Yan HUANG, Jian-ping LI, Peng WANG, 2019. Unusual phenomenon of optimizing the Griewank function with the increase of dimension. *Frontiers of Information Technology & Electronic Engineering*, 20(10):1344-1360. https://doi.org/10.1631/FITEE.1900155

Unusual phenomenon of optimizing the Griewank function with the increase of dimension

Key words: Griewank; Two-scale structure; Multi-scale quantum harmonic oscillator algorithm; Quantum tunnel effect

Corresponding authors: Jian-ping LI; Peng WANG
E-mail: jpli2222@uestc.edu.cn; qhoalab@163.com
Yan HUANG, http://orcid.org/0000-0003-3896-5636

Motivation

1. The Griewank function is a typical multimodal benchmark function composed of a quadratic convex function and an oscillatory nonconvex function.

2. Different from most test functions, an unusual phenomenon appears when optimizing the Griewank function.

3. The Griewank function first becomes more difficult and then easier to optimize with the increase of dimension.

Main idea

1. The Griewank function is composed of a quadratic convex function with a unique global minimum, and an oscillatory nonconvex function with numerous local minima.

2. With the increase of dimension, the comparative importance of these two parts alters.

3. The two-scale structure is the core difficulty in the optimization process.

4. Structural, mathematical, and quantum analyses can be used to reveal the reason behind this phenomenon.

Method

1. Three analytical methods are proposed to interpret this phenomenon, including structural, mathematical, and quantum analyses.

2. Frequency transformation and amplitude transformation are implemented on the Griewank function to make a generalization.

Major results

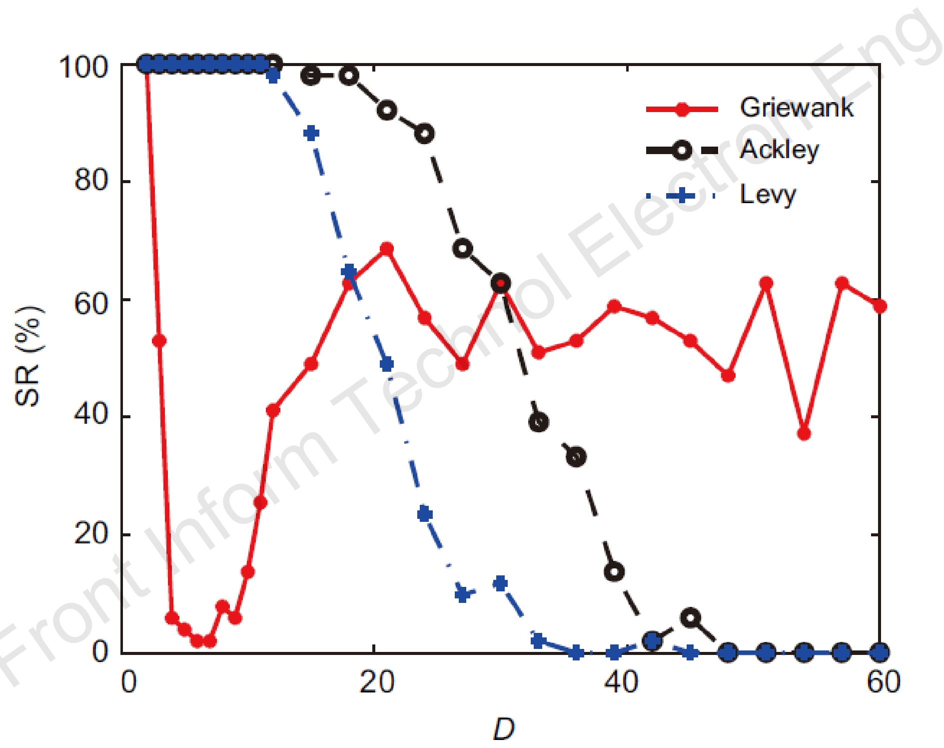

Fig. 3 Success rates (SRs) of Griewank, Ackley, and Levy, optimized with SPSO2011

Major results (Cont'd)

