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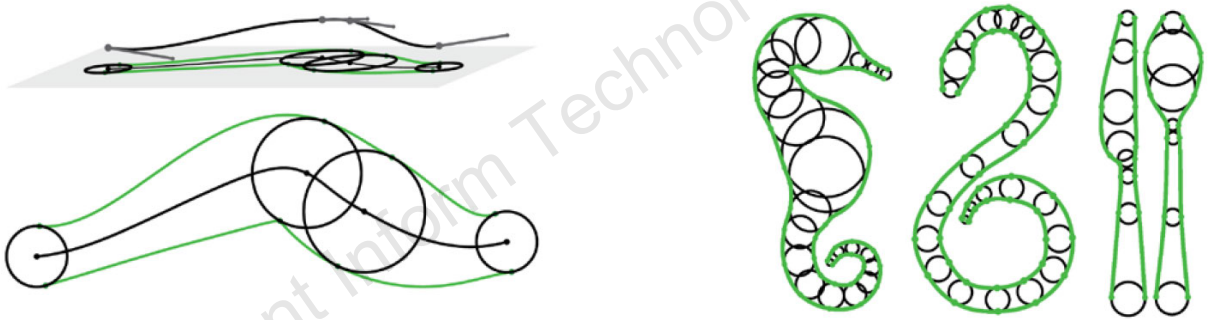
Applying Rational Envelope curves for skinning purposes



Key words: Medial axis transform; Envelope; Interpolation; Skinning; Circle

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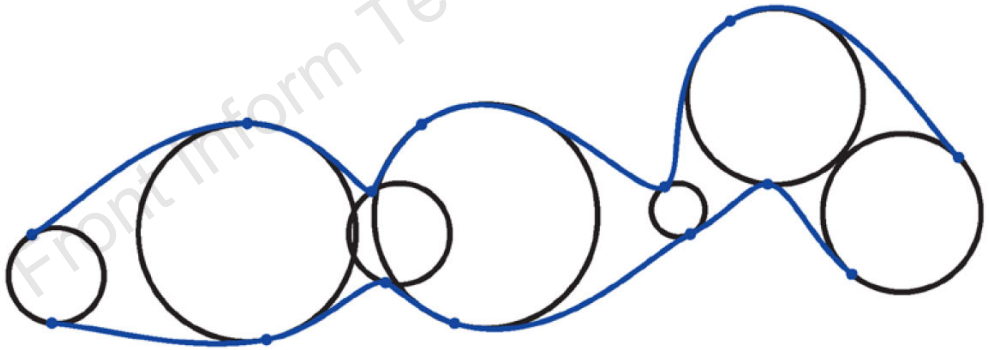
Motivation

- Curves in $\mathbb{R}^{2,1}$ may be considered as the medial axis transform of a planar domain
  - Minkowski Pythagorean hodograph (MPH) curves (Moon, 1999)
  - Rational Envelope (RE) curves (Bizzarri et al., 2016)
- MPH and RE curves describe domains with rational boundaries
- Using the so-called envelope formula (Choi et al., 1997), we can obtain the boundary/envelope of a planar object

Main idea

Our main idea is to **apply RE curves for skinning purposes, so we can use the envelopes for modeling**

