Muhammad Faisal FATEH, Aneela ZAMEER, Sikander M. MIRZA, Nasir M. MIRZA, Muhammad Saeed ASLAM, Muhammad Asif Zahoor RAJA, 2019. Differential evolution based computation intelligence solver for elliptic partial differential equations. *Frontiers of Information Technology & Electronic Engineering*, 20(10):1445-1456. https://doi.org/10.1631/FITEE.1900221

Differential evolution based computation intelligence solver for elliptic partial differential equations

Key words: Deferential evolution; Boundary value problems; Partial differential equation; Finite difference scheme; Numerical computing

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Motivation

- A literature review reveals a rich diversity of numerical techniques that have been developed over time for the solution of elliptic partial differential equations (PDEs).
- Homogeneous elliptic PDEs are usually called the Laplace equation, and occur in many fields, such as electrostatics, gravitation, steady state flow of inviscid fluids, steady state heat conduction electrostatics, and elasticity theory.
- Stochastic numerical solutions have attracted growing interest due to their ability to provide accurate solutions for stiff systems like elliptic PDEs.

Main idea

A novel stochastic computational paradigm is presented for approximating the solution of elliptic PDEs using discretization strength of the finite difference scheme to transform the PDEs into nonlinear systems of equations and their solutions by the efficacy of the differential evolution based optimization scheme.

Method

1. **Discretization:** The elliptic PDEs are transformed into systems of nonlinear equations by exploiting the finite difference method based discretization process.
2. **Fitness function:** Exploit the approximate theory in the mean square error sense to formulate the residual error based fitness/ object/merit function of the transformed nonlinear systems of equations.
3. **Optimization:** The residual errors of elliptic differential equations are optimized by exploiting the global and local search ability of the differential evolution based solver.

Major results

Solution of the proposed algorithm for elliptic PDEs for case 1

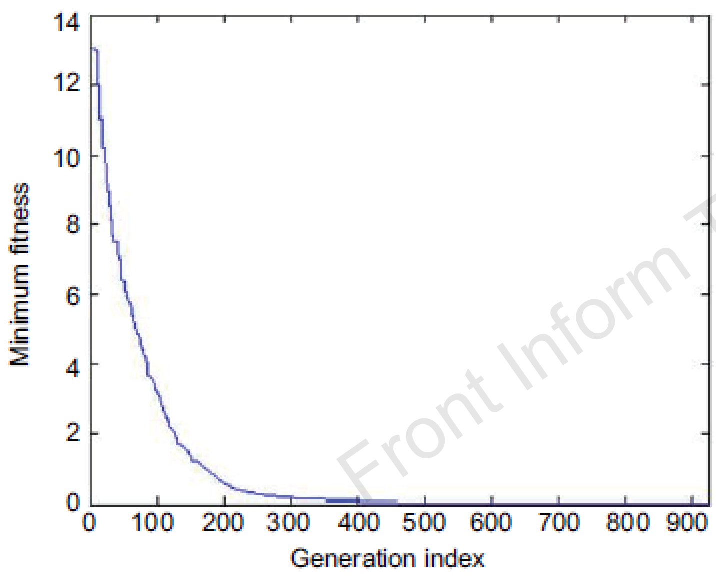

Fig. 1 Convergence curve for case 1 (Laplace equation with Dirichlet boundary conditions)

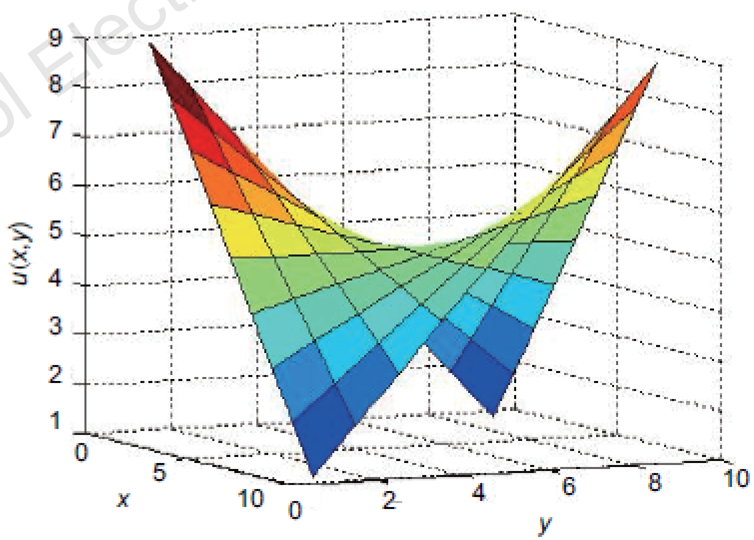

Fig. 2 Plot of the proposed solution for case 1 (Laplace equation with Dirichlet boundary conditions)

Major results (Cont'd)

Table 1 **Proposed solution for $u(x, y)$ for $1 \le x, y \le 9$ with $h = 1$ in case 1 (Laplace equation with Dirichlet boundary conditions)**

Node	Experimental value								
	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	2.7500	3.5000	4.2500	5.0000	5.7500	6.5000	7.2500	8
3	3	3.5001	4.0001	4.5001	5.0001	5.5001	6.0000	6.5000	7
4	4	4.2500	4.5001	4.7501	5.0001	5.2501	5.5001	5.7500	6
5	5	5.0001	5.0001	5.0001	5.0001	5.0001	5.0001	5.0001	5
6	6	5.7500	5.5001	5.2501	5.0001	4.7501	4.5001	4.2501	4
7	7	6.5000	6.0000	5.5000	5.0001	4.5001	4.0001	3.5001	3
8	8	7.2500	6.5000	5.7500	5.0000	4.2500	3.5000	2.7500	2
9	9	8	7	6	5	4	3	2	1

Major results (Cont'd)

Table 2 **Comparison with exact solution $u(x, y)$ for $1 \leq x, y \leq 9$ with $h = 1$ through the absolute error in case 1 (Laplace equation with Dirichlet boundary conditions)**

Node	Absolute error ($\times 10^{-5}$)									Node	Absolute error ($\times 10^{-5}$)								
	1	2	3	4	5	6	7	8	9		1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	0	0	0	0	6	0	3.4	8.7	10	13	14	13	7.7	0
2	0	3.3	2.5	4.3	4.1	1.7	0.67	0.76	0	7	0	0.9	2.7	2.4	5.0	6.7	7.3	5.9	0
3	0	5.2	8.3	8.1	7.4	5.3	3.9	1.6	0	8	0	1.6	0.48	1.0	0.92	2.2	3.0	4.3	0
4	0	2.9	9.4	12	10	9.4	6.8	2.6	0	9	0	0	0	0	0	0	0	0	0
5	0	5.4	10	15	16	15	11	6.6	0										

Conclusions

- A novel differential evolution based numerical commuting methodology has been developed for the solution of elliptic PDEs. The results clearly show the effectiveness of the proposed DE methodology for various benchmark problems.
- The CPU time and amount of histories required for convergence clearly show that DE is an effective computing framework to be exploited for solving PDEs. Also, DE can easily be applied to both linear and nonlinear problems.
- The random initialization scheme is easy to use and requires minimum information of the nature of the problem. A smaller step size yields a more accurate answer, but it requires higher computational cost.
- High convergence rates, wide scope of applicability, inherently parallel nature, and robustness are among the attractive features of the proposed DE solver for elliptic PDEs.