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Event-based H_∞ control for piecewise-affine systems subject to actuator saturation

Key words: Event-triggered control; Piecewise-affine system; Linear matrix inequality; Actuator saturation; H_∞ performance

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Motivation

1. Traditional periodic control strategies have some negative effects on the control performance, such as wasting the communication resource and high energy consumption. To overcome these problems, event-triggered control has been proposed.

2. Actuator saturation is a common issue in control systems because of the natural physical limitations of actuators.

3. Piecewise-affine systems are an important class of hybrid systems with wide applications. However, there are few studies about event-triggered control for piecewise-affine (PWA) systems.

Main idea

1. We propose an event-based state feedback control approach for piecewise-affine systems.
2. By considering saturation information, a novel event-triggered strategy is proposed.
3. We transform the event-triggered H_∞ controller design problem into feasibility problem of linear matrix inequalities.
4. We provided optimization approaches to optimize the H_∞ performance and the domain of attraction.

Method

The discrete-time PWA system:

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{A}_i \boldsymbol{x}(k) + \boldsymbol{B}_{1i} \mathrm{sat}(\boldsymbol{u}(k)) + \boldsymbol{B}_{2i} \boldsymbol{\omega}(k) + \boldsymbol{a}_i, \\ \boldsymbol{z}(k) = \boldsymbol{C}_i \boldsymbol{x}(k) + \boldsymbol{D}_{1i} \mathrm{sat}(\boldsymbol{u}(k)) + \boldsymbol{D}_{2i} \boldsymbol{\omega}(k) \ . \end{cases}$$

The state feedback control law:

$$\begin{cases} \hat{\boldsymbol{u}}(k) = \boldsymbol{K}_i \hat{\boldsymbol{x}}(k), \ i \in \mathcal{I}, \\ \hat{\boldsymbol{u}}(0) = \boldsymbol{0} \ . \end{cases}$$

The event-triggering condition:

$$\hat{\boldsymbol{u}}(k) = \begin{cases} \boldsymbol{u}(k), & \|\boldsymbol{e}(k)\| \geq \sigma \|\boldsymbol{K}_i \boldsymbol{x}(k)\|, \\ \hat{\boldsymbol{u}}(k-1), & \|\boldsymbol{e}(k)\| < \sigma \|\boldsymbol{K}_i \boldsymbol{x}(k)\|. \end{cases}$$

Method (Cont’d)

The control structure:

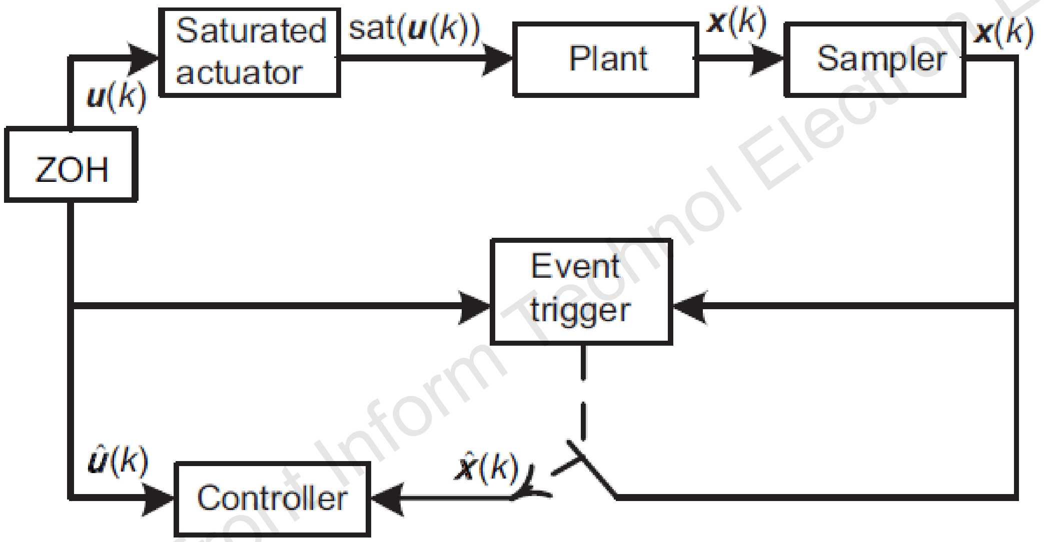
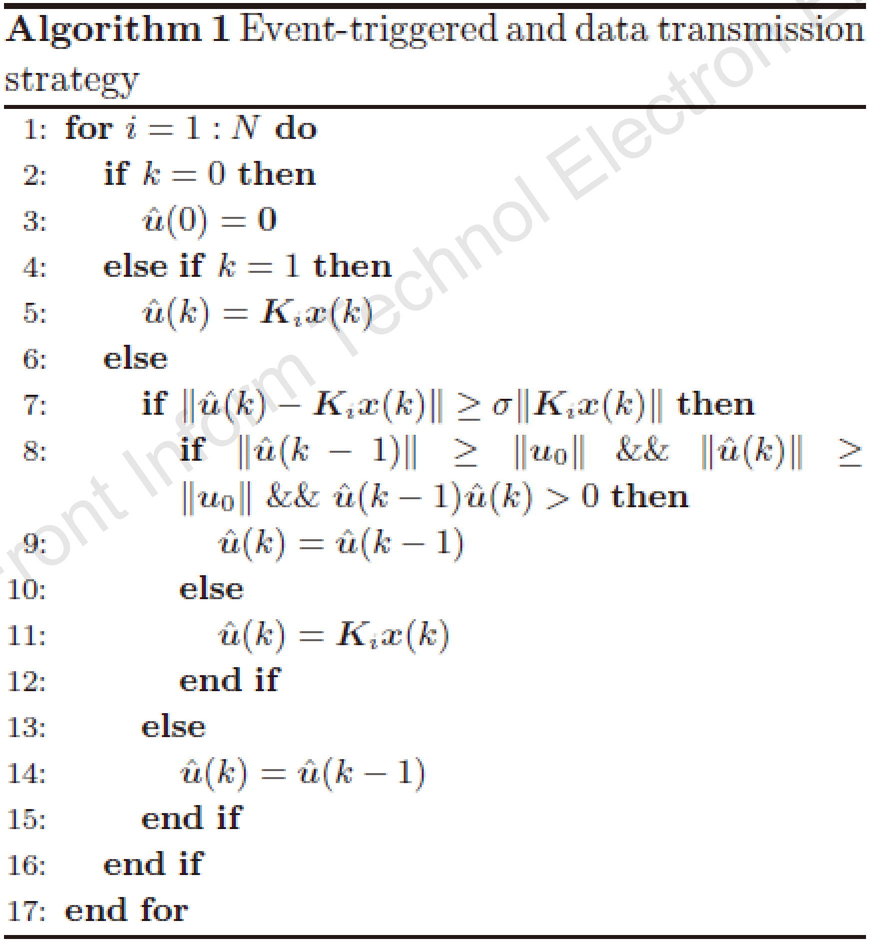


Fig. 1 Structure of the event-based control loop (ZOH: zero-order holder)

Method (Cont’d)

The proposed event-triggered strategy:



Algorithm 1 Event-triggered and data transmission strategy

```
for $i = 1 : N$ do
    if $k = 0$ then
        $\hat{u}(0) = 0$
    else if $k = 1$ then
        $\hat{u}(k) = K_i x(k)$
    else
        if $\|\hat{u}(k) - K_i x(k)\| \geq \sigma \|K_i x(k)\|$ then
            if $\|\hat{u}(k-1)\| \geq \|u_0\|$ && $\|\hat{u}(k)\| \geq \|u_0\|$ && $\hat{u}(k-1)\hat{u}(k) > 0$ then
                $\hat{u}(k) = \hat{u}(k-1)$
            else
                $\hat{u}(k) = K_i x(k)$
            end if
        else
            $\hat{u}(k) = \hat{u}(k-1)$
        end if
    end if
end for
```

Major results

Example 1 The chaotic map-*T* system:

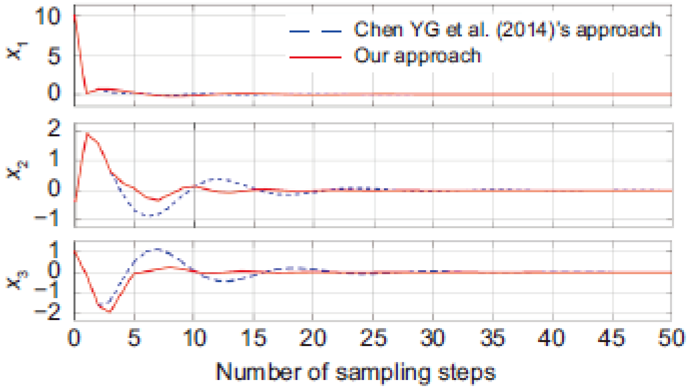


Fig. 2 State responses of the system when $\sigma = 0.9$

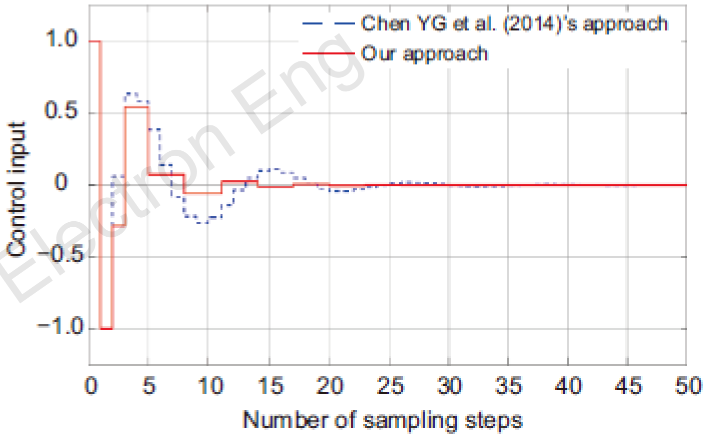


Fig. 3 Control input of the system when $\sigma = 0.9$

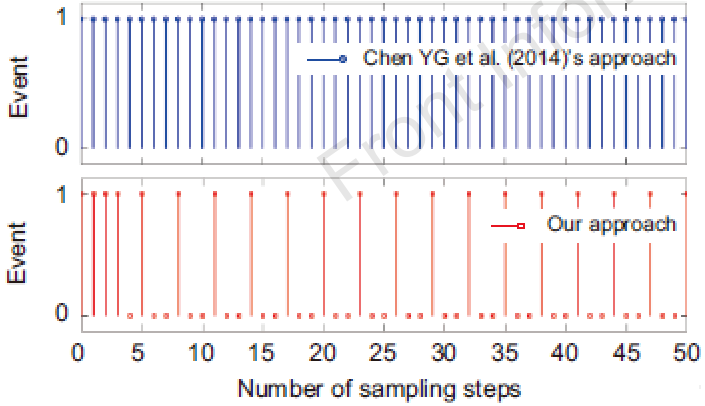


Fig. 4 Simulation results of the events when $\sigma = 0.9$

"1" means that the event is triggered and "0" means that it is not triggered

Table 1 H_∞ performance index $\gamma_{\min}$ and transmission rate (TR) for various values of σ

σ	$\gamma_{\min}$	TR (%)
0.1	0.0406	94.12
0.3	0.0795	74.51
0.5	0.2616	66.67
0.7	6.0865	52.94
0.9	11.7065	39.22

Major results (Cont'd)

Example 2 The single-link robot arm control system:

$$\ddot{\theta}(t) = -\frac{MgL}{J}\sin(\theta(t)) - \frac{R}{J}\dot{\theta}(t) + \frac{1}{J}u(t) + \omega(t).$$

