Shu-peng LAI, Meng-lu LAN, Ya-xuan LI, Ben M. CHEN, 2019. Safe navigation of quadrotors with jerk limited trajectory. *Frontiers of Information Technology & Electronic Engineering*, 20(1):107-119. https://doi.org/10.1631/FITEE.1800719

Safe navigation of quadrotors with jerk limited trajectory

Key words: Quadrotor; Unmanned aerial vehicle; Motion planning

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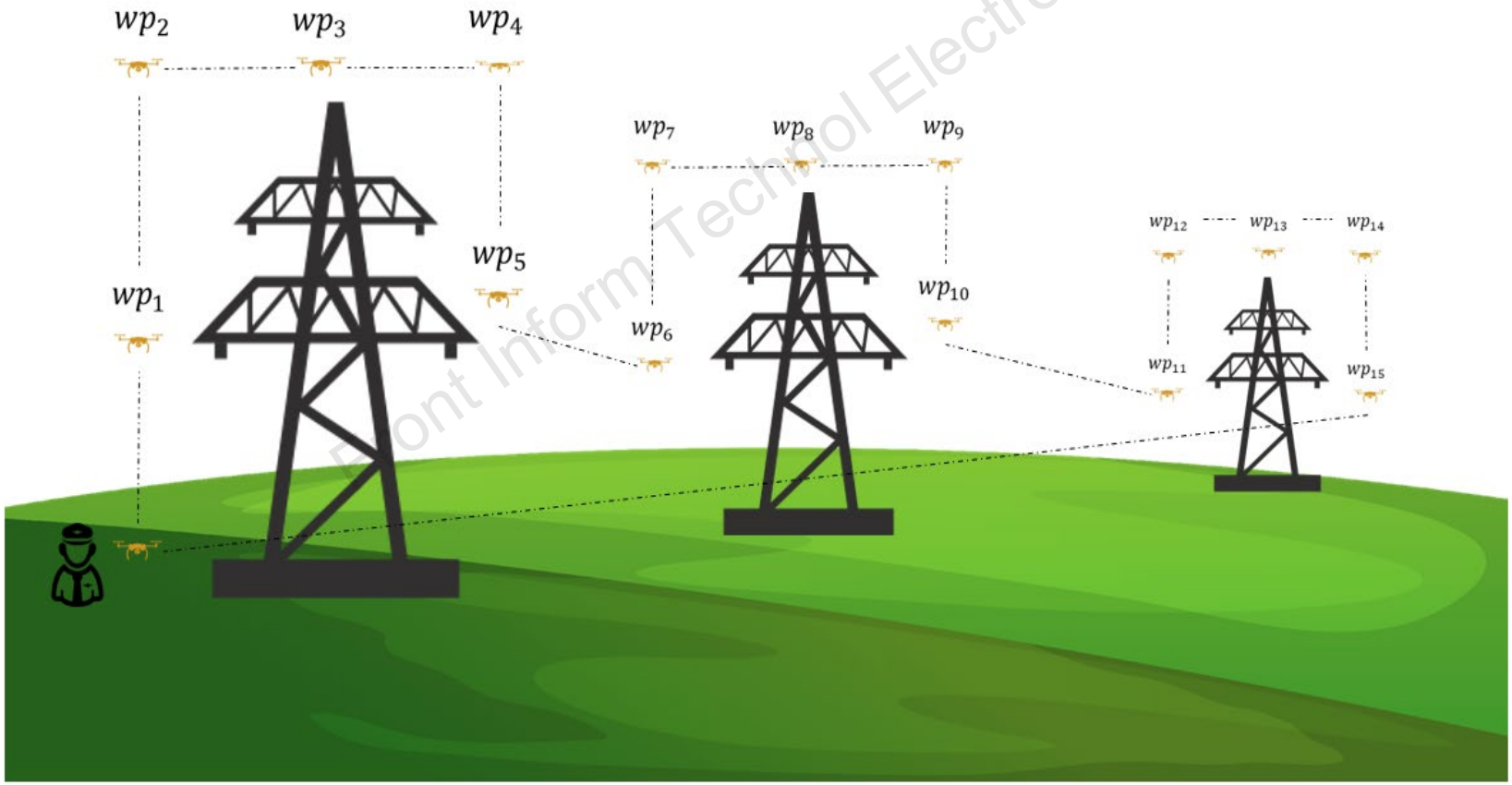
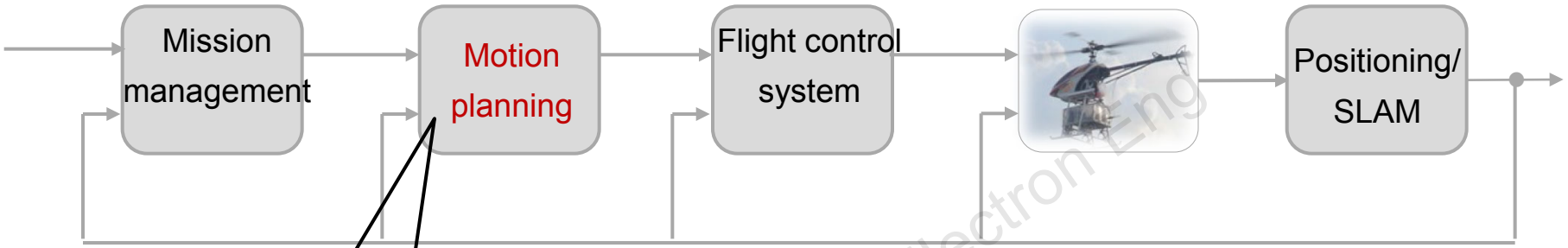
Motivation

Quadrotors are usually tasked to fly in low-altitudes and obstacle-strewn environments. The ability to navigate safely in such an environment is crucial for all other applications.