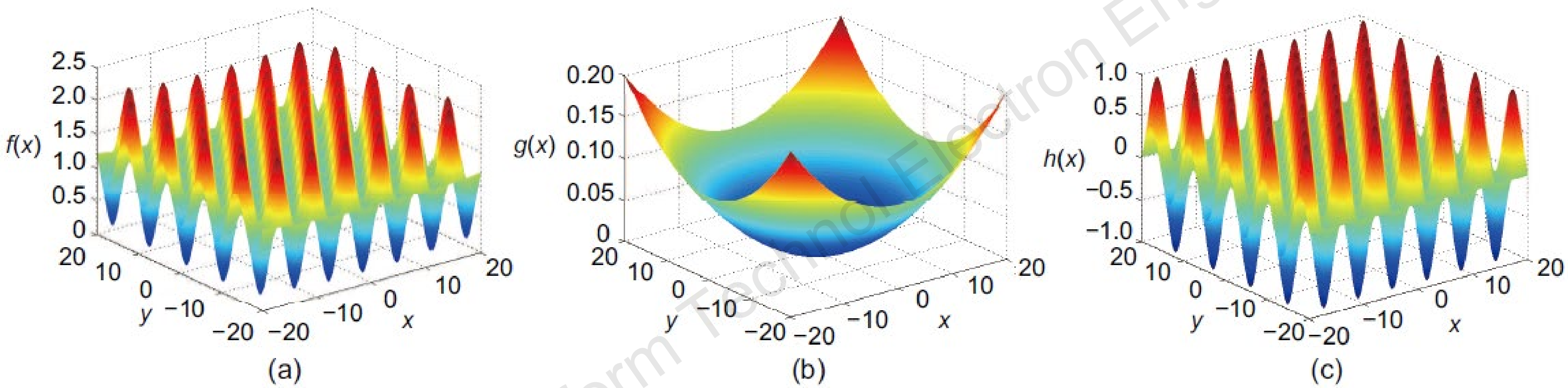

Fig. 2 Three-dimensional images of the Griewank function and its components: (a) Griewank ($f(x)$); (b) $g(x)$; (c) $h(x)$

Major results (Cont'd)

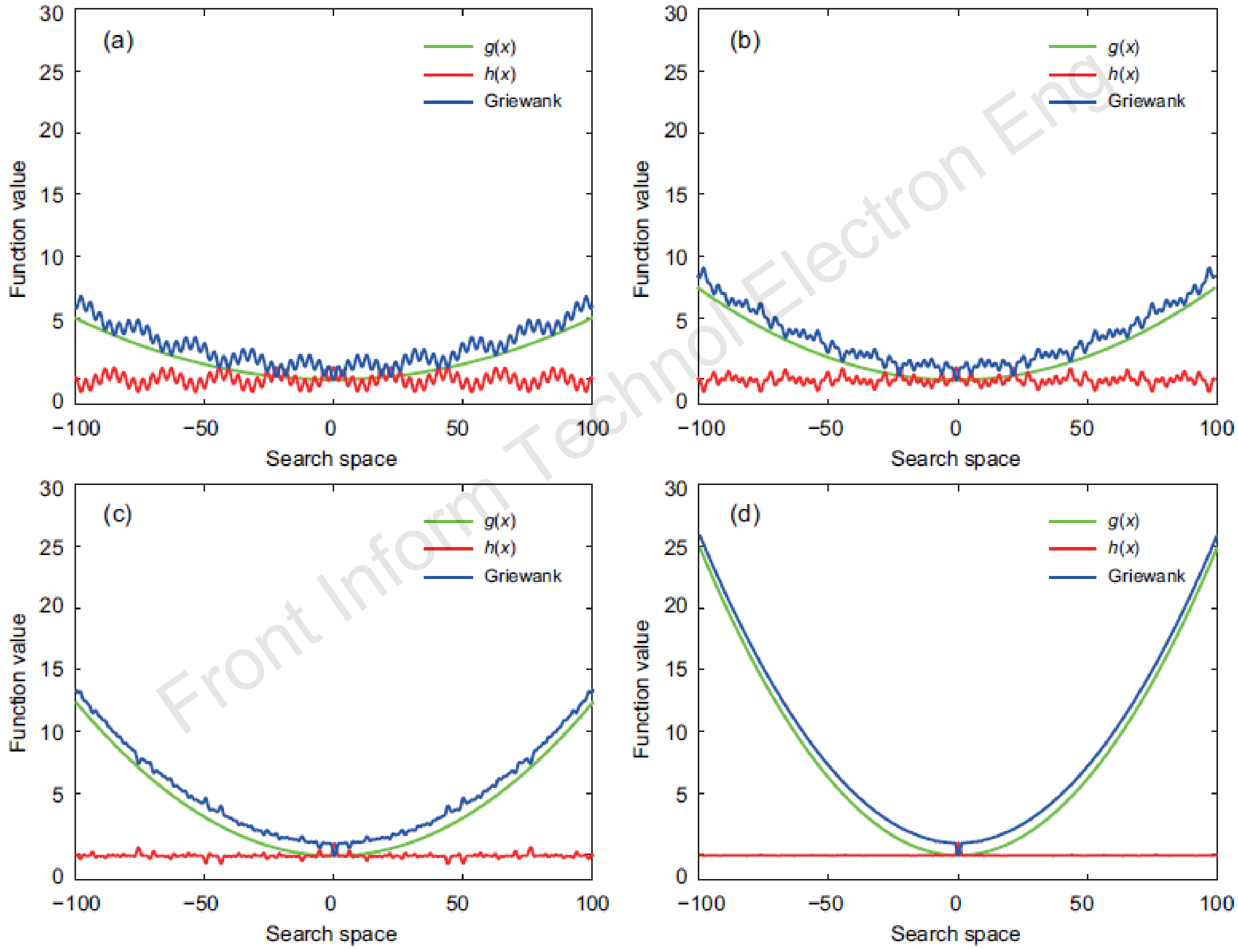

Fig. 4 Projecting an n-dimensional Griewank function to a plane: (a) $n = 2$; (b) $n = 3$; (c) $n = 5$; (d) $n = 10$. References to color refer to the online version of this figure