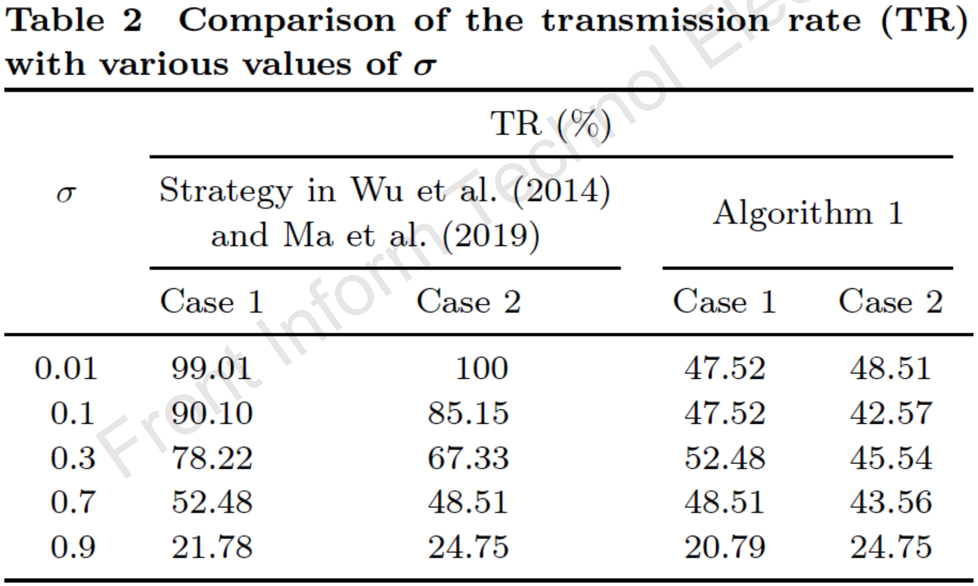


Table 2 Comparison of the transmission rate (TR) with various values of σ

σ	TR (%)			
	Strategy in Wu et al. (2014) and Ma et al. (2019)		Algorithm 1	
	Case 1	Case 2	Case 1	Case 2
0.01	99.01	100	47.52	48.51
0.1	90.10	85.15	47.52	42.57
0.3	78.22	67.33	52.48	45.54
0.7	52.48	48.51	48.51	43.56
0.9	21.78	24.75	20.79	24.75

Case 1: $\omega(t)$ =0; Case 2: $\omega(t)$ =0.5sin(4t)

Major results (Cont'd)

