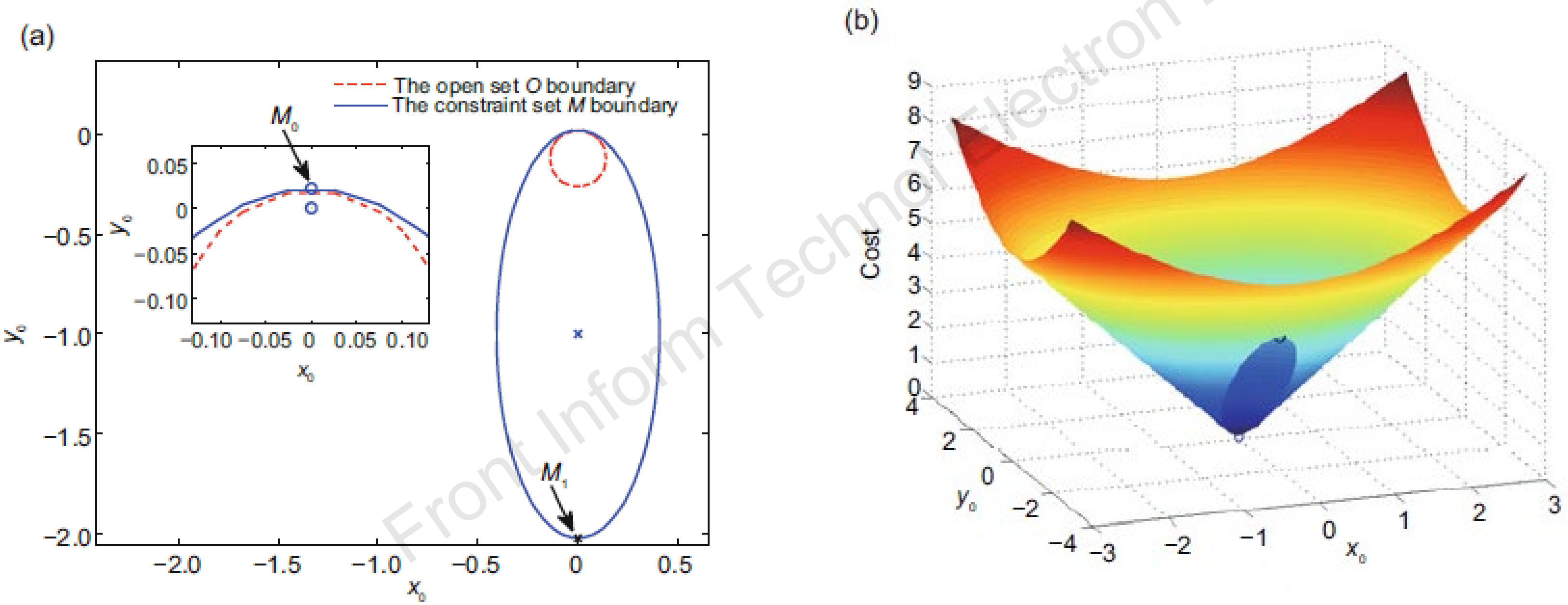

Fig. 1 Hodograph plane (a) and the 3D graph of the cost function $\Delta v(x_0, y_0)$ (b)

Numerical results (Cont'd)

