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Dynamics of a neuron exposed to integer- and fractional-order discontinuous external magnetic flux

Key words: Fitzhugh-Nagumo; Chaos; Fractional order; Magnetic flux

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Motivation

1. For many years, modeling of neurons has been a hot topic.
2. One of the basic neuron models is the Hodgkin and Huxley (HH) model.
3. A simplified two-dimensional form of the HH model is the FitzHugh-Nagumo model.
4. The calculation of fractional-order systems has been an interesting topic since the 17th century.
5. In this paper, a modified FitzHugh–Nagumo neuron (MFNN) model is proposed.
6. The integer-order MFNN system (case-A) and a fractional-order MFNN system (case-B) are investigated using the proposed model to investigate the effect of differentiator.

Main idea and method

The improved FNN model

$$\begin{cases} \dot{x} = -kx(x-a)(x-1) - xy + I_e + k_0\rho(\phi)x, \\ \dot{y} = \left(\varepsilon + \dfrac{\mu_1 y}{x+\mu_2}\right)(-y - kx(x-a-1)), \\ \dot{\phi} = k_1 x - k_2 z + \phi_e, \end{cases} \quad (1)$$

The magnetic flux effects are discontinuous as the ion concentration thresholds keep shifting.

$$q(\phi) = \beta\phi + 0.5(\alpha - \beta)\left(|\phi+1| - |\phi-1|\right), \quad (2)$$

$$\rho(\phi) = \frac{d^\sigma q(\phi)}{d^\sigma \phi}. \quad (3)$$

Main idea and method

$$\begin{cases} \dot{x} = -kx(x-a)(x-1) - xy + I_e + k_0 D^\sigma q(\phi_k)x, \\ \dot{y} = \left(\varepsilon + \dfrac{\mu_1 y}{x+\mu_2}\right)(-y - kx(x-a-1)), \\ \dot{\phi} = k_1 x - k_2 z + \phi_e, \\ D^\sigma q(\phi_k) = (\beta + 0.5(\alpha-\beta)(\mathrm{sgn}(\phi_{k-1}+1) \\ \qquad -\mathrm{sgn}(\phi_{k-1}-1)))\, h^\sigma - \sum_{j=1}^{N} B_j^\sigma \phi(t_{k-j}), \end{cases} \quad (4)$$

Dynamical properties of the MFNN are separately derived for case A (integer order) and case B (fractional order).

Major results

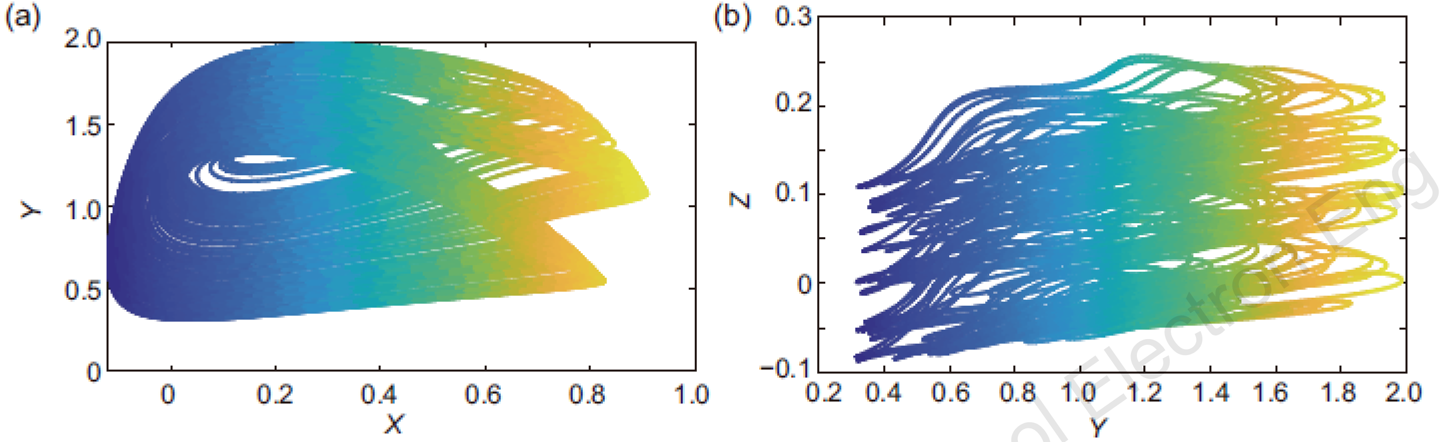

Fig. 1 The 2D phase portraits of the modified Fitzhugh-Nagumo neuron system with integer-order magnetic flux in XY (a) and YZ (b) planes