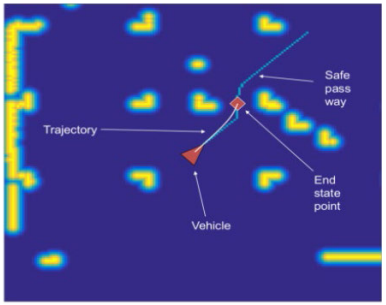
System structure

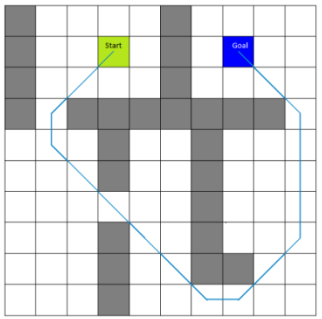


Nominal path plan consists of line-segments from the user or other algorithms.

Trajectory generation:

smooth and feasible trajectory.





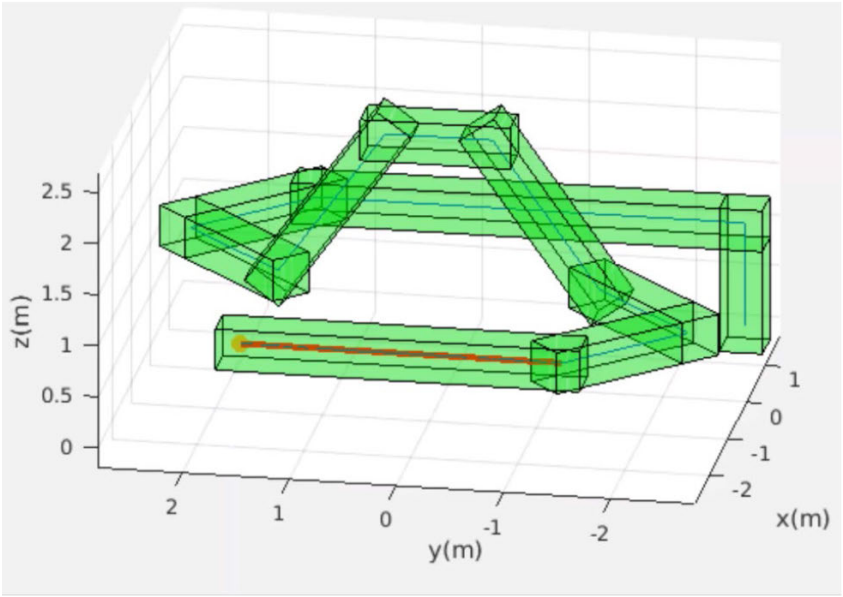
Main idea

Incremental trajectory generation in safe corridor:

1. An **efficient** algorithm that generates **3D** velocity, acceleration, and jerk-limited trajectory.

2. Instant reaction to change environment or mission.

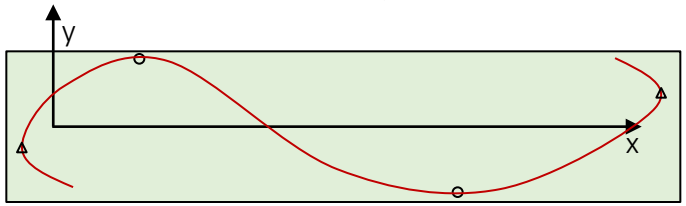
3. Navigating the nominal path plan:
   (1) safely (safe flying corridor)
   (2) smoothly (C^2 continuity)
   (3) efficiently (computing & flying)



Method overview: safe flying corridor (SFC)

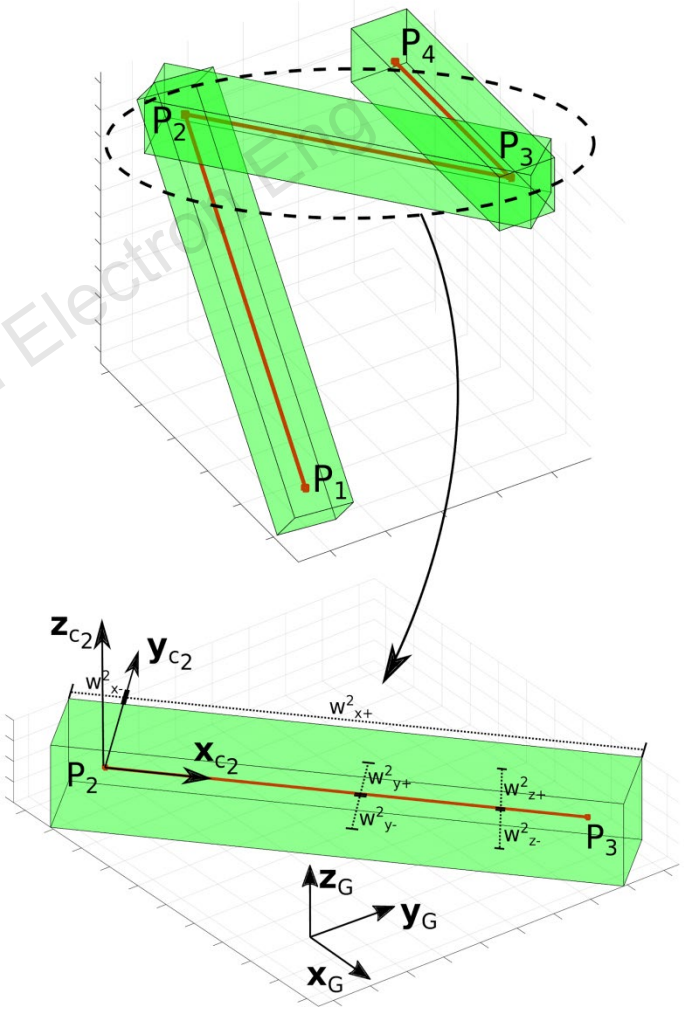
A serries of inter-connected bounding boxes:

1. Encapsulate the nominal path plan;
2. Aligned with each line-segment;
3. Defines a bounding-box frame;
4. Easy inside checking.