Major results (Cont'd)

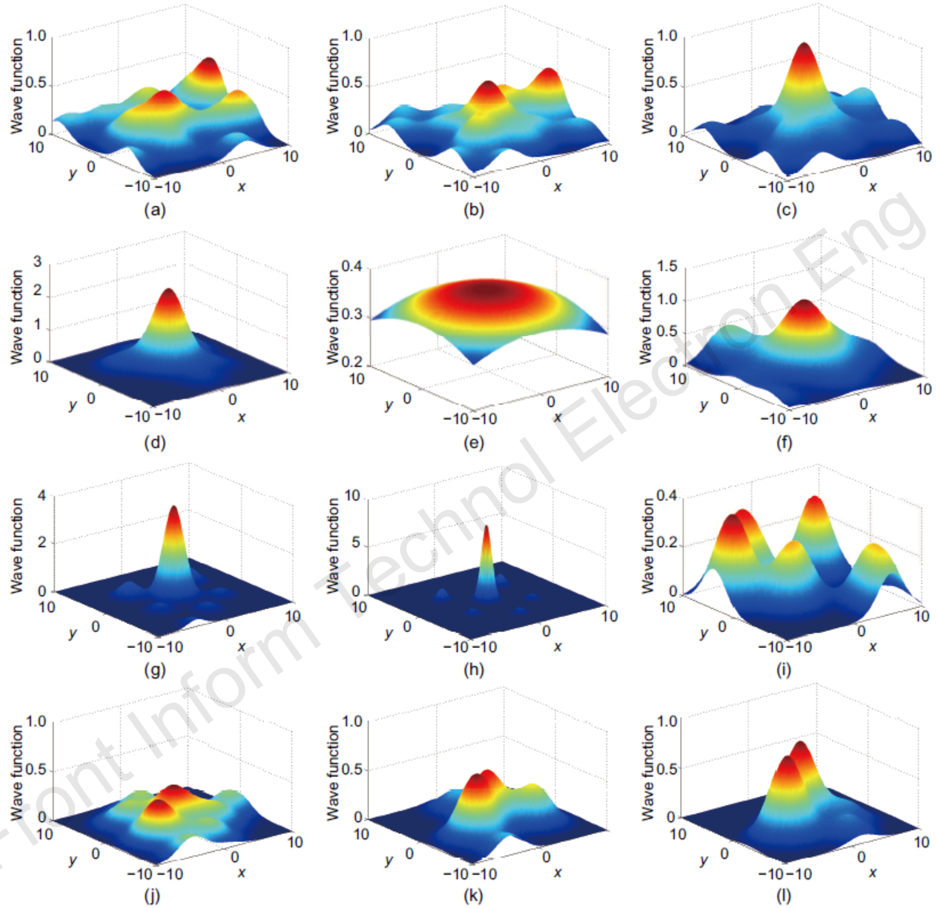

Fig. 9 Wave functions of the Griewank function optimized by MQHOA: (a) initial status (k=20, σ_0=2, i=1, σ=2); (b) competitive sampling areas appear (k=20, σ_0=2, i=400, σ=2); (c) aggregate and approach to the optimal solution (k=20, σ_0=2, i=450, σ=2); (d) the global optimal solution is obtained accurately (k=20, σ_0=2, i=20 000, σ=2); (e) initial status, very flat in shape (k=20, σ_0=20, i=1, σ=20); (f) competitive sampling areas appear (k=20, σ_0=20, i=505, σ=2.5); (g) the optimal area becomes obvious (k=20, σ_0=20, i=7611, σ=1.25); (h) the global optimal solution is obtained accurately (k=20, σ_0=20, i=7613, σ=0.625); (i) initial status, the sampling area is very scattered (k=10, σ_0=2, i=1, σ=2); (j) sampling points gather gradually (k=10, σ_0=2, i=250, σ=2); (k) three competitive sampling areas appear (k=10, σ_0=2, i=2500, σ=2); (l) two competitive sampling areas compete with each other, and the global optimal solution is not obtained accurately (k=10, σ_0=2, i=20 000, σ=2)

Conclusions

1. The Griewank function first becomes more difficult and then easier to optimize with the increase of dimension.

2. Structural, mathematical, and quantum analyses on the Griewank function explain the reason for this phenomenon.

3. MQHOA is used to optimize the generalized Griewank function and it performs well.