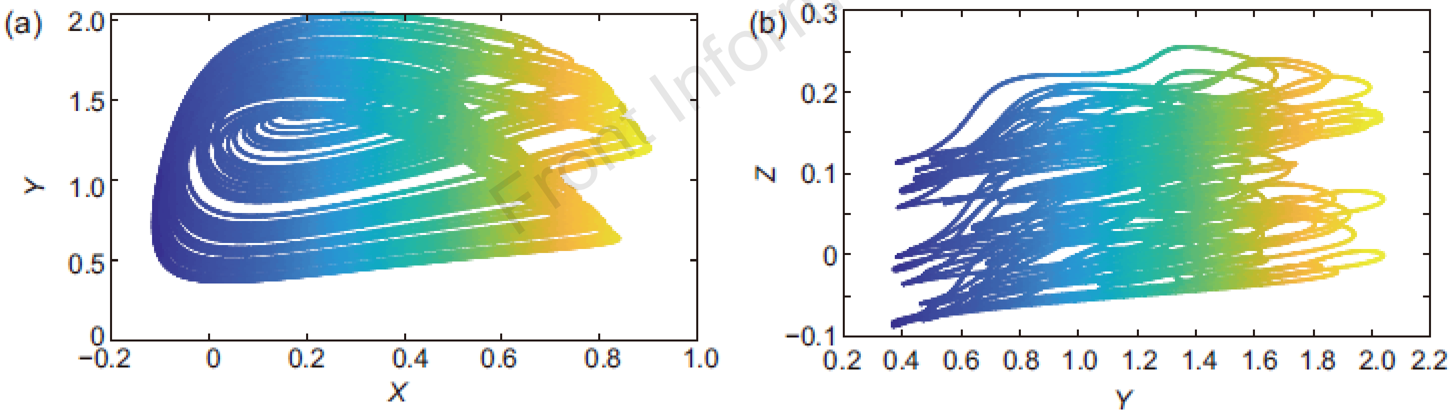

Fig. 4 The 2D phase portraits of the modified Fitzhugh-Nagumo neuron system with fractional-order magnetic flux in XY (a) and YZ (b) planes

