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Switching-based stabilization of aperiodic sampled-data Boolean control networks with all subsystems unstable

Key words: Aperiodic sampled-data control; Boolean control networks; Unstable subsystem; Discretized Lyapunov function; Dwell time

Corresponding author: Jian-quan LU
E-mail: jqluma@seu.edu.cn
ORCID: http://orcid.org/0000-0003-4423-6034

Introduction

1. To the best of our knowledge, there is no work on the global stability of switched Boolean networks (SBNs) containing all unstable subsystems. Moreover, how to design a switching policy to achieve stability of the switched system containing all subsystems unstable has always been a challenging issue.

2. The proposed method in this paper can not only solve the global stability problem of the Boolean control network (BCN) under aperiodic sampled-data control (ASDC) when all subsystems of the transformed SBN are unstable, but also adapt to work on the global stability of SBNs containing all subsystems unstable.

Introduction (Cont'd)

3. Compared with direct research on the global stability of SBNs with all unstable subsystems, the problem considered in this study is more complex. Although we transform this problem into studying the global stability of the transformed SBN containing all subsystems unstable, the switching instant must be the sampling instant of the original BCN under ASDC.

Main idea

The central thought in this paper is that switching behavior also has good stabilization; i.e., the SBN can also be stable with appropriate switching laws designed, even if all subsystems are unstable.

Method

1. Using semi-tensor product (STP) of matrices, a BCN under ASDC can be transformed into a SBN, whose subsystems are all unstable.

2. Some results for global stability of BCNs under ASDC have been obtained by means of a discretized Lyapunov function and dwell time.

Major results

Consider the following BCN under ASDC:

$$X(t+1) = f\big(X(t), U(t)\big), \tag{1}$$

$$U(t) = e\big(X(t_k)\big), t_k \leq t < t_{k+1}. \tag{2}$$

By STP, we have

$$\boldsymbol{x}(t+1) = \boldsymbol{M}\boldsymbol{u}(t)\boldsymbol{x}(t), \tag{3}$$

$$\boldsymbol{u}(t) = \boldsymbol{E}\boldsymbol{x}(t_k), t_k \leq t < t_{k+1}. \tag{4}$$

Major results (Cont'd)

1. Convert a BCN under ASDC into an SBN

Denote $h_k \triangleq t_{k+1} - t_k$ as the k^{th} sampling interval, where $h_k \in Z_h \triangleq \{i_1, i_2, \ldots, i_l\}$ $(i_1 < i_2 < \cdots < i_l)$ and i_j $(j = 1,2, \ldots, l)$ are positive integers. By STP, system (3) under ASDC (4) can be translated into an SBN, which can be described as follows:

$$\boldsymbol{x}(t_{k+1}) = \left(\boldsymbol{MW}_{[2^n,2^m]}\right)^{h_k} \boldsymbol{x}(t_k) \boldsymbol{\Phi}_m^{h_k-1} \boldsymbol{u}(t_k) \triangleq \boldsymbol{F}_{\sigma(t_k)} \boldsymbol{x}(t_k), \qquad (5)$$

where the switching signal $\sigma(t_k) \in Z_\sigma \triangleq \{1,2, \ldots, l\}$.

Major results (Cont'd)

Thus, the switching time sequence is given below:

$$0 = t_0 = t_{k^{(0)}} < t_{k^{(1)}} < t_{k^{(2)}} < \cdots < t_{k^{(i)}} < t_n,$$

where $t_{k^{(j)}} \in \{t_0, t_1, \ldots, t_n\}$ $(j = 0,1, \ldots, i)$, and $t_0, t_1, \ldots, t_n$ are sampling instants.

2. Global stability

Definition 1 System (3) is said to be globally stable at $\boldsymbol{\delta}_{2^n}^{2^n}$, if for any initial state $\boldsymbol{x}(0) \in \Delta_{2^n}$, the corresponding trajectory $\boldsymbol{x}(t)$ converges to $\boldsymbol{\delta}_{2^n}^{2^n}$.

Major results (Cont'd)

Lemma 1 System (2) is globally stable at $\boldsymbol{\delta}_{2^n}^{2^n}$ if and only if the corresponding Eq. (5) is globally stable at $\boldsymbol{\delta}_{2^n}^{2^n}$ and $\boldsymbol{\delta}_{2^n}^{2^n} = \boldsymbol{ME}\boldsymbol{\delta}_{2^n}^{2^n}\boldsymbol{\delta}_{2^n}^{2^n}$.

Here, we assume that system (1) considered in this part satisfies $\boldsymbol{\delta}_{2^n}^{2^n} = \boldsymbol{ME}\boldsymbol{\delta}_{2^n}^{2^n}\boldsymbol{\delta}_{2^n}^{2^n}$.

Let $Z \triangleq \{k^{(0)}, k^{(1)}, \ldots, k^{(i)}\}$. Define $\tau_{k^{(j)}} = k^{(j+1)} - k^{(j)}$ as the dwell time, where $j = 0,1,\ldots,i-1$ and $\tau_{k^{(j)}} \in [\tau_{\min}, \tau_{\max}]$ with $\tau_{\min} = \inf_{k^{(j)} \in Z} \tau_{k^{(j)}}$ and $\tau_{\max} = \sup_{k^{(j)} \in Z} \tau_{k^{(j)}}$.

Major results (Cont'd)

Definition 2 The set of vectors $\{\boldsymbol{\beta}_{a,q} | a \in Z_\sigma, q = 0,1,\ldots,L\}$ is defined as a set of Lyapunov coefficients of Eq. (5) if $\forall r = 1,2,\ldots,2^n - 1$ and $\forall a, b \in Z_\sigma$, the following equations/inequalities are satisfied:

$$\boldsymbol{\beta}_{a,q}^{\mathrm{T}} \boldsymbol{\delta}_{2^n}^{2^n} = 0, q = 0,1,\ldots,L, \tag{6}$$

$$\boldsymbol{\beta}_{a,q}^{\mathrm{T}} \boldsymbol{\delta}_{2^n}^{r} > 0, q = 0,1,\ldots,L, \tag{7}$$

$$\left[\boldsymbol{\beta}_{a,q}^{\mathrm{T}}(\boldsymbol{F}_a - \boldsymbol{I}) + \frac{1}{h}\left(\boldsymbol{\beta}_{a,q+1}^{\mathrm{T}} - \boldsymbol{\beta}_{a,q}^{\mathrm{T}}\right)\boldsymbol{F}_a - \lambda_a \boldsymbol{\beta}_{a,q}^{\mathrm{T}}\right] \boldsymbol{\delta}_{2^n}^{r} < 0, q = 0,1,\ldots,L-1, \tag{8}$$

$$\left[\boldsymbol{\beta}_{a,q+1}^{\mathrm{T}}(\boldsymbol{F}_a - \boldsymbol{I}) + \frac{1}{h}\left(\boldsymbol{\beta}_{a,q+1}^{\mathrm{T}} - \boldsymbol{\beta}_{a,q}^{\mathrm{T}}\right)\boldsymbol{F}_a - \lambda_a \boldsymbol{\beta}_{a,q+1}^{\mathrm{T}}\right] \boldsymbol{\delta}_{2^n}^{r} < 0, q = 0,1,\ldots,L-1, \tag{9}$$

$$\boldsymbol{F}_a \boldsymbol{\delta}_{2^n}^{2^n} = \boldsymbol{\delta}_{2^n}^{2^n}, \tag{10}$$

$$\left[\boldsymbol{\beta}_{a,L}^{\mathrm{T}}(\boldsymbol{F}_a - \boldsymbol{I}) - \lambda_a \boldsymbol{\beta}_{a,L}^{\mathrm{T}}\right] \boldsymbol{\delta}_{2^n}^{r} < 0, \tag{11}$$

$$\boldsymbol{\beta}_{b,0}^{\mathrm{T}} \le \mu_b \boldsymbol{\beta}_{a,L}^{\mathrm{T}}, 0 < \mu_b < 1, \mathrm{a} \neq b, \tag{12}$$

where $\lambda_a > 0$ and $h = \left\lfloor \frac{\tau_{\min}}{L} \right\rfloor$.

Major results (Cont’d)

Theorem 1 Consider system (3) under Eq. (4). If there exist a set of Lyapunov coefficients $\{\boldsymbol{\beta}_{a,q} | a \in Z_\sigma, q = 0,1,\ldots,L\}$ as defined in Definition 2 and a constant $\tau_{\max} \geq \tau_{\min}$, such that for any $a, b \in Z_\sigma, a \neq b$, the following inequality holds：

$$\ln \mu_b + \tau_{\max} \ln(\lambda_a + 1) < 0\,,$$

then system (3) is globally stable at $\boldsymbol{\delta}_{2^n}^{2^n}$.

Conclusions

In this paper, the global stability of BCNs under ASDCs has been studied. Using STP, we converted a BCN under ASDC into an SBN, whose subsystems are all unstable. Some results for global stability of BCNs under ASDCs have been obtained by means of a discretized Lyapunov function and dwell time. The validity has been demonstrated by a biological example.