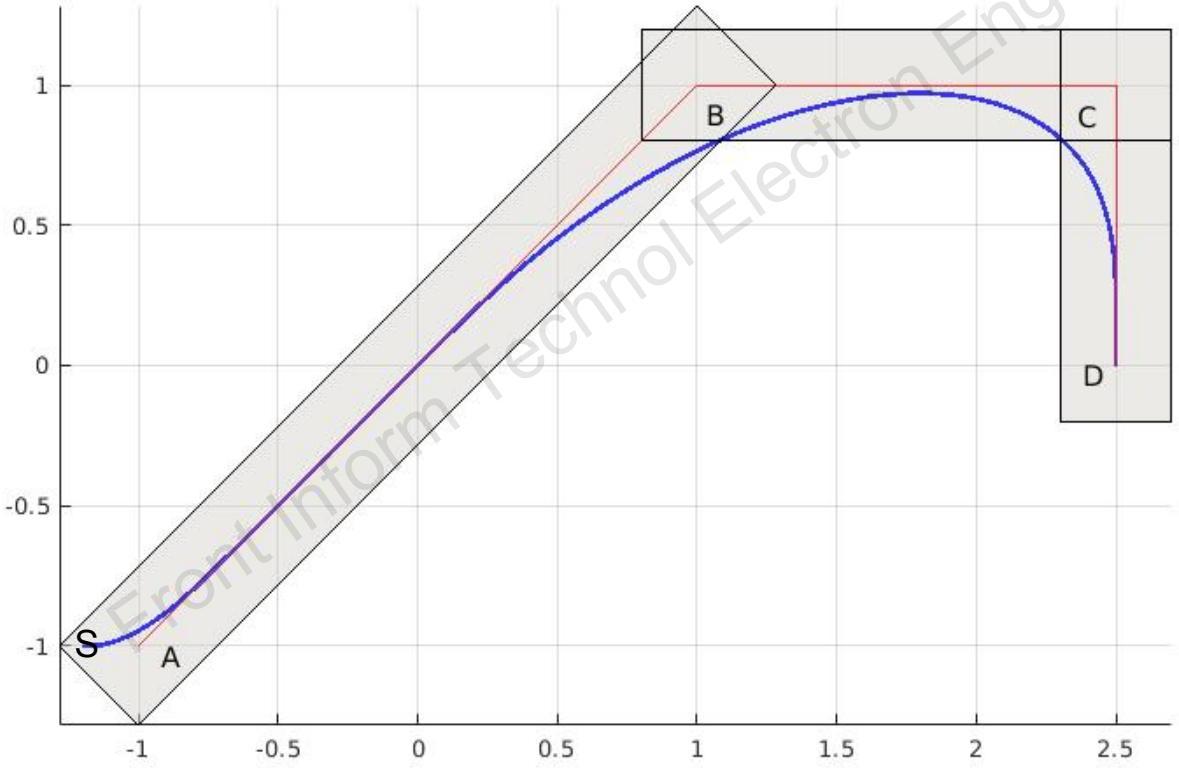
◦ *y* axis min/max

▵ *x* axis min/max



Method overview: SFC navigation

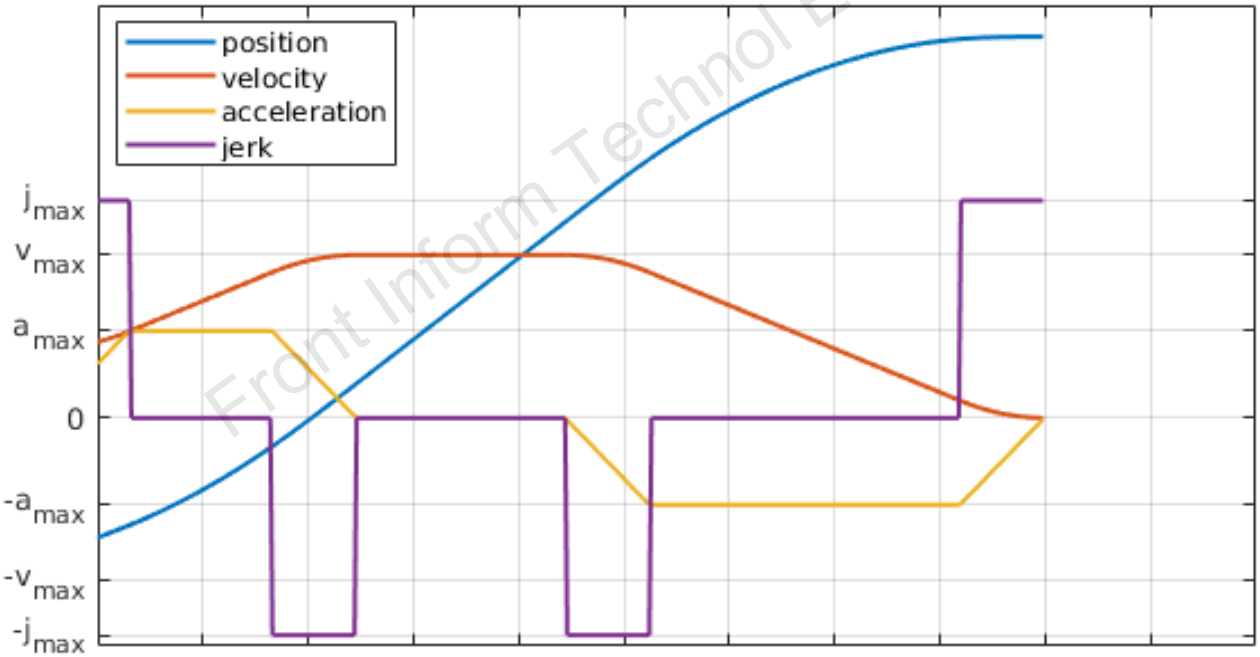
Online and incremental,
sample and throw



