Xiang-shan Kong, Shu-ling Wang, Hai-tao Li, Fuad E. Alsaadi, 2020. New developments in control design techniques of logical control networks. *Frontiers of Information Technology & Electronic Engineering*, 21(2):220-233. https://doi.org/10.1631/FITEE.1900397

New developments in control design techniques of logical control networks

Key words: Logical control network; Control design; Semi-tensor product of matrices

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Motivation

- Multi-valued logical systems have wide applications in networked evolutionary game, combinational circuit design, feedback shift registers, fuzzy control, and so on.
- By virtue of specific control design techniques and designing suitable control policies, one can achieve various control objectives such as reachability, stabilization, synchronization, and optimal control.
- The control of multi-valued logical systems has been a challenging problem for a long time until the establishing of the algebraic state space representation (ASSR) approach.

Introduction

- Some preliminary results on the semi-tensor product method and logical control networks are reviewed.
- We move on to some new developments for control design techniques of logical control networks, including the reachable set approach, the pinning control technique, the control Lyapunov function approach, the event-triggered control technique, and the sampled-data control technique.

Method

1. Establishing the algebraic state space representation (ASSR) approach (Cheng et al., 2011).
2. Control design techniques of logical control networks, including the reachable set approach, the pinning control technique, the control Lyapunov function approach, the event-triggered control technique, and the sampled-data control technique.

Major results

- Reachable set approach

$$\begin{cases} \boldsymbol{x}(t+1) = \boldsymbol{L}\boldsymbol{u}(t)\boldsymbol{x}(t), \\ \boldsymbol{y}(t) = \boldsymbol{G}\boldsymbol{x}(t), \end{cases} \tag{5}$$

Definition 3 Consider system (5) given a state $\boldsymbol{\delta}_{k^n}^{\gamma} \in \Delta_{k^n}$. Let $E_l^{(\mathrm{R})}(\boldsymbol{\delta}_{k^n}^{\gamma})$ denote the set of all the initial states $\boldsymbol{x}_0 \in \Delta_{k^n}$ that can be driven to the state $\boldsymbol{\delta}_{k^n}^{\gamma}$ in l steps by a control sequence $\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(l-1) \in \Delta_{k^m}$. $E_l^{(\mathrm{R})}(\boldsymbol{\delta}_{k^n}^{\gamma}) = \Big\{\boldsymbol{x}_0 \in \Delta_{k^n}$: there exists a control sequence $\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(l-1) \in \Delta_{k^m}$, such that $\boldsymbol{x}(l; \boldsymbol{x}_0, \boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(l-1)) = \boldsymbol{\delta}_{k^n}^{\gamma}\Big\}$. $E_l^{(\mathrm{R})}(\boldsymbol{\delta}_{k^n}^{\gamma})$ is called the l^{th} reachable set of system (5).

Major results (Cont'd)

- Pinning control

Assuming that the pinned-nodes are given as $\{1, \cdots, s\}$, the controllability of the following system is considered:

$$\begin{cases} x_i(t+1) = \hat{f}_i(u_i(t), x_1(t), \cdots, x_n(t)), i = 1, \cdots, s, \\ x_j(t+1) = f_j(x_1(t), \cdots, x_n(t)), j = s+1, \cdots, n, \end{cases} \tag{9}$$

where $x_i \in \mathcal{D}_k$, $i = 1, \cdots, n$, $u_i \in \mathcal{D}_k$, $i = 1, \cdots, s$ are k-valued logical variables, and $\hat{f}_i : \mathcal{D}_k^{n+1} \to \mathcal{D}_k$, $i = 1, \cdots, s$, $f_j : \mathcal{D}_k^n \to \mathcal{D}_k$, $j = s+1, \cdots, n$ are k-valued functions.

Major results (Cont'd)

- Sampled-data control

Definition 4 Let a set of sampling points be given as $S = \{t_h : h \in \mathbb{N}\}$, $t_0 = 0$. $\{\boldsymbol{U}(t) : t \in \mathbb{N}\} \subseteq C_u$ is said to be a sampled-data control with regard to S, if $\boldsymbol{U}(t) = \boldsymbol{U}(t_h)$, $t \in [t_h, t_h + 1)|_{\mathbb{N}}$ holds for any $h \in \mathbb{N}$. A sampled-data control with regard to S is called a uniform sampled-data control, if there exists an integer $\rho \in \mathbb{Z}_+$, such that $t_{h+1} - t_h = \rho$ holds for any $h \in \mathbb{N}$, where the integer ρ is called the sampling period.

Major results (Cont'd)

- Event-triggered control

Let a nonempty set $M \subseteq \Delta_{k^n}$, an initial state $\boldsymbol{x}(0) = \delta_{k^n}^{\theta}$ and a time-variant control $\boldsymbol{u}(t) = \boldsymbol{\Phi}(t, \boldsymbol{x}(0))\boldsymbol{x}(t)$ be given. The event-triggered condition was formulated as

$$d_H(\Lambda(t+1), M) > 0, \tag{26}$$

where $\Lambda(t+1) = Col(Blk_\theta(\ltimes_{i=t}^{0}(\boldsymbol{L}\boldsymbol{\Phi}(i, \boldsymbol{x}(0))\boldsymbol{R}_{k^n}^{P})))$, and $d_H(\Lambda(t+1), M)$ represents the Hausdorff distance[2] between $\Lambda(t+1)$ and M. Denote the sequence of triggering time by $t_1 < t_2 < \cdots < t_\tau < \cdots$. Correspondingly, one can obtain a sequence of feedback control updates as $\boldsymbol{\Phi}(t_1, \boldsymbol{x}(0))$, $\boldsymbol{\Phi}(t_2, \boldsymbol{x}(0))$, $\cdots$, $\boldsymbol{\Phi}(t_\tau, \boldsymbol{x}(0))$, $\cdots$. Then, the event-triggered controllers can be designed as

$$\boldsymbol{u}(t) = \boldsymbol{\Phi}(t_s, \boldsymbol{x}(0))\boldsymbol{x}(t), t \in [t_s, t_{s+1}) \cap \mathbb{N},$$

where $s = 0, 1, \cdots, \tau, \cdots$ with $t_0 := 0$.

Major results (Cont'd)

- Control Lyapunov function approach

$$\begin{cases} \boldsymbol{x}(t+1) = \boldsymbol{L}\boldsymbol{u}(t)\boldsymbol{x}(t), \\ \boldsymbol{y}(t) = \boldsymbol{G}\boldsymbol{x}(t), \end{cases} \qquad (5)$$

Definition 8 Consider system (5) and let an equilibrium $\boldsymbol{x}_e = \boldsymbol{\delta}_{k^n}^{\xi} \in \Delta_{k^n}$ be given. $V(x) : \Delta_{k^n} \to \mathbb{R}$ is called a CLF of system (5), if (1) $\exists \boldsymbol{u}^* \in \Delta_{k^m}$, such that $V(\boldsymbol{L}\boldsymbol{u}^*\boldsymbol{x}_e) - V(\boldsymbol{x}_e) = 0$, and (2) $\forall \boldsymbol{x} \in \Delta_{k^n}$ satisfying $\boldsymbol{x} \neq \boldsymbol{x}_e$, $\exists\ \boldsymbol{u}_x \in \Delta_{k^m}$ such that $V(\boldsymbol{L}\boldsymbol{u}_x\boldsymbol{x}) - V(\boldsymbol{x}) > 0$.

Conclusions

- We have reviewed a series of new developments for control design techniques of LCNs.
- Reducing the computational complexity of existing results is an interesting topic.
- Extending the application fields of LCNs is also an interesting topic.