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Complete synchronization of coupled Boolean networks with arbitrary finite delays

Key words: Boolean networks; Synchronization; Time delay

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Motivation

1. Recently, synchronization of Boolean networks has been studied extensively because of its wide applications in chemistry, economy, biology, and so on.

2. In the real world, time delays are unavoidable and transmission between different nodes and Boolean networks can be distinctive in many real-world systems. Therefore, it is meaningful to consider them in Boolean networks.

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Main idea

1. An algorithm is designed, which can be used to calculate two important matrices.

2. Based on two matrices calculated by the algorithm, a necessary and sufficient condition has been proposed to judge whether the system is synchronized.

3. The algorithm and the necessary and sufficient condition are validated by numerical simulations.

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Method

1. Design an algorithm to establish the relationship between the initial value and the value at any time.

2. Derive the necessary and sufficient condition for the system to achieve synchronization based on the properties of matrices and semi-tensor products.

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Major results

Algorithm 1 Construction of Q and Ω

1: Obtain the algebraic representation of the system

2: Obtain the expression of $\ltimes_{j=1}^{M}\left(\widehat{\boldsymbol{X}}_j(t)\widehat{\boldsymbol{Y}}(t)\right)$ with $\ltimes_{k=0}^{\tau M}\left(\ltimes_{j=1}^{M}\boldsymbol{X}_j(t-k)\right)$ and $\widehat{\boldsymbol{Y}}(t)$

3: Obtain matrices $\boldsymbol{Q}$ and $\boldsymbol{\Omega}$

4: Obtain the expression $\ltimes_{j=1}^{M}\boldsymbol{X}_j(t+1) = \boldsymbol{Q}\boldsymbol{\Omega}^t \ltimes_{s=0}^{\tau M}\left(\ltimes_{j=1}^{M}\boldsymbol{X}_j(-s)\right)$

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Major results (Cont'd)

Theorem 2 System synchronization occurs if and only if there exists a positive integer d satisfying

$$\mathbf{Col}(Q\Omega^{d-1}) \subseteq \left\{ \delta_{2^{MN}}^{\rho_i} : \rho_i = 1 + \frac{(i-1)(2^{MN}-1)}{2^N-1} \right\},$$

where $d_0 = \min\{m : m \geq 1, Q\Omega^{m-1} = Q\Omega^{n-1}, n > m\}$, $1 \leq d \leq d_0$, and $i = 1, 2, \ldots, 2^N$.

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Major results (Cont'd)

An example of a synchronous system:

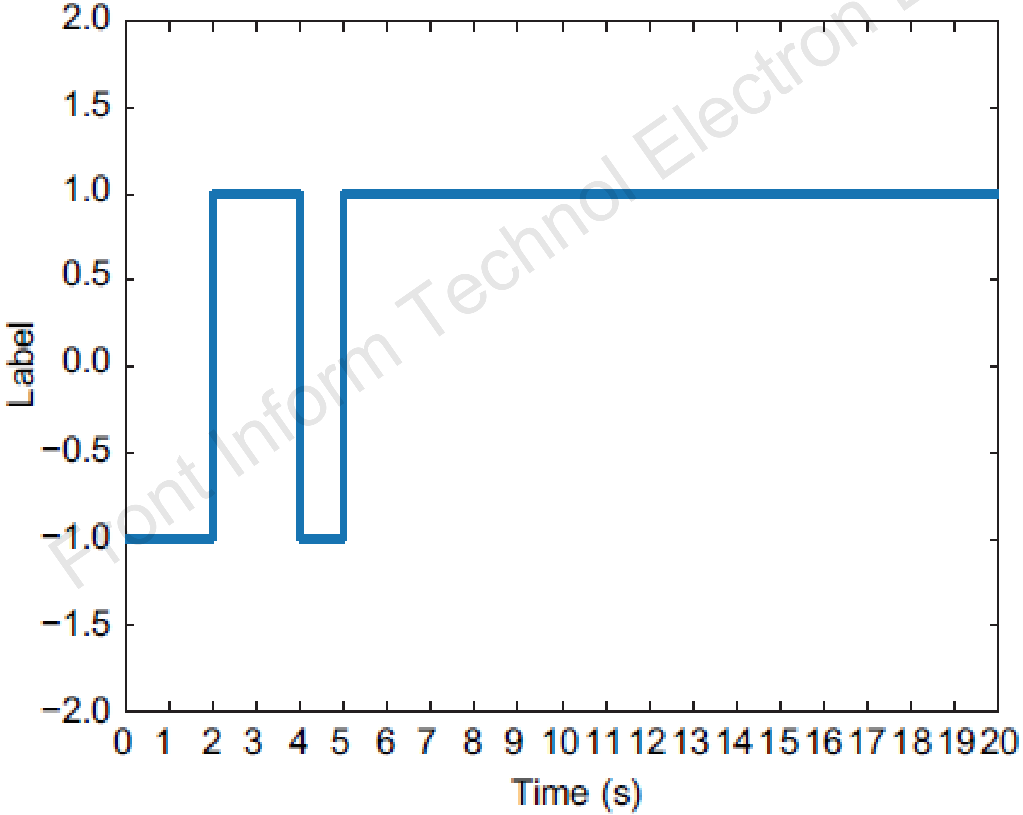

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Fig. 1 States of the system in Example 1

Major results (Cont'd)

An example of an asynchronous system:

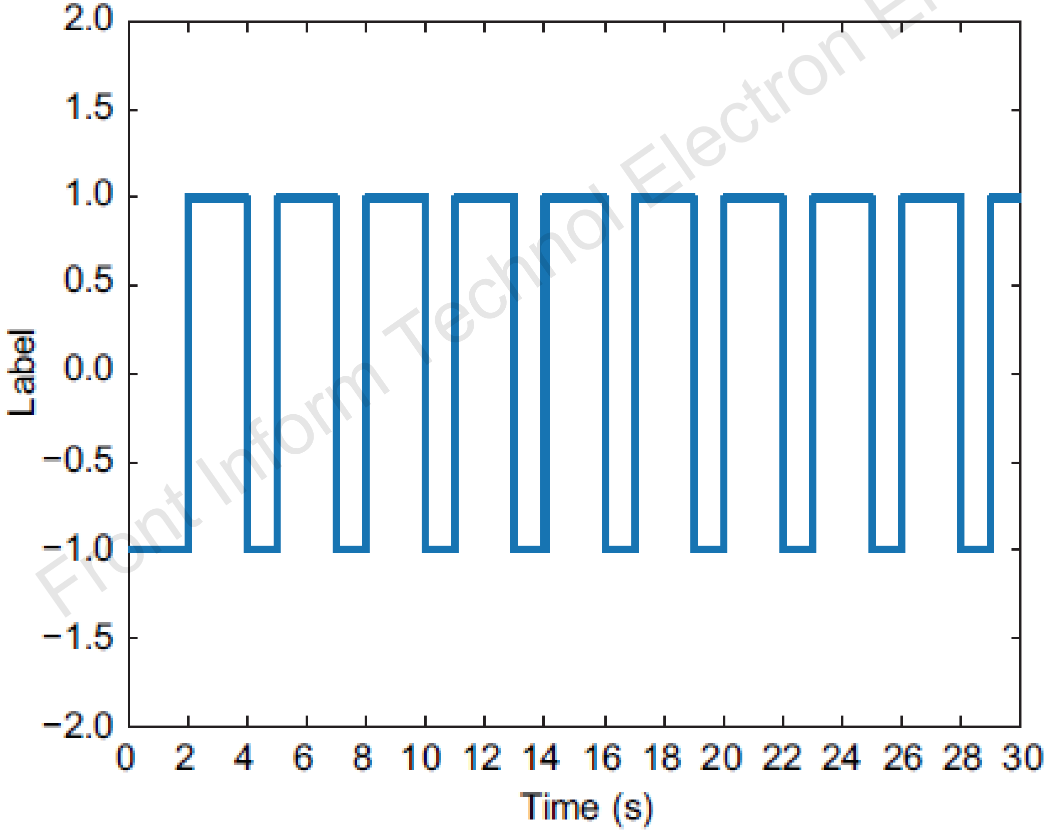

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Fig. 2 States of the system in Example 2

Conclusions

1. An algorithm has been designed to establish the relationship between the initial value and the value at any time.

2. The necessary and sufficient condition has been proposed to determine whether the system achieves synchronization.

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