1. How to generate the **feasible** trajectory **fast enough**?
2. How to test its safety **efficiently**?

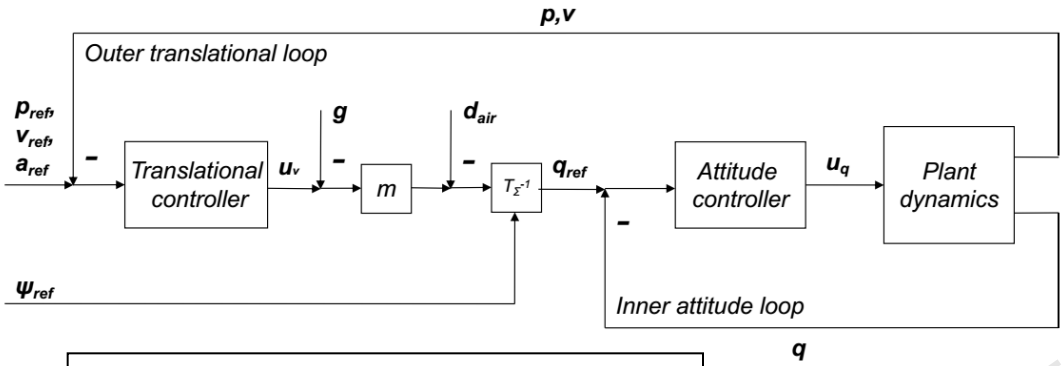
The jerk limited trajectory

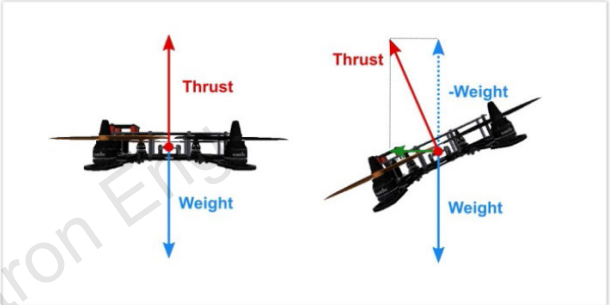
1. Not only limited jerk, but also limited acceleration and velocity.
2. Effectiveness for quadrotors is proved.



The jerk limited trajectory for a single axis

The jerk limited trajectory: why effective?





Inner loop constraints:

Limited body rate $\omega_{\max}$

and total thrust $f_{\max}$

Sufficient condition

Outer loop constraints:

The limited acceleration and jerk

$$\sqrt{\dddot{x}^2+\dddot{y}^2+\dddot{z}^2} \le (\ddot{z}_{\min}+g)\omega_{\max}.$$

$$\|f\| = \sqrt{\ddot{x}^2+\ddot{y}^2+(\ddot{z}+g)^2} \le \frac{f_{\max}}{m},$$

$$\ddot{z} \ge \ddot{z}_{\min} \ge \frac{f_{\min}}{m} - g,$$

Decoupling

Sufficient condition

Final constraints:

The limited acceleration and jerk

$$-v_{\max} \leqslant v(t) \leqslant v_{\max},$$

$$-a_{\max} \leqslant a(t) \leqslant a_{\max},$$

$$-u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max},$$

Limit the velocity for safety

Single axis constraints:

The limited acceleration and jerk

$$-a_{\max} \leqslant a(t) \leqslant a_{\max},$$

$$-u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max},$$

The jerk limited trajectory: compared with TOC

The jerk limited trajectory problem considers a system characterized by

$$\begin{pmatrix} \dot{p} \\ \dot{v} \\ \dot{a} \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} p \\ v \\ a \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_{\mathrm{j}}$$

with $-v_{\max} \leqslant v(t) \leqslant v_{\max}, \quad -a_{\max} \leqslant a(t) \leqslant a_{\max}, \quad -u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max}$

The traditional time optimal control (TOC) or bang-bang control problem considers

$$\dot{x}(t) = Ax(t) + Bu(t)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ a \end{bmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y \\ v \end{pmatrix} = \begin{pmatrix} y \\ \dot{y} \end{pmatrix} \tag{4.6}$$

Note that $v = \dot{y}$ is the velocity of the system. Let the control input be constrained as follows:

$$| u(t) | \leq u_{\max}$$

The jerk limited trajectory: velocity set-point

Problem:

From an arbitrary state to a desired velocity such that:

$$v(0) = v_0, \quad v(T) = v_{\mathrm{ref}}$$

$$a(0) = a_0, \quad a(T) = 0$$

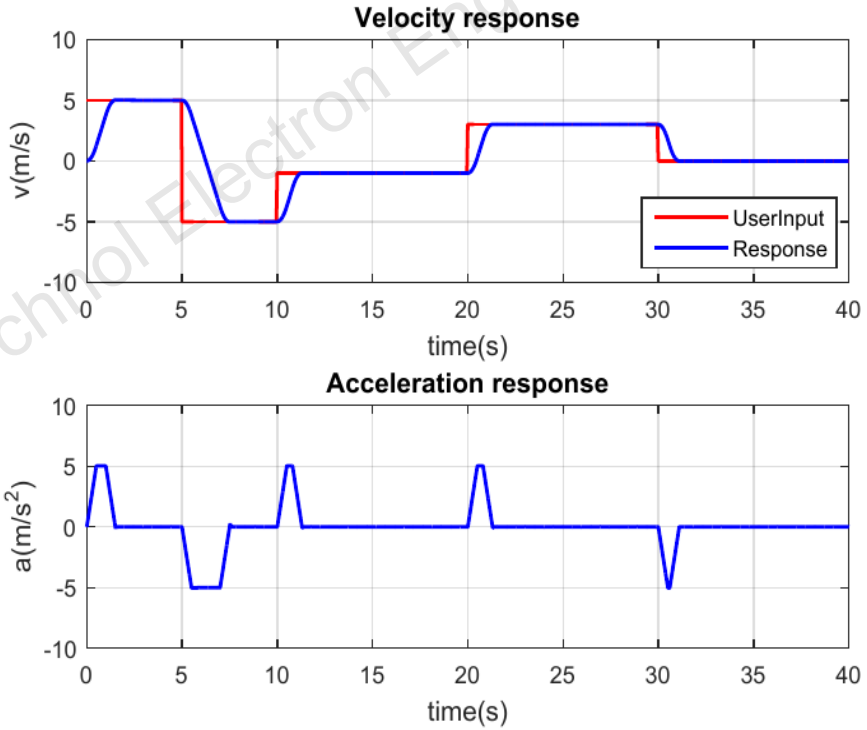
$$\dot{v}(t) = a(t)$$

$$\dot{a}(t) = u_{\mathrm{j}}(t)$$

$$-a_{\max} \leqslant a(t) \leqslant a_{\max}, \quad \forall t \in [0, T]$$

$$-u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max}, \quad \forall t \in [0, T]$$

An example:



It is shown that the trajectory consists of at most three segments with $u = \pm u_{j\max}$ and 0.

