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Controllability of fractional-order damped systems with time-varying delays in control

Key words: Controllability; Fractional-order damped systems; Time-varying delays; Gramian matrix

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Motivation

- Fractional-order calculus is a useful tool for describing a process with hereditary and memory properties and has attracted more and more attention. Fractional control theory is a basic topic of application-oriented fractional-order calculus in information science.
- The fractional damped system is an extension of the integer-order damped system, and it is a meaningful and interesting theory. Considering that the delay often occurs in practical systems, we investigate the fractional damped system with time-varying delays.

Main idea

- The controllability of a linear fractional-order damped system can be described by its coefficient matrix.
- The small disturbance shall not change the controllability of the controllable linear system.

Method

1. Find the solution expression of the fractional-order damped system with time-varying delays;
2. Define the controllability Gramian matrix;
3. Construct the control input function.

Major results

- The linear fractional damped system (1) is controllable on $[0,T]$ if and only if the controllability Gramian matrix $\boldsymbol{W}$ is positive definite.

- System

$$\begin{cases} {}_0^C D_t^p \boldsymbol{y}(t) - \boldsymbol{A}\,{}_0^C D_t^p \boldsymbol{y}(t) = \sum_{i=1}^{M} \boldsymbol{B}_i \boldsymbol{u}[t-\tau_i], t \geq 0, \\ \boldsymbol{y}(0) = \boldsymbol{y}_0, \boldsymbol{y}'(0) = \boldsymbol{y}_0', \\ \boldsymbol{u}(t) = \boldsymbol{\psi}(t), -\tau_M \leq t \leq 0. \end{cases}$$

is controllable on $[0,T]$ if and only if

$$\begin{aligned} \operatorname{rank}[&\boldsymbol{B}_0, \boldsymbol{A}\boldsymbol{B}_0, \boldsymbol{A}^2\boldsymbol{B}_0, \cdots, \boldsymbol{A}^{n-1}\boldsymbol{B}_0, \\ &\boldsymbol{B}_1, \boldsymbol{A}\boldsymbol{B}_1, \boldsymbol{A}^2\boldsymbol{B}_1, \cdots, \boldsymbol{A}^{n-1}\boldsymbol{B}_1, \\ &\boldsymbol{B}_M, \boldsymbol{A}\boldsymbol{B}_M, \boldsymbol{A}^2\boldsymbol{B}_M, \cdots, \boldsymbol{A}^{n-1}\boldsymbol{B}_M] = n \end{aligned}$$

Major results

- Let f be a continuous function satisfying

$$\lim_{\|(\boldsymbol{y},\boldsymbol{u})\|\to\infty} \frac{\|f(t,\boldsymbol{y},\boldsymbol{u})\|}{\|(\boldsymbol{y},\boldsymbol{u})\|} = 0$$

uniformly on $t \in [0, T]$. Let the linear fractional system (1) be controllable. Then the nonlinear system

$$\begin{cases} {}_0^C D_t^p \boldsymbol{y}(t) - \boldsymbol{A}\,{}_0^C D_t^p \boldsymbol{y}(t) = \sum_{i=1}^{M} \boldsymbol{B}_i \boldsymbol{u}[t-\tau_i] + f(t,\boldsymbol{y},\boldsymbol{u}), t \geq 0, \\ \boldsymbol{y}(0) = \boldsymbol{y}_0, \boldsymbol{y}'(0) = \boldsymbol{y}_0', \\ \boldsymbol{u}(t) = \boldsymbol{\psi}(t), -\tau_M(0) \leq t \leq 0 \end{cases}$$

is controllable on $[0, T]$.

Conclusions

- The linear fractional damped system is controllable on $[0, T]$ if and only if the controllability Gramian matrix is positive definite.
- When the delay is independent of time, the controllability can be equivalently characterized by the rank of a matrix.
- For the nonlinear system, under the controllability of its corresponding linear system, the controllability can be ensured by a restriction on the nonlinear term.