Example 2 Dynamic optimization

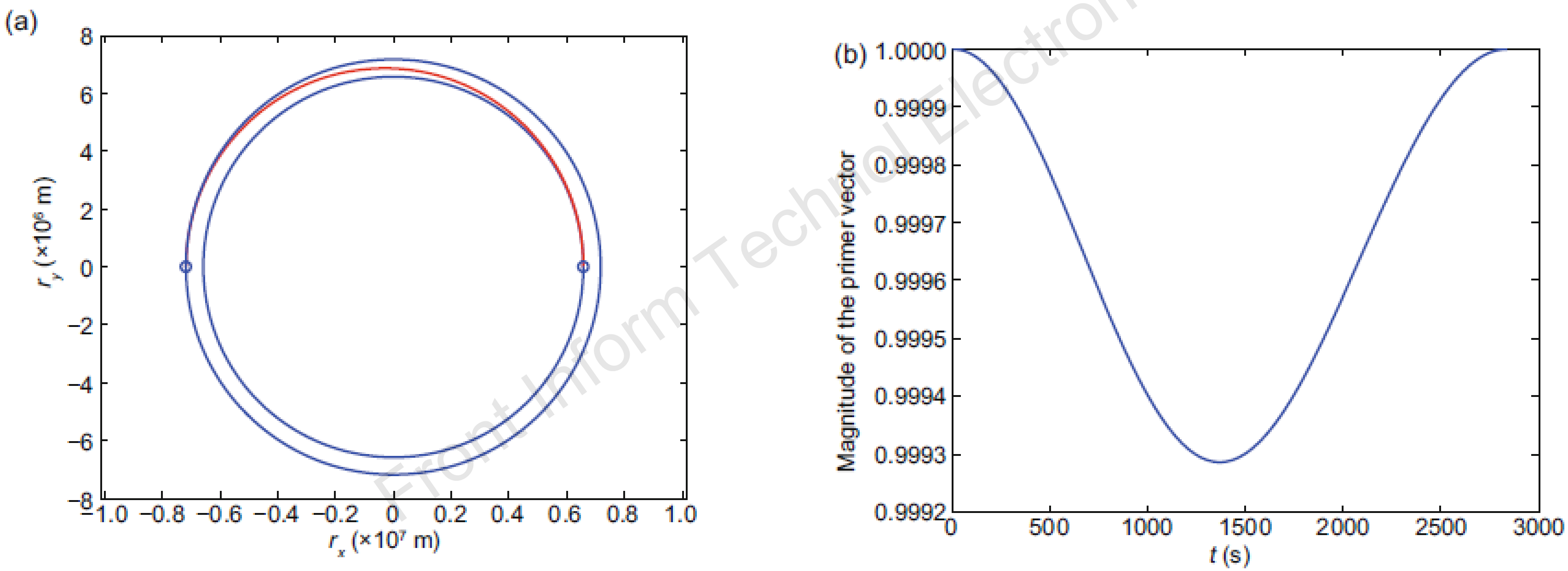

Fig. 2 Hohmann transfer (a) and the magnitude of the primer vector in Example 2 for Problem 1 (b)

Numerical results (Cont'd)

Example 3 Dynamic optimization

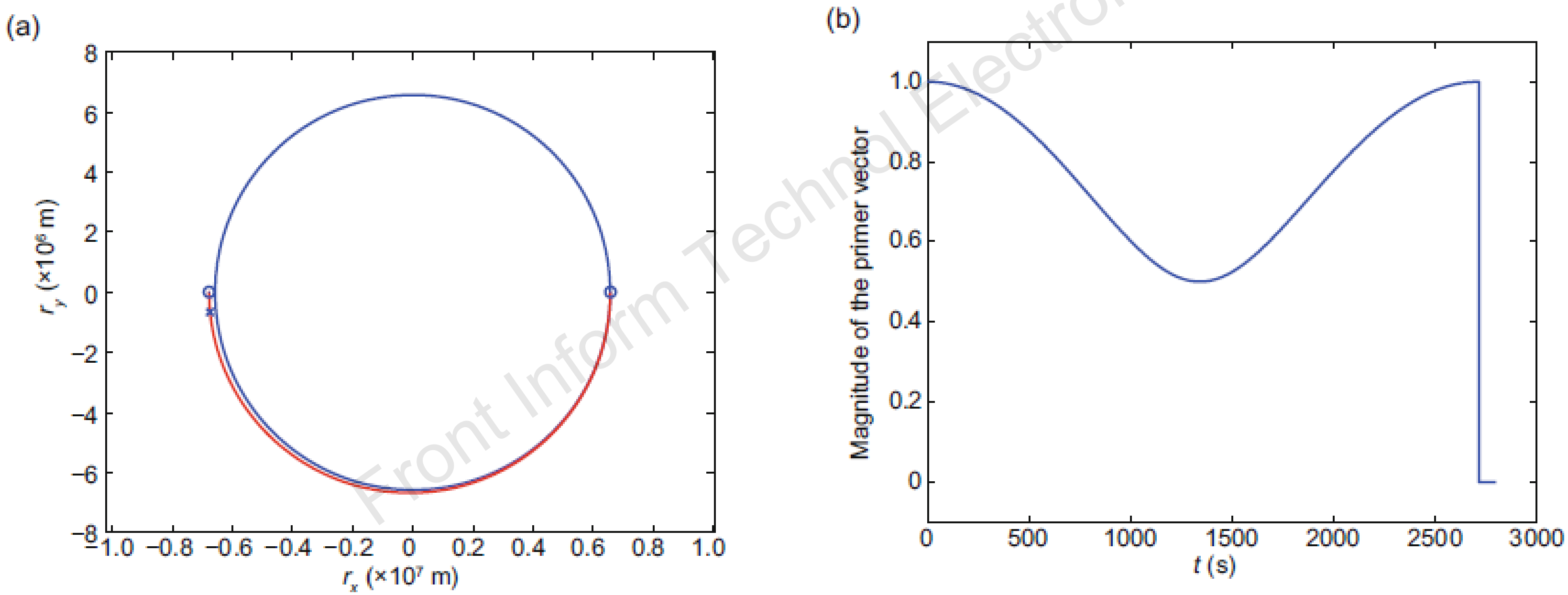

Fig. 5 Orbital transfer (a) and the magnitude of the primer vector corresponding to the local minimum in Example 3 for Problem 2 (b)

Conclusions

By formulating the Hohmann transfer problem as static and dynamic constrained optimization problems, the solution to the Hohmann transfer problem is re-discovered and its global minimum is analytically verified.