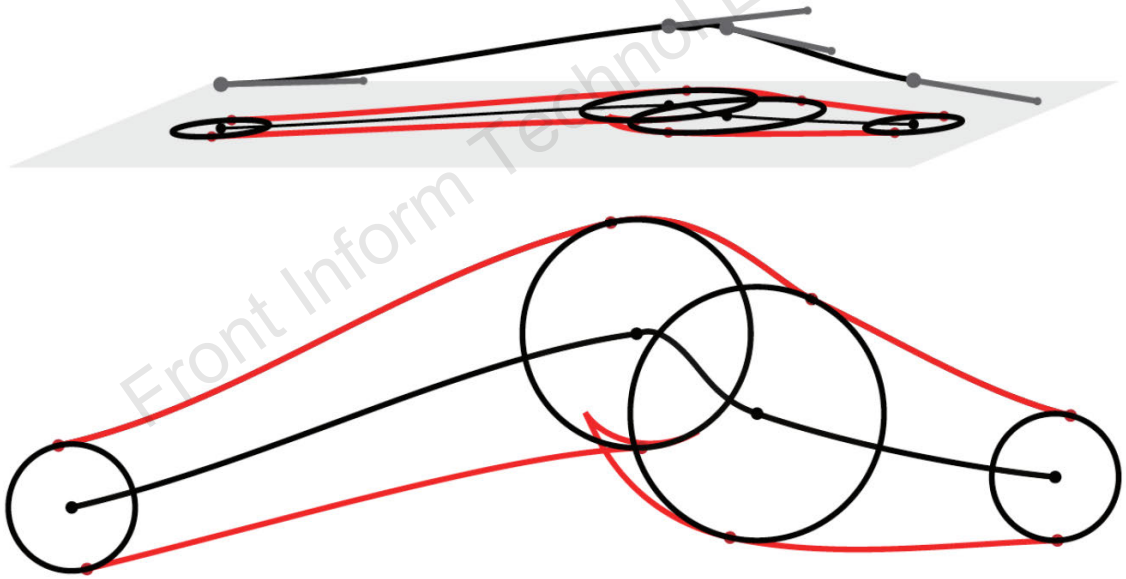
Skinning: constructing a pair of at least G^1 continuous splines for a predefined sequence of circles that touches each circle at one point



Modeling based on circles: skinning with the method of Kunkli and Hoffmann (2010)

Main idea: challenges

Arbitrarily choosing the tangent vectors may provide **inappropriate** results: the touching points may be inside the circles

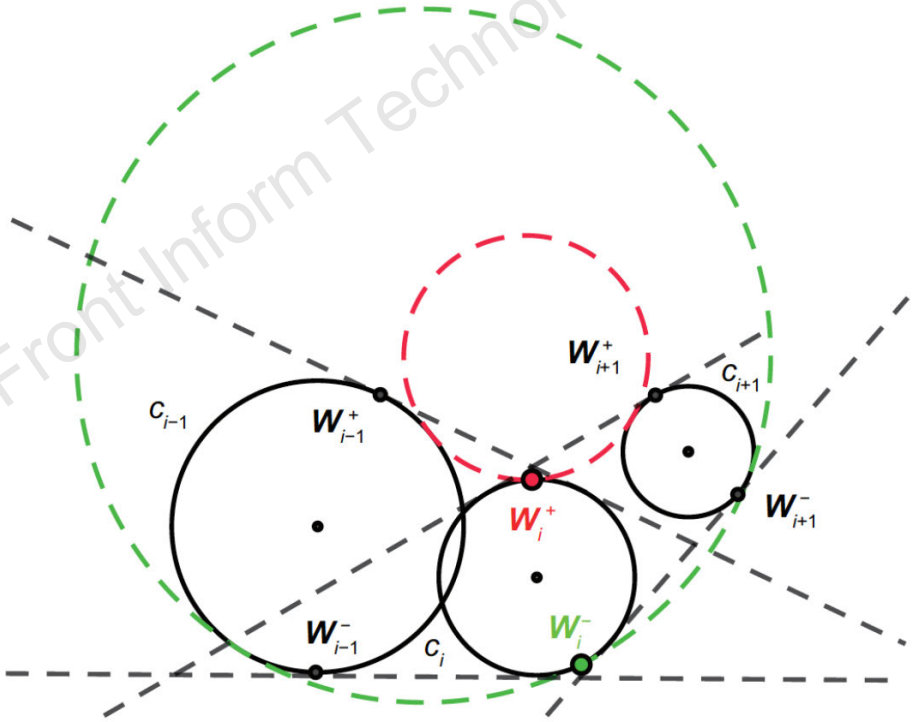


Question: how to choose the tangent vectors?