Major results

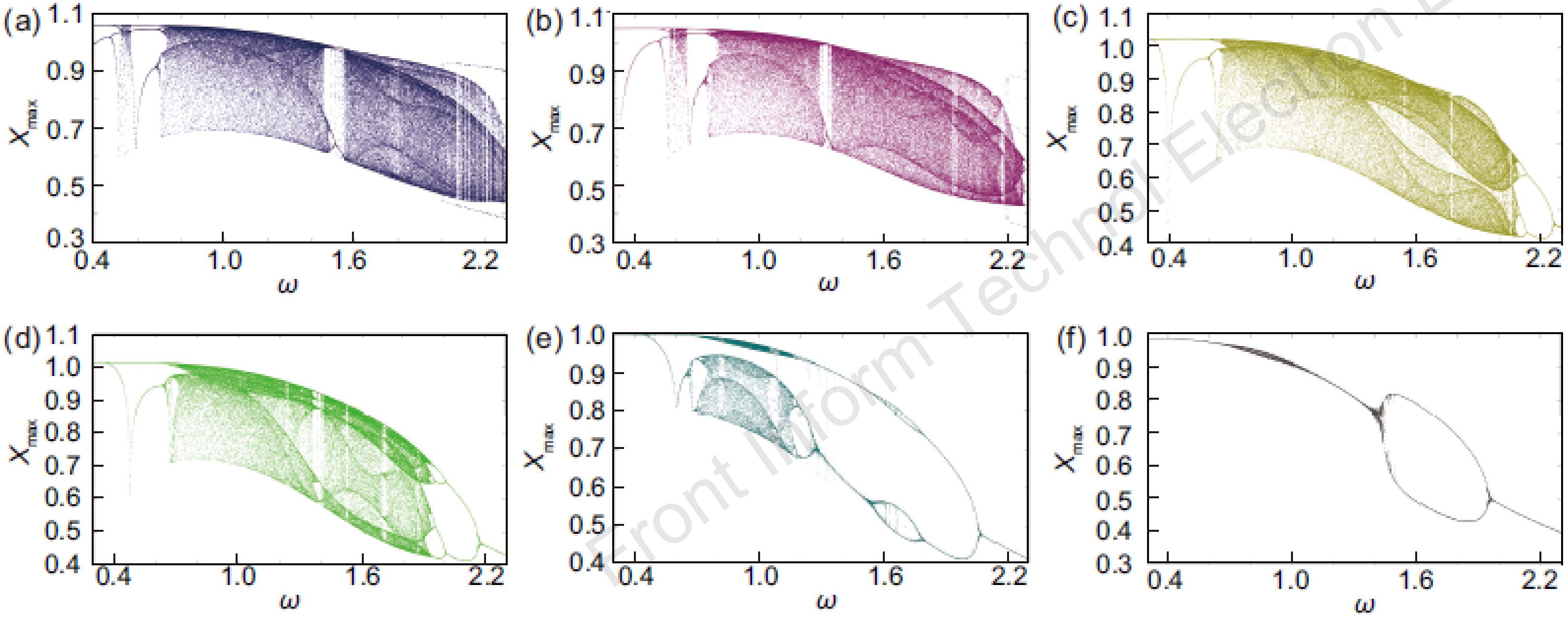

Fig. 3 Bifurcation of the modified Fitzhugh-Nagumo neuron system in case A with respect to changing ω for $\alpha = 0.1$ (a), $\alpha = 0.2$ (b), $\alpha = 0.4$ (c), $\alpha = 0.5$ (d), $\alpha = 0.6$ (e), and $\alpha = 0.7$ (f)

Major results

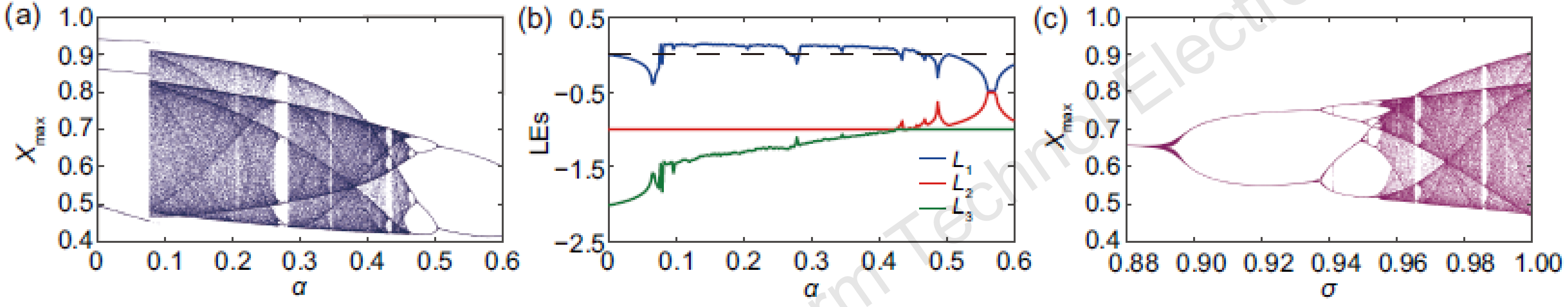

Fig. 5 (a) and (b) are the bifurcation and Lyapunov exponents of the modified Fitzhugh-Nagumo neuron system in case B with respect to changing α, respectively, and (c) is the bifurcation of the modified Fitzhugh-Nagumo neuron system in case B with respect to changing σ

Conclusions

1. The dynamical behaviors of neurons are very important.

2. A modified FitzHugh–Nagumo neuron model has been proposed.

3. The model has been studied with two approaches.

4. In the first approach (case A), the magnetic flux has been considered of an integer order, while in the second approach (case B), it has been considered of a fractional order.

5. Electromagnetic induction and radiation can cause variation in the distribution of memductance and induction. Thus, studies of fractional-order magnetic flux are important.

6. Fractional-order setting can be used to investigate non-uniform diffusion.

7. The results showed that only the integer-order system has an antimonotonicity feature.

8. The initial condition of the magnetic flux variable has been very important in the dynamical behavior of the MFNN model.