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Data recovery with sub-Nyquist sampling: fundamental limit and a detection algorithm

Key words: Nyquist-Shannon sampling theorem; Sub-Nyquist sampling; Minimum Euclidean distance; Under-determined linear problem; Time-variant Viterbi algorithm

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Motivation

1. While the Nyquist rate serves as a lower bound to sample a general bandlimited signal with no information loss, the sub-Nyquist rate may also be sufficient for sampling and recovering signals under certain circumstances.
2. Sub-Nyquist sampling has been widely studied, but mainly focused on signals whose dimensions can be reduced either in the time domain or in the frequency domain.
3. The sub-Nyquist sampling of a linearly modulated signal, whose dimension cannot be reduced, is studied in this paper. We investigate the performance limit and data recovery method.

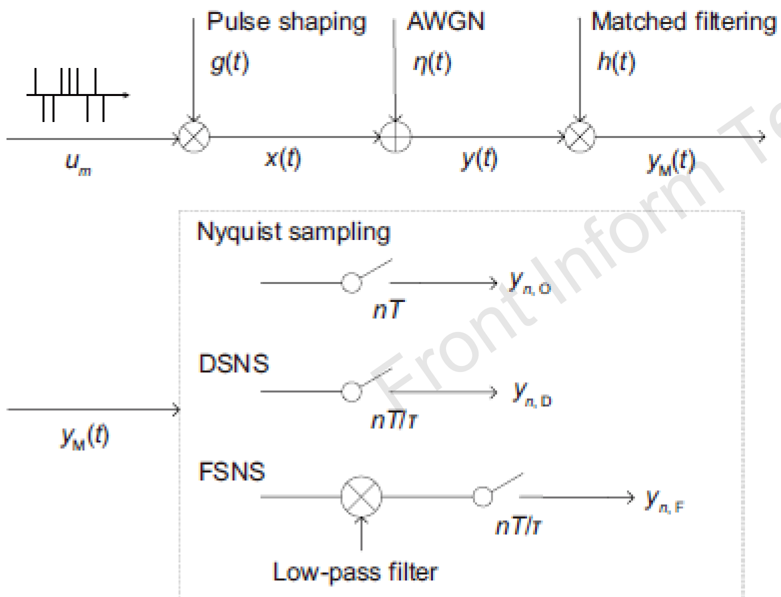
Main idea

1. Two sub-Nyquist sampling schemes, direct sub-Nyquist sampling (DSNS) and low-pass filtered sub-Nyquist sampling (FSNS), are used to sample the linearly modulated baseband signal.
2. The minimum normalized Euclidean distances between sample sequences indicate the best performances that data recovery algorithms can achieve.
3. Data recovery in sub-Nyquist sampling of the linearly modulated baseband signal is a time-variant and under-determined problem. A modified time-variant Viterbi algorithm is used for data recovery.

System model

Two sub-Nyquist sampling schemes vs. Nyquist sampling

- Sub-Nyquist sampling: a linear time-variant system
- Nyquist sampling: a linear time-invariant system



DSNS:

$$y_{n,\mathrm{D}} = \sqrt{\frac{E_\mathrm{s}}{T}} \sum_m u_m \,\mathrm{sinc}\left(\frac{n}{\tau} - m\right) + \eta_{n,\mathrm{D}}$$

FSNS:

$$y_{n,\mathrm{F}} = \sqrt{\frac{E_\mathrm{s}}{T}} \tau \sum_m u_m \,\mathrm{sinc}(n - m\tau) + \eta_{n,\mathrm{F}}$$

Nyquist sampling:

$$y_{n,\mathrm{O}} = \sqrt{\frac{E_\mathrm{s}}{T}} u_n + \eta_{n,\mathrm{O}}$$

Minimum Euclidean distance

The minimum normalized Euclidean distances between sample sequences indicate the best performance that data recovery can achieve.

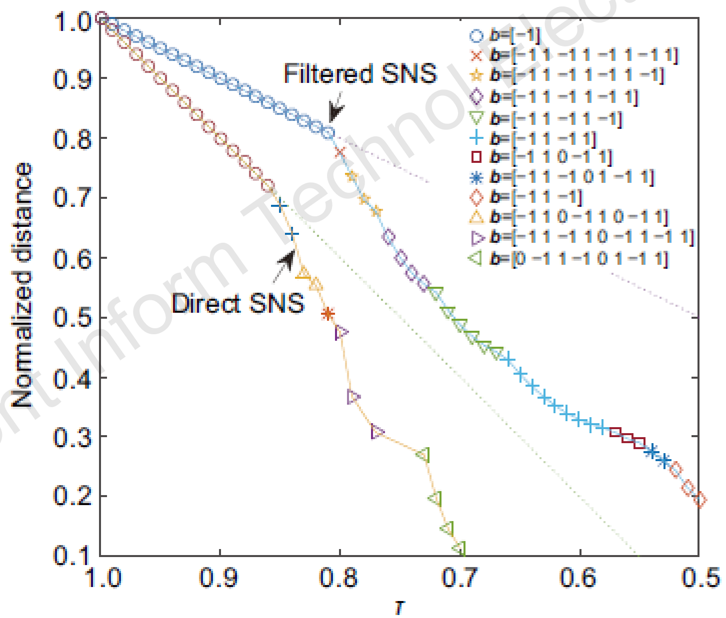


Fig. 3 Minimum of the normalized distance of two distinct sample sequences in DSNS and FSNS. Reprinted from Luo and Zhang (2019), Copyright 2019, with permission from IEEE

Data recovery algorithm

With a modified time-variant Viterbi algorithm, data can be recovered from the under-determined and time-variant linear problem of sub-Nyquist sampling.

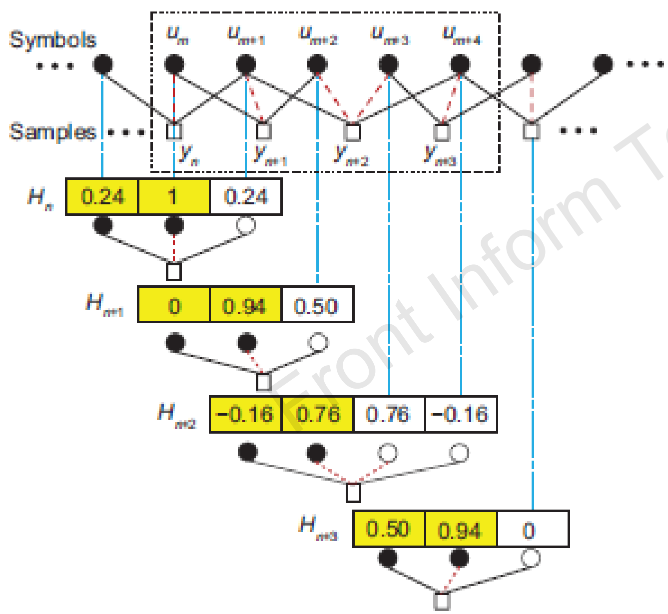


Fig. 4 Structure of the time-variant inter-symbol interference with $\tau = 0.8$ and $2L = 2$. Reprinted from Luo and Zhang (2019), Copyright 2019, with permission from IEEE

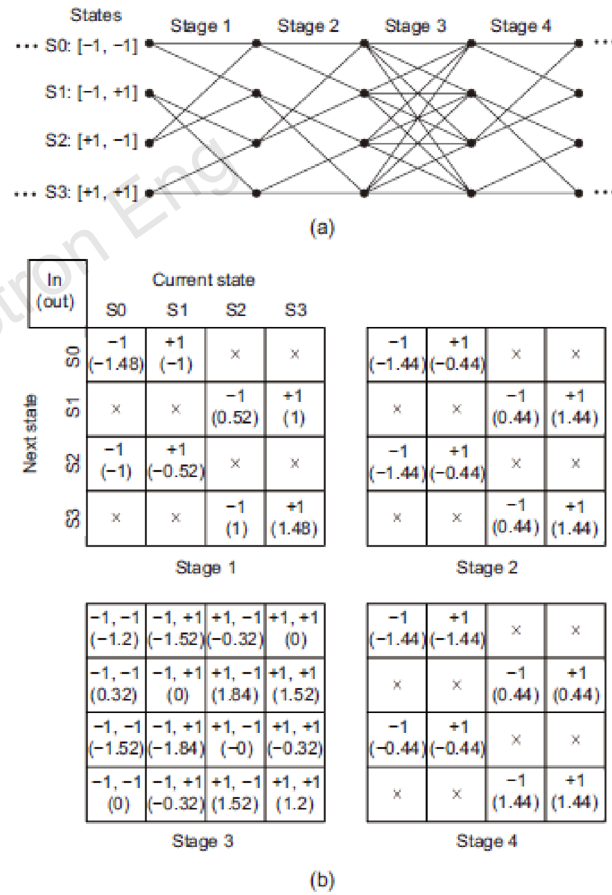


Fig. 5 The trellis graph (a) (reprinted from Luo and Zhang (2019), Copyright 2019, with permission from IEEE) and state transition (b) of a time-variant VA with $\tau = 0.8$ and $2L = 2$. In (b), each term in the form represents a trellis branch connecting the current state and the next state, the inputs and outputs denote the input symbol values and the predicted sample values respectively, and the "×" in the form stands for an impossible state transition

Simulation results

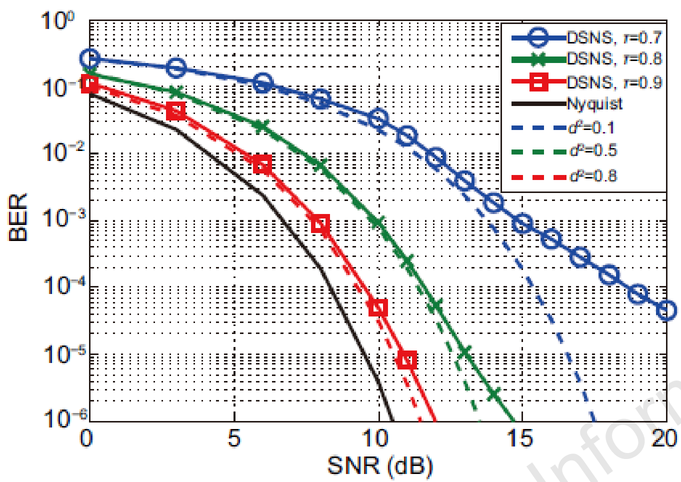


Fig. 6 BER of DSNS with sinc pulse shaping. Reprinted from Luo and Zhang (2019), Copyright 2019, with permission from IEEE. The solid lines show the simulated BERs, and the dotted lines are the theoretical BERs calculated based on the minimum normalized Euclidean distances in Fig. 3

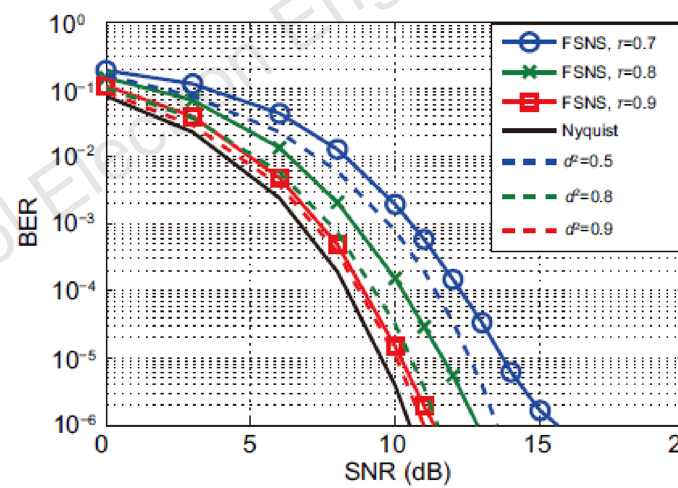


Fig. 7 BER of FSNS with sinc pulse shaping. Reprinted from Luo and Zhang (2019), Copyright 2019, with permission from IEEE. The solid lines show the simulated BERs, and the dotted lines are the theoretical BERs calculated based on the minimum normalized Euclidean distances in Fig. 3

Conclusions

1. Performance limits of DSNS and FSNS have been estimated by calculating the minimum Euclidean distances. DSNS has the minimum Euclidean distance of $2\tau-1$ within the sampling period $\tau\in[0.855, 1]$, while FSNS has the minimum Euclidean distance of τ within the sampling period $\tau\in[0.802, 1]$.

2. The modified time-variant Viterbi algorithm can effectively recover data from sub-Nyquist sampling. The simulation performance approaches theoretical limits as the truncation length grows.

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