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Output tracking of delayed logical control networks with multi-constraint

Key words: Logical control networks; Multi-constraint; Output tracking; Stabilization; State-dependent delay; Semi-tensor product

Corresponding author: Jun-e FENG
E-mail: fengjune@sdu.edu.cn
ORCID: https://orcid.org/0000-0003-3881-3042

Motivation

As a powerful tool for studying the gene regulatory networks (GRNs), logical modelling frameworks are easy to simulate and are little computationally taxing.

Because of some time consuming processes, such as DNA translation and RNA translation, time delay is inevitable in the gene regulatory process. Moreover, some virulence genes are expected to avoid synthesis, which may lead to a dangerous situation. This motivates us to investigate the delayed logical control networks (DLCNs) with constraints.

Main idea

Using the semi-tensor product (STP) method, we first convert the dynamics of the DLCN into a bilinear discrete-time system and then construct an equivalent augmented system.

Based on the augmented system, the output tracking problem can be transformed into a set stabilization problem. By modifying the state transition matrix, some necessary and sufficient conditions for output tracking of the system is obtained.

Modeling process

Original system:

$$\begin{cases} a_i(t+1) = f_i\left(A\left(t-\mu\big(A(t)\big)\right), U(t)\right), & i = 1, 2, \dots, n, \\ b_j(t) = h_j\left(A\left(t-\mu\big(A(t)\big)\right)\right), & j = 1, 2, \dots, p, \end{cases}$$

where $A(t) = (a_1(t), a_2(t), \dots, a_n(t))$, $\mu(A(t))=g(A(t)) \in [0{:}\mu^*]$.

New system:

$\mu(a(t))=Ga(t)$, $\begin{cases} x(t+1) = \Theta u(t) x(t) \\ u(t) = \dot{\Phi} x(t) \end{cases}$.

Constraint sets :

$$\Gamma_u = \{\delta_{k^m}^{\nu_1}, \delta_{k^m}^{\nu_2}, \ldots, \delta_{k^m}^{\nu_s}\} \subseteq \Delta_{k^m},$$

$$\Gamma_a = \{\delta_{k^n}^{\gamma_1}, \delta_{k^n}^{\gamma_2}, \ldots, \delta_{k^n}^{\gamma_\zeta}\} \subseteq \Delta_{k^n}.$$

$$\Gamma_x = \{\delta_{k^{(\mu^*+1)n}}^{\xi_1}, \delta_{k^{(\mu^*+1)n}}^{\xi_2}, \ldots, \delta_{k^{(\mu^*+1)n}}^{\xi_{\zeta(\mu^*+1)}}\} \subseteq \Delta_{k^{(\mu^*+1)n}}.$$

Reference signal: $b_{\mathrm{r}} = \delta_{k^P}^{\epsilon}$

$$\Omega = \{\delta_{k^{(\mu^*+1)n}}^{\xi} : Col_\xi(\widehat{H}) = \delta_{k^P}^{\epsilon}, \ \xi \in C_x\}.$$

Major results

Theorem 1 The outputs of a system with multi-constraint track the reference signal b_{r} via state-feedback control, if and only if the constrained augmented system is Ω-stabilizable.

Theorem 2 The outputs of a system with multi-constraint track the reference signal b_{r} via state feedback control, if and only if there exists an integer $\eta \in [1 : \zeta^{\mu^*+1} - z]$ and a kernel-attractor set $N \subseteq \Omega$, such that

$$\mathrm{Row}_1\,(Q_N^{\eta}) > 0.$$

Conclusions

Using the STP method, the dynamics of the DLCN is converted into a bilinear discrete-time system, and an equivalent augmented system is constructed. Then the output tracking problem is transformed into a set stabilization problem.

By modifying the state transition matrix, some necessary and sufficient conditions for output tracking of the system are presented. In addition, the state-feedback controller is designed to drive the outputs of the constrained DLCN to track the reference signal.