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Differential coupling contributes to synchronization via a capacitor connection between chaotic circuits

Key words: Synchronization; Voltage coupling; Chaotic circuit; Cpacitor coupling

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Motivation

1. Resistor is often used to bridge connection between nonlinear circuits for a synchronization approach, while Joule heat consumed by the coupling resistor could cause damage on these circuits.

2. Capacitor, inductor, and memristor are important electric devices, and it is important to explore whether a capacitor can realize synchronization between nonlinear circuits without consuming Joule heat.

Main idea

1. Capacitor can be used to connect two nonlinear circuits by triggering time-varying electric field in the coupling capacitor, and field energy flow is passed and pumped to modulate the coupled circuits for reaching synchronization without consuming Joule heat.

2. Physical mechanism for capacitor coupling can be understood as a kind of electric field coupling.

Method

1. Two nonlinear circuits are connected via a capacitor, and dimensionless dynamical systems are approached to detect whether the synchronization can be realized.

2. Symmetrical and cross coupling are considered. Scale transformation is applied to the circuits, and thus dynamical equations can be approached.

3. Error functions for variables and phase series are calculated to detect complete synchronization and phase synchronization, respectively.

Major results

1. The coupled circuits are connected by a capacitor.

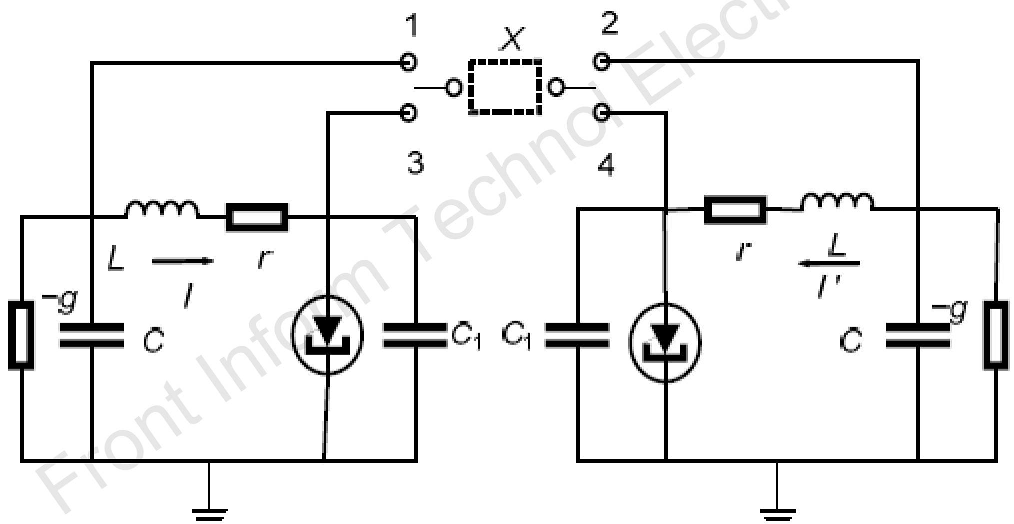

Fig. 3 Schematic of two PR circuits under different coupling channels and styles

X denotes the coupling devices, which can be resistors or capacitors. i=1, 3, j=2, 4 represent the output ends

Major results (Cont'd)

2. Scale transformation and dimensionless system:

$$\begin{cases} x = \dfrac{I}{I_0} - 1,\ y = \dfrac{V - Ir - U_0}{\omega L I_0},\ z = \dfrac{U}{U_0} - 1,\ \tau = t\omega, \\ \omega = \sqrt{\dfrac{1 - gr}{LC}},\ \delta = \dfrac{U_0}{\omega I_0 L},\ 2\gamma = \dfrac{gL - rC}{\omega LC}, \\ \alpha = \dfrac{rU_0}{\omega^2 L^2 I_0},\ \beta = -1 + \dfrac{gU_0}{\omega^2 I_0 LC},\ \mu = \dfrac{I_0}{\omega C_1 U_0}. \end{cases} \tag{2}$$

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}\tau} = y - \delta z, \\ \dfrac{\mathrm{d}y}{\mathrm{d}\tau} = -x + 2\gamma y + \alpha z + \beta, \\ \dfrac{\mathrm{d}z}{\mathrm{d}\tau} = \mu(x + z - z^3). \end{cases} \tag{3}$$

Major results (Cont’d)

3. Circuit equations under capacitor coupling:

$$
\begin{cases}
L\dfrac{\mathrm{d}I}{\mathrm{d}t}=V-U-rI,\\
C\dfrac{\mathrm{d}V}{\mathrm{d}t}=gV-I-\lambda_1 I_X,\\
C_1\dfrac{\mathrm{d}U}{\mathrm{d}t}=I-I_1-\lambda_3 I_X,
\end{cases}
\quad
\begin{cases}
L\dfrac{\mathrm{d}\hat{I}}{\mathrm{d}t}=\hat{V}-\hat{U}-r\hat{I},\\
C\dfrac{\mathrm{d}\hat{V}}{\mathrm{d}t}=g\hat{V}-\hat{I}+\lambda_2 I_X,\\
C_1\dfrac{\mathrm{d}\hat{U}}{\mathrm{d}t}=\hat{I}-I_1+\lambda_4 I_X,
\end{cases}
\tag{4}
$$

$$
I_X=\begin{cases}
I_g=\pm g_X(V_i-V_j),\\
I_C=\pm C_X\dfrac{\mathrm{d}(V_i-V_j)}{\mathrm{d}t},
\end{cases}
\tag{5}
$$

Conclusions

1. Capacitor coupling provides evidence for electric field coupling and explains differential coupling and control on nonlinear dynamical systems.

2. Symmetrical field coupling can enhance complete synchronization when the capacitance for the coupling capacitor is beyond a certain threshold, which can pump and save more energy from the two coupled circuits.

3. Cross coupling via a capacitor can support phase synchronization.

4. It gives important guidance to understand signal exchange between neurons and neural circuits.