Method

Inverse approach:

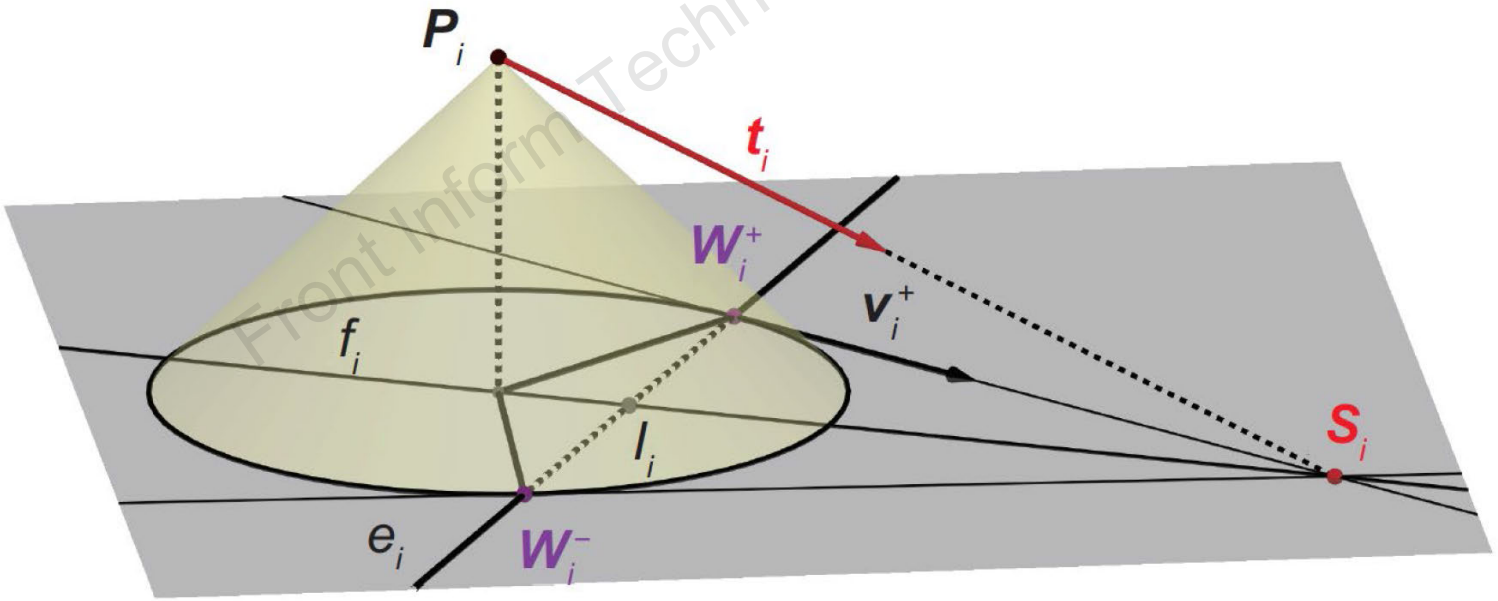
1. Localize the touching points $\boldsymbol{W}_i^+$ and $\boldsymbol{W}_i^-$
2. Reconstruct the tangent vector



Method (Cont'd)

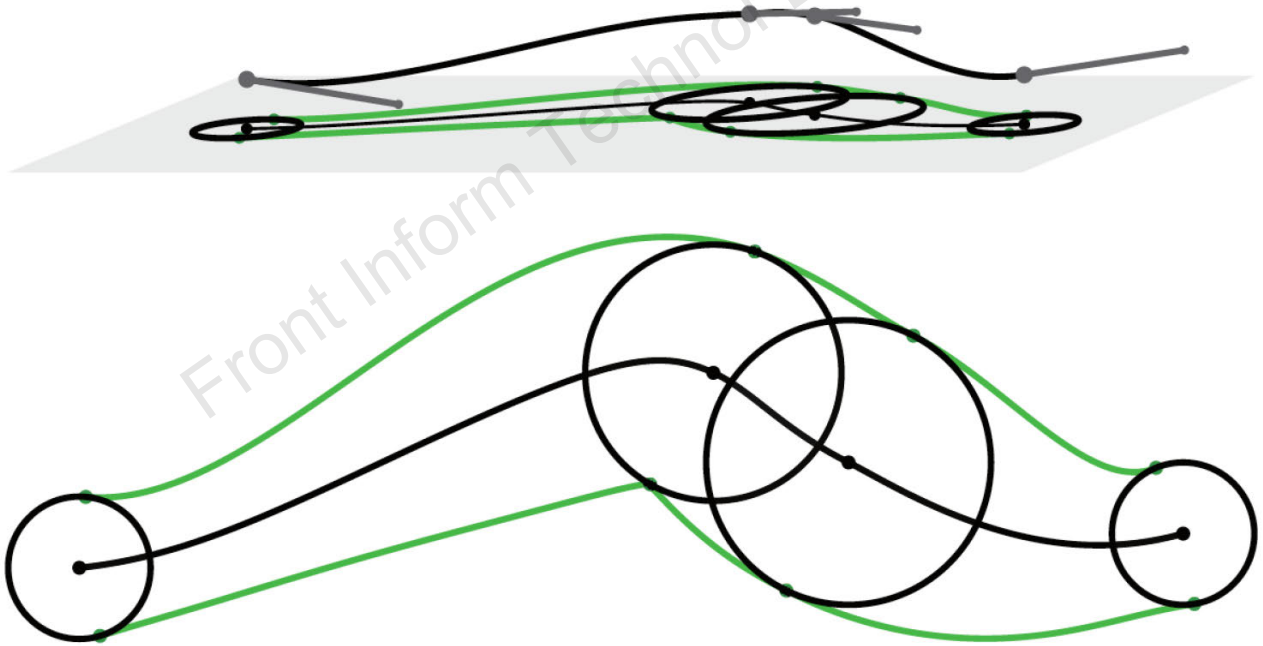
Inverse approach:

1. Localize the touching points $\boldsymbol{W}_i^+$ and $\boldsymbol{W}_i^-$
2. Reconstruct the tangent vector



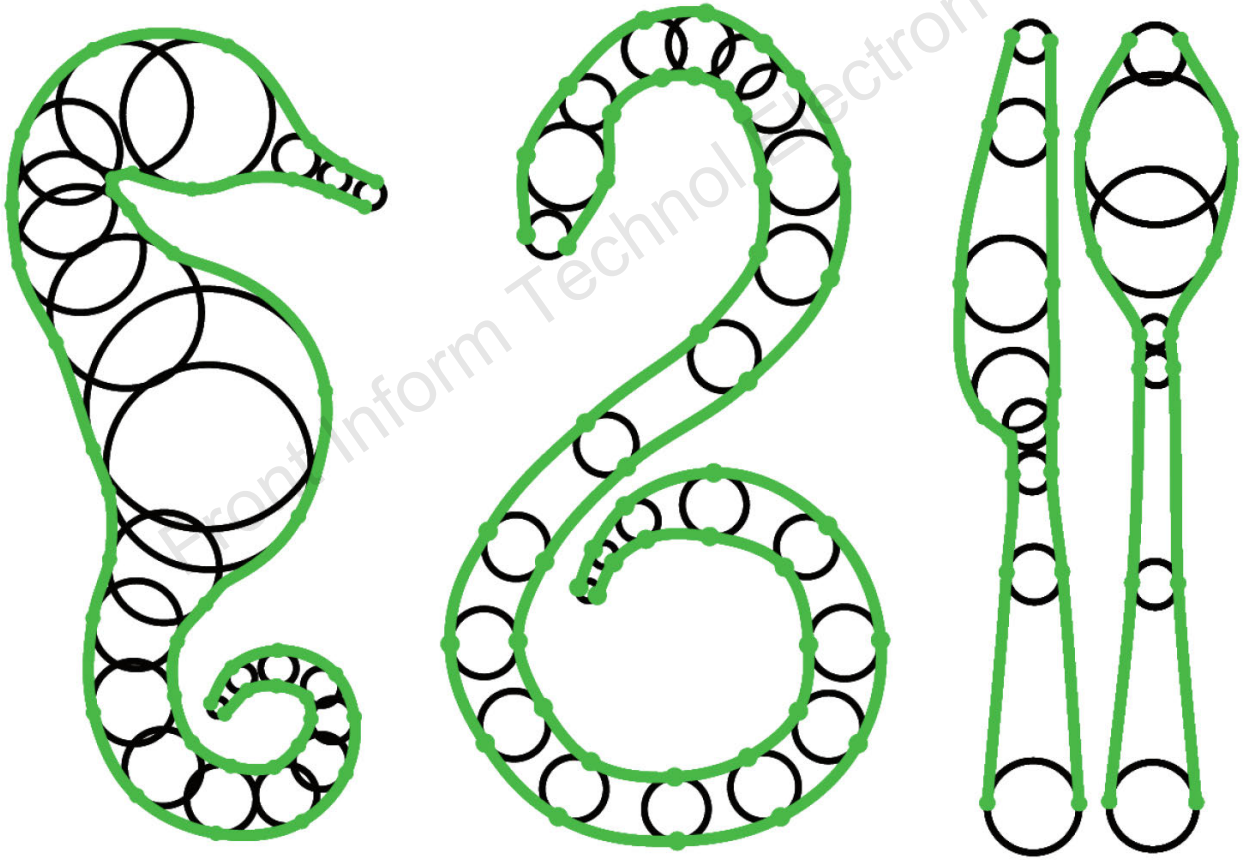
Major results

We solve the problematic scenarios: we can now use RE curves for skinning purposes



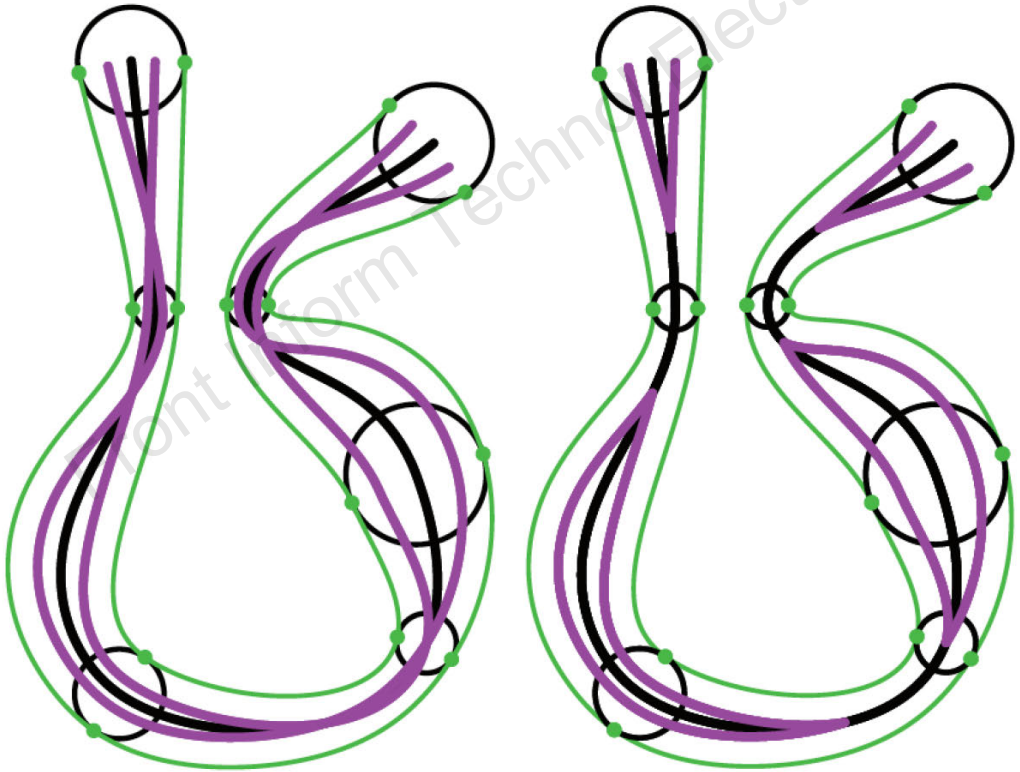
Major results (Cont'd)

We can create various circle-based models



Major results (Cont'd)

Advantage of our method: may be used in CNC machining by efficient trimming of the offset curves



Conclusions

We offer a new application area for RE curves: skinning a discrete set of circles

- Possibility for modeling
- Numerically stable calculations
- Efficient trimming of the offset curves, thus usable in CNC machining

Most important references

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