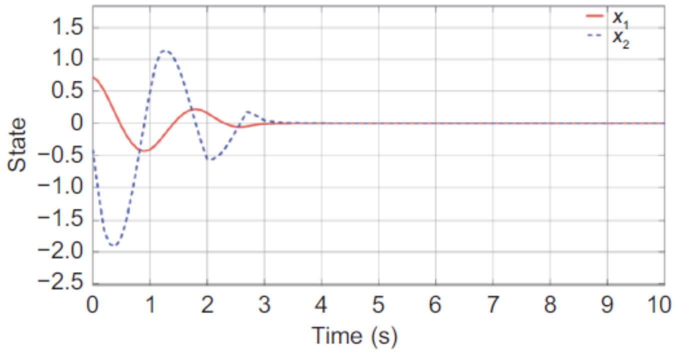


Fig. 6 State response of the system in case 1 when $\sigma = 0.7$

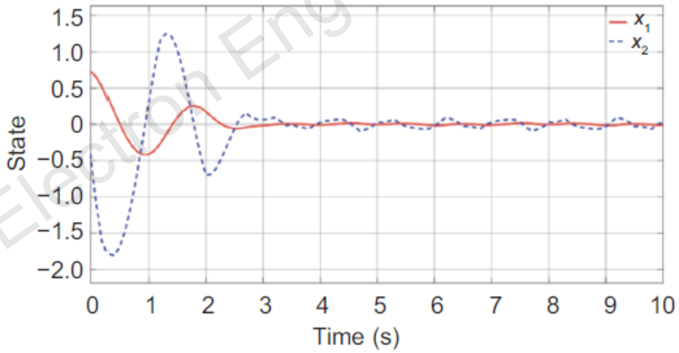


Fig. 8 State response of the system in case 2 when $\sigma = 0.7$

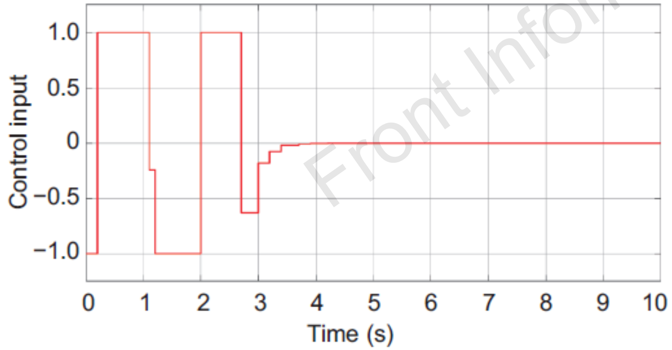


Fig. 7 Control input of the system in case 1 when $\sigma = 0.7$

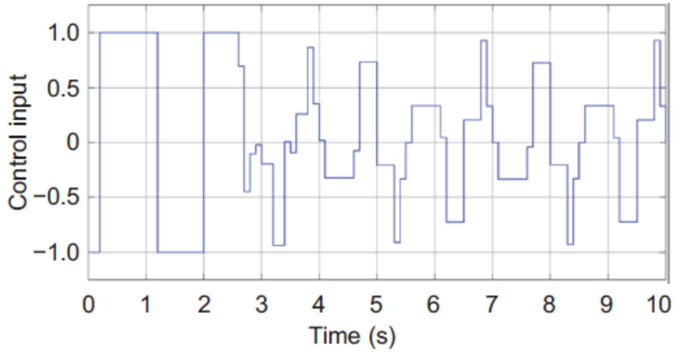


Fig. 9 Control input of the system in case 2 when $\sigma = 0.7$

Major results (Cont'd)

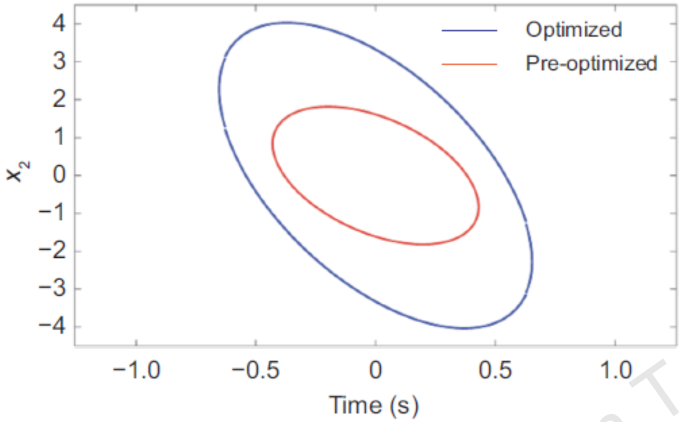


Fig. 10 Estimate of the domain of attraction when $\sigma = 0.7$

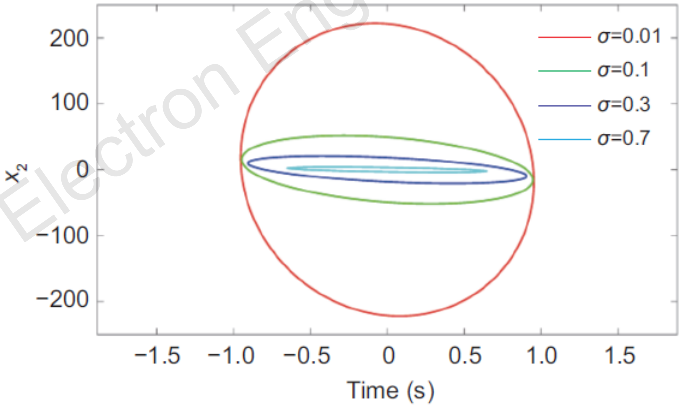


Fig. 11 Estimate of the domain of attraction with different values of σ

Conclusions

1. Event-triggered H_∞ control for discrete-time PWA systems subject to actuator saturation has been proposed .
2. A novel event-triggered strategy that considers the saturation information has been proposed, which can save more communication resources.
3. The event-triggered H_∞ control problem has been cast into LMIs, which can be efficiently solved by available software.