The jerk limited trajectory: velocity set-point

Intuition on the solution:

Covered area (acc)=ΔVelocity

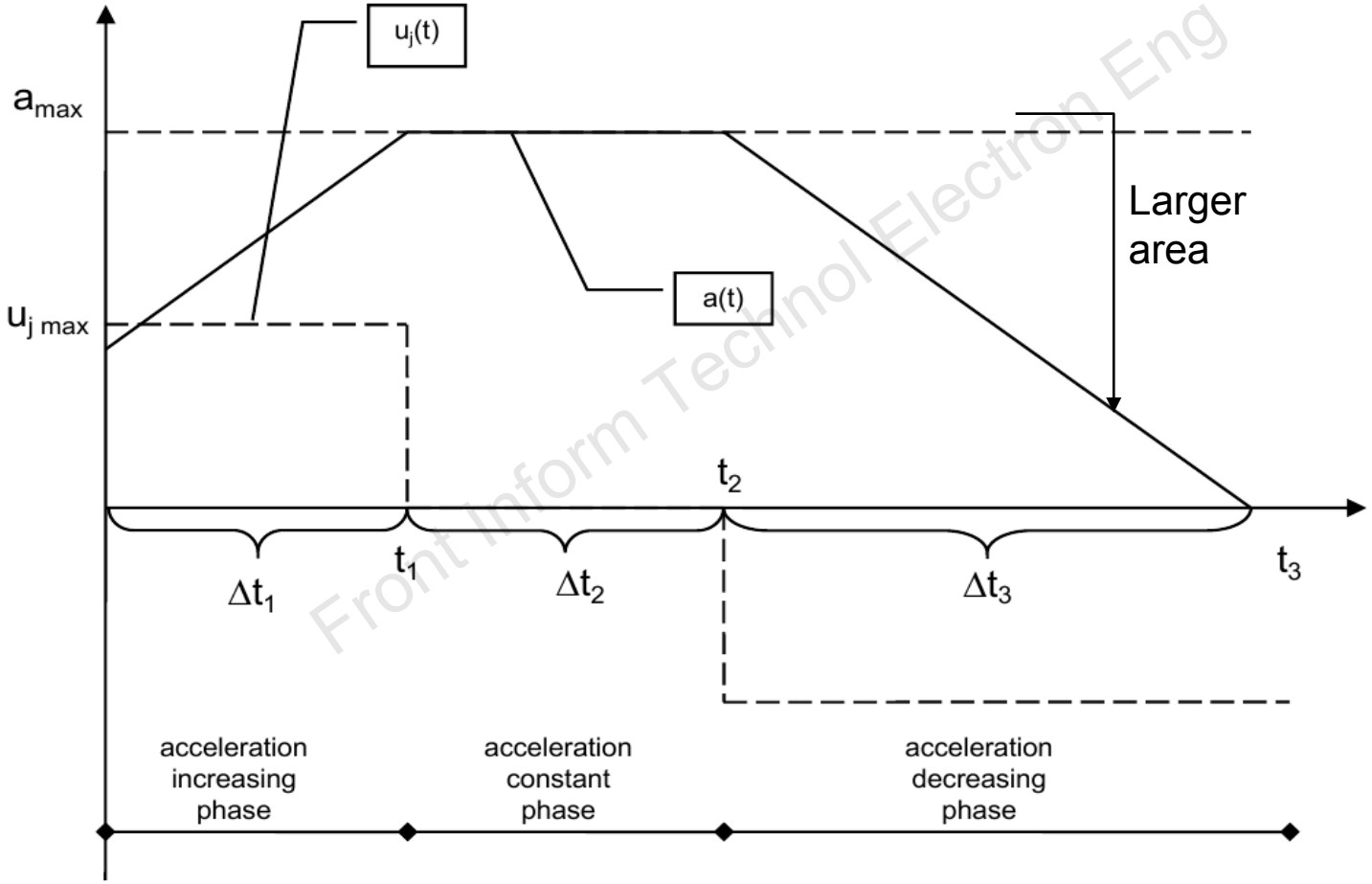


Figure 4.12: Trapezoidal acceleration profile

The jerk limited trajectory: position set-point

Problem:

