Xue-feng ZHANG, Hui YAN, Hao HE, 2020. Multi-focus image fusion based on fractional-order derivative and intuitionistic fuzzy sets. *Frontiers of Information Technology & Electronic Engineering*, 21(6):834-843.
https://doi.org/10.1631/FITEE.1900737

Multi-focus image fusion based on fractional-order derivative and intuitionistic fuzzy sets

Key words: Image fusion; Fractional-order derivative; Intuitionistic fuzzy sets; Multi-focus images

Corresponding author: Xue-feng ZHANG
E-mail: zhangxuefeng@mail.neu.edu.cn
ORCID: https://orcid.org/0000-0002-2831-5747

Motivation

1. As an important branch of information fusion, image fusion is mainly to fuse the information of multiple images in matrix form into a single image. The information of the fused image is more comprehensive, and more suitable for human visual observation.
2. Spatial frequency (SF), which refers to the quality of the image region focus, has been introduced into image fusion. However, SF suffers from poor performance in some practical image fusion tasks.
3. Intuitionistic fuzzy set (IFS) takes account of the information of membership, non-membership, and hesitation degrees at the same time, so it is more flexible and practical than traditional fuzzy sets (FSs) in dealing with fuzziness and uncertainty. Many image fusion problems have been solved using IFS.

Main idea

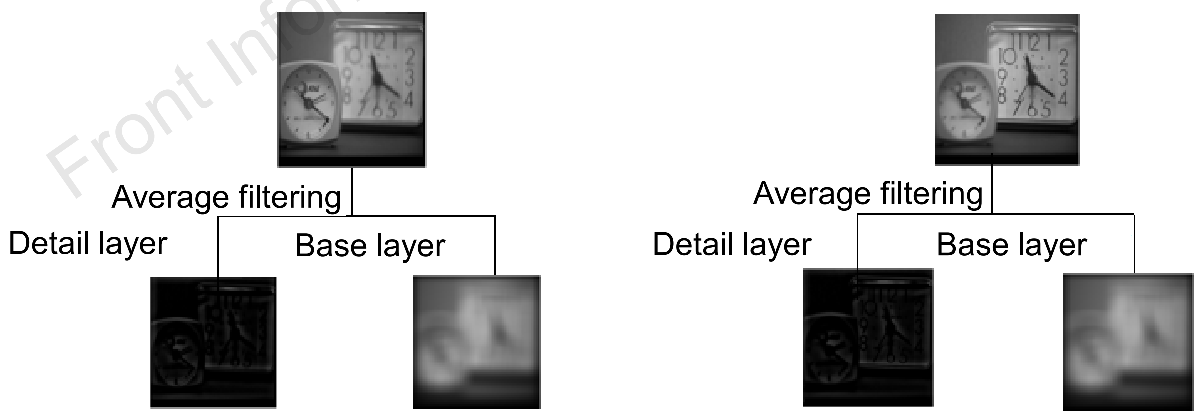
1. Two-scale representations of the original images are obtained according to an average filter to better preserve the details of the original images.
2. To reflect the clarity of images, the fractional-order spatial frequency (FSF) is built which is an integration of fractional-order derