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Cascading decomposition of Boolean control networks: a graph-theoretical method

Key words: Boolean control networks; Semi-tensor product; Cascading decomposition; Graphic condition

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Motivation

- Cascading decomposition, as an essential and special decomposition form of Boolean control networks (BCNs), is a theoretically interesting and practically useful issue.
- How to construct a logical coordinate transformation to realize cascading state-space decomposition is still an open problem.
- The cascading state-space decomposition problem (SSDP) of BCNs discussed in the literature does not involve the decomposition of inputs, and the cascading SSDP with cascading inputs is also very interesting and important.

Main idea

- The state transition graph of BCNs can reflect all information of BCNs. Thus, the cascading decomposition form must have a graph-theoretic characterisation .
- The cascading decomposition form corresponds to some special nested structure.
- Motivated by the graph-theoretic characterisation of decomposition w.r.t. inputs (Zou and Zhu, 2014), the cascading form of BCNs must match some special vertex partition.

Method

- We use perfect vertex partition to describe a system free from external variables.
- We use the nested structure to describe the cascading decomposition form.
- To construct a logical coordinate transformation for solving two types of cascading decomposition problems, we construct logical matrices $\boldsymbol{Q}_i$.

Major results

Theorem 1 Considering Eq. (8), the Type-I cascading decomposition problem is solvable by a coordinate transformation $\boldsymbol{z} = \boldsymbol{T}\boldsymbol{x}$ if and only if the state transition diagram of Eq. (8) has a set of NPEVPs $\{\mathcal{S}^{1,2,\ldots,p}, \mathcal{S}^{1,2,\ldots,p-1}, \ldots, \mathcal{S}^{1,2}, \mathcal{S}^{1}\}$ satisfying $\mathcal{S}^{1,2\ldots p} \sqsubset \mathcal{S}^{1,2,\ldots,p-1} \sqsubset \ldots \sqsubset \mathcal{S}^{1,2} \sqsubset \mathcal{S}^{1}$, where for any $i \in [1,p]$, $\mathcal{S}^{1,2,\ldots,i} = \{S_l^{1,2,\ldots,i}\}_{l=1}^{2^{N_i}}$ with $|S_l^{1,2,\ldots,i}| = 2^{n-N_i}$ is a common PEVP of $\mathcal{G}_j$, $j = 1, 2 \ldots, 2^m$.

Major results

Theorem 2 For any $i \in [1, p]$, there exists a logical coordinate transformation $\boldsymbol{z} = \boldsymbol{T}_i \boldsymbol{x}$ such that under the $\boldsymbol{z}$ coordinate frame BCN becomes Eq. (36) if and only if the diagram $\mathcal{G}_j^{1,2,\ldots,i}$ has a common PEVP $\mathcal{S}^i := \{S_l^i\}_{l=1}^{2^{N_i}}$ with $|S_l^i| = 2^{n-N_i}$.

Theorem 3 Considering Eq. (8), the Type-II cascading decomposition problem is solvable by a coordinate transformation $\boldsymbol{z} = \boldsymbol{T}\boldsymbol{x}$ if and only if there exists an NPEVP $\mathcal{S}^{1,2,\ldots,p} \sqsubset \mathcal{S}^{1,2,\ldots,p-1} \sqsubset \ldots \sqsubset \mathcal{S}^{1,2} \sqsubset \mathcal{S}^1$, where for any i, $\mathcal{S}^{1,2,\ldots,i} = \{S_l^{1,2,\ldots,i}\}_{l=1}^{2^{N_i}}$ with $|S_l^{1,2,\ldots,i}| = 2^{n-N_i}$ is a common PEVP of $\mathcal{G}_j^{1,2,\ldots,i}$, $j = 1, 2, \ldots, 2^{M_i}$.

Conclusions

- This paper has provided a graph perspective to study cascading SSDP of BCNs for the first time, and given a simple and clear graphic description for the solvability of cascading SSDP.
- To realize cascading state-space decomposition, based on the graphic condition, we design an algorithm to construct a logical coordinate transformation $\boldsymbol{z}=\boldsymbol{T}\boldsymbol{x}$.
- We propose a new cascading SSDP with cascading inputs, and similar results are derived for this case.