From an arbitrary state to a desired point such that

$$p(0) = p_0, \quad p(T) = p_{\text{ref}}$$

$$v(0) = v_0, \quad v(T) = 0$$

$$a(0) = a_0, \quad a(T) = 0$$

$$\dot{p}(t) = v(t)$$

$$\dot{v}(t) = a(t)$$

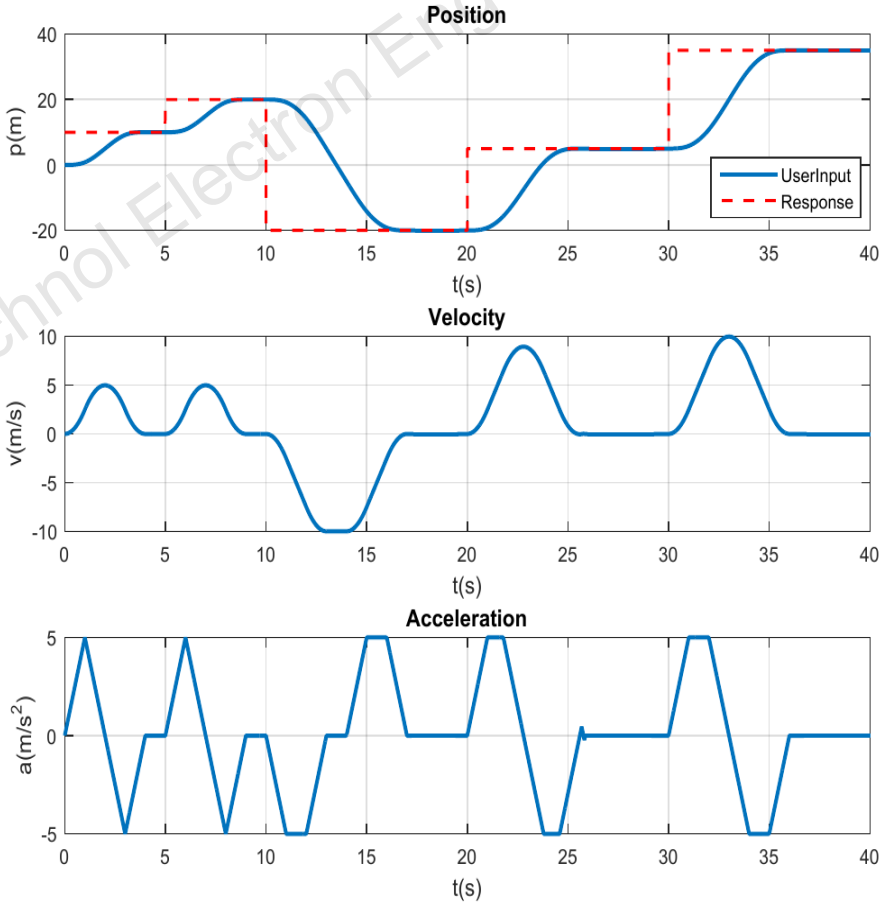
$$\dot{a}(t) = u_j(t)$$

$$-v_{\max} \leqslant v(t) \leqslant v_{\max}, \quad \forall t \in [0, T]$$

$$-a_{\max} \leqslant a(t) \leqslant a_{\max}, \quad \forall t \in [0, T]$$

$$-u_{\text{j}\max} \leqslant u_{\text{j}}(t) \leqslant u_{\text{j}\max}, \quad \forall t \in [0, T]$$

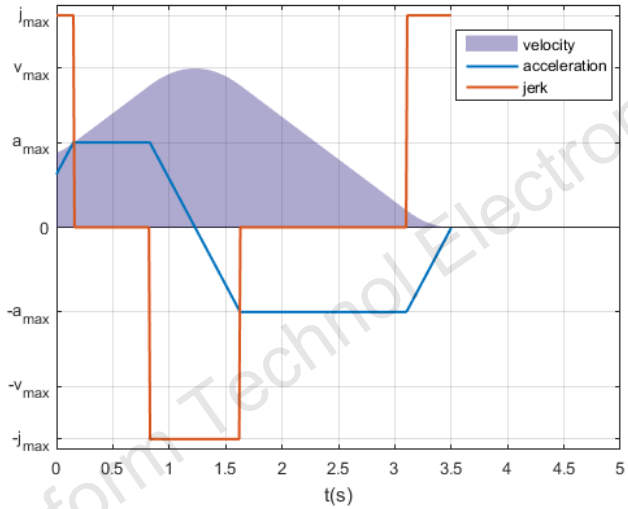
An example:



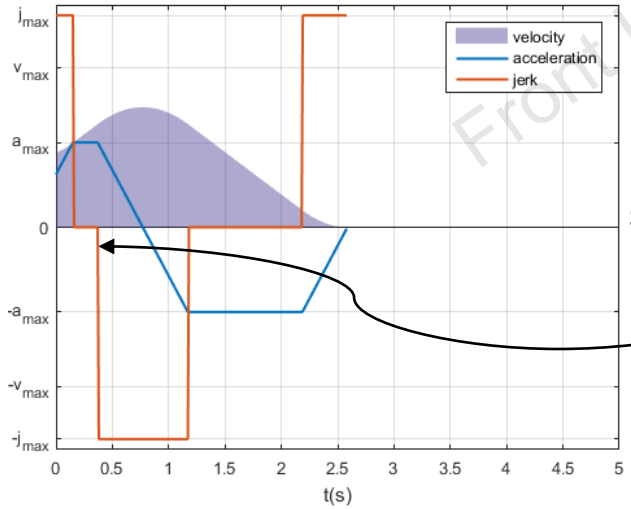
The jerk limited trajectory: position setpoint

Intuition on the solution:

Covered area (vel)=ΔPosition

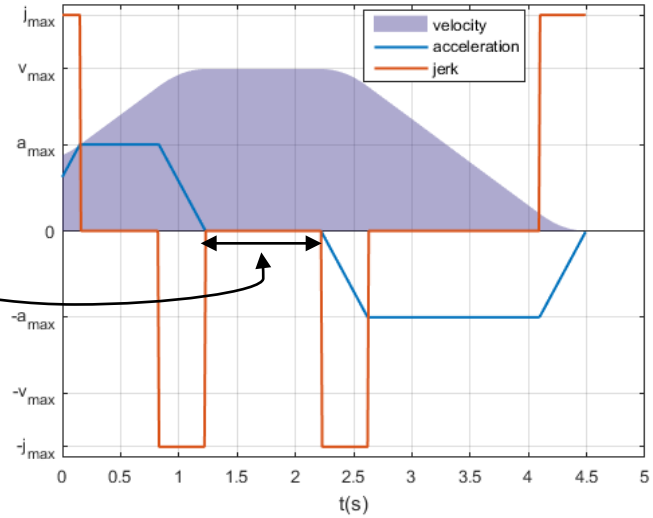


Needing smaller area



Determining switching moment

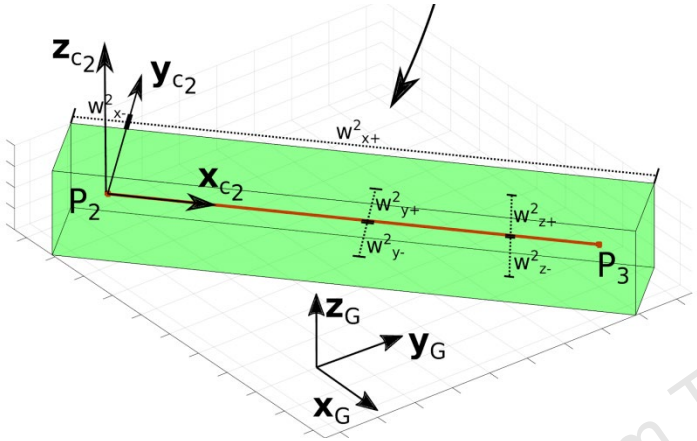
Needing larger area

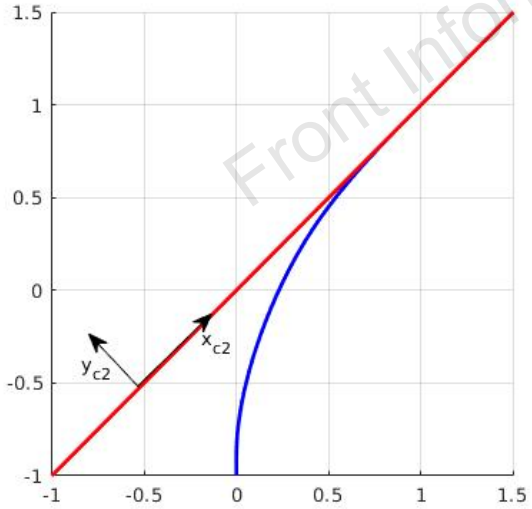


Determining cruise time

The jerk limited trajectory: trajectory along lines

For the trajectory to "converge" to a given line segment:





On the *y* and *z* axes:
Set the position set-point to zero.

On the *x* axis:
Set the position set-point to $|P_3–P_2|$.

Result:
The trajectory converges to the desired line-segment path.

The jerk limited trajectory: how to control?

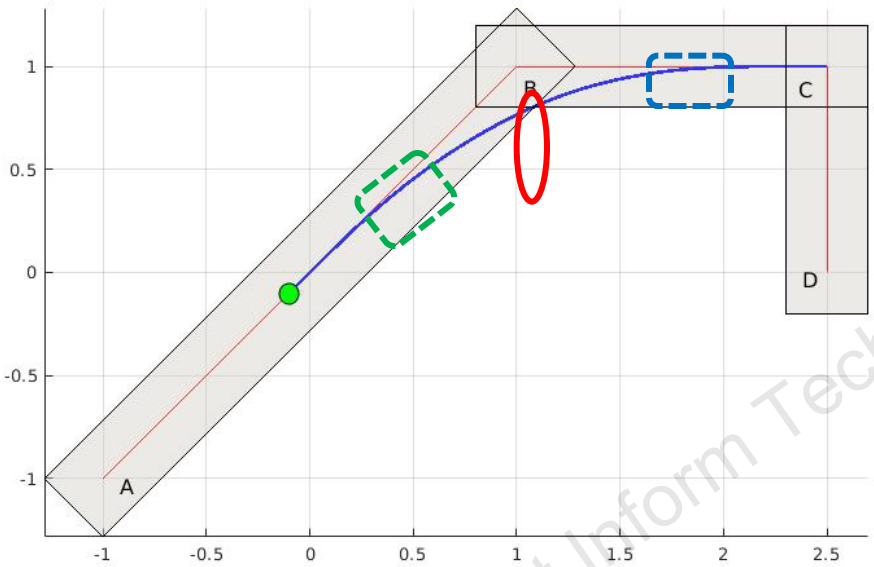
The robust and perfect tracking (RPT) controller:

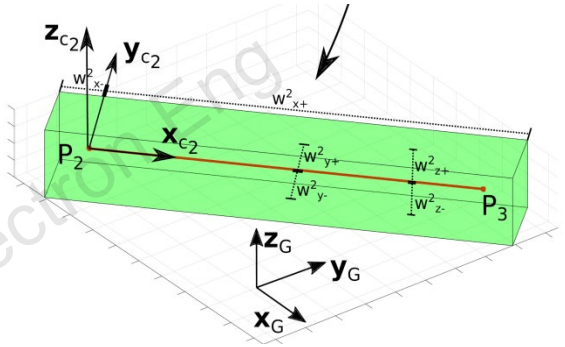
$$a_{x,n} = -\begin{bmatrix} \frac{\omega_{n,x}^2}{\varepsilon_x^2} & \frac{2\zeta_x\omega_{n,x}}{\varepsilon_x} \end{bmatrix}\begin{pmatrix} x_n \\ u_n \end{pmatrix} + \left(\frac{\omega_{n,x}^2}{\varepsilon_x^2}\right)x_{n,r} + \left(\frac{2\zeta_x\omega_{n,x}}{\varepsilon_x}\right)u_{n,r} + a_{x,n,r}$$

$$a_{y,n} = -\begin{bmatrix} \frac{\omega_{n,y}^2}{\varepsilon^2} & \frac{2\zeta_y\omega_{n,y}}{\varepsilon} \end{bmatrix}\begin{pmatrix} y_n \\ v_n \end{pmatrix} + \left(\frac{\omega_{n,y}^2}{\varepsilon^2}\right)y_{n,r} + \left(\frac{2\zeta_y\omega_{n,y}}{\varepsilon}\right)v_{n,r} + a_{y,n,r}$$

$$a_{z,n} = -\begin{bmatrix} \frac{\omega_{n,z}^2}{\varepsilon^2} & \frac{2\zeta_z\omega_{n,z}}{\varepsilon} \end{bmatrix}\begin{pmatrix} z_n \\ w_n \end{pmatrix} + \left(\frac{\omega_{n,z}^2}{\varepsilon^2}\right)z_{n,r} + \left(\frac{2\zeta_z\omega_{n,z}}{\varepsilon}\right)w_{n,r} + a_{z,n,r}$$

Safety check





Advantages:

1. Finite checking time regardless of the length of the corridor;
2. Guaranteed soundness.

Inside a single bounding box?

1. Project into the bounding-box frame;
2. Check the extreme value in individual axes.

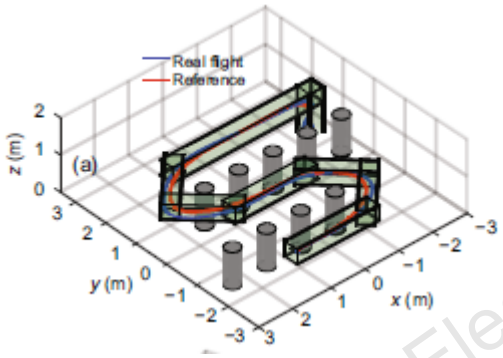
Inside multiple bounding boxes?

1. Find split points;
2. Split the trajectory;
3. Check the split parts individually.

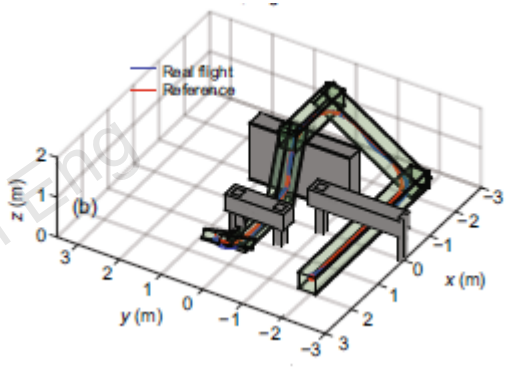
Major results

Experimental results

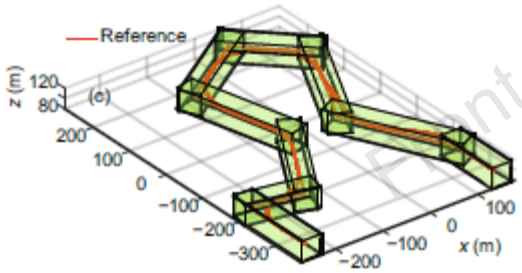
Fig. 14 Platform used for real flight experiments



Experiment A: horizontal maneuver



Experiment B: vertical maneuver



Experiment C: long distance fly (simulation)

Table 2 State constraints

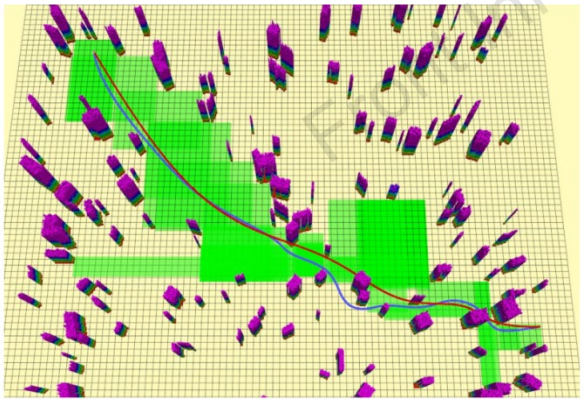
Experiment	$v_{\mathrm{h}\max}$ (m/s^2)	$v_{\mathrm{v}\max}$ (m/s)	$v_{\mathrm{v}\min}$ (m/s)	$a_{\mathrm{h}\max}$ (m/s)	$a_{\mathrm{v}\max}$ (m/s^2)
A	4	0.8	−0.8	2.2	0.8
B	3	0.8	−0.8	1.2	0.8
C	15	0.8	−0.8	2.2	0.8

Experiment	$a_{\mathrm{v}\min}$ (m/s^2)	$j_{\mathrm{h}\max}$ (m/s^3)	$j_{\mathrm{v}\max}$ (m/s^3)	$j_{\mathrm{v}\min}$ (m/s^3)
A	−0.8	3	3	−3
B	−0.8	3	3	−3
C	−0.8	3	3	−3

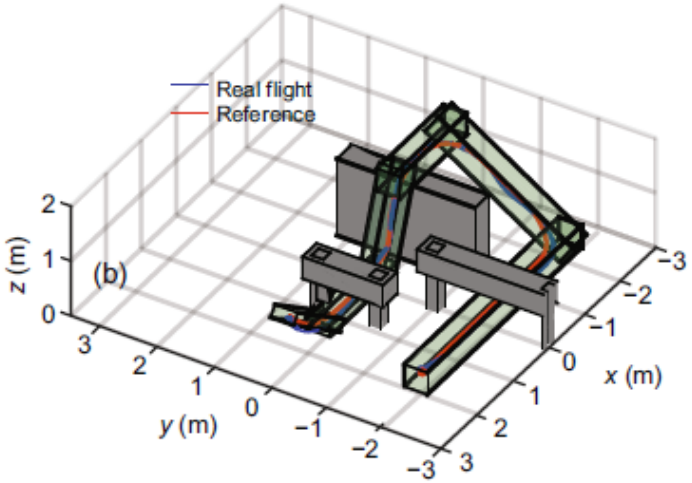
Experimental results: handling of constraints

Reference	Tilted corridor	State constraints	Soundness
Chen et al. (2016)	?	✔	✔
Liu et al. (2017)	✔	✔	?
Tang et al. (2016)	✔	?	✔
Shen et al. (2018)	?	✔	✔
Proposed method	✔	✔	✔

* All other methods, except the proposed one, use the spline approach, and require the solving of one or multiple non-linear programming problems.



Axes aligned corridor



Tilted corridor

Experimental results: computational efficiency

Our method

1. Incrementally generating the trajectory during execution;

2. For vehicle with low-power onboard PC;

3. Faster reaction to changes.

Spline-based SFC method

Generating the entire trajectory before execution.

Table 2 Time consumption: smooth navigation

Total time	Trajectory generation	Checking
3.91 us	2.68 us	0.9 us

Concluding remarks

1. We have proposed a rather down-to-earth solution for navigating a quadrotor along the nominal path plan

(1) safely, (2) smoothly, and (3) efficiently,

with its physical constraints on velocity, acceleration, and jerk.

2. The proposed approach

(1) saves flying time, and (2) is computationally efficient.

References

Chen J, Liu T, Shen S, 2016. Online generation of collision-free trajectories for quadrotor flight in unknown cluttered environments. IEEE Int Conf on Robotics and Automation, p.1476-1483.

Liu S, Watterson M, Mohta K, 2017. Planning dynamically feasible trajectories for quadrotors using safe flight corridors in 3-D complex environments. *IEEE Rob Autom Lett*, 2(3):1688-1695.

Tang S, Kumar V, 2016. Safe and complete trajectory generation for robot teams with higher-order dynamics. IEEE/RSJ Int Conf on Intelligent Robots and Systems, p.1894-1901.

Gao F, Wu W, Lin Y, 2018. Online safe trajectory generation for quadrotors using fast marching method and Bernstein basis polynomial. IEEE Int Conf on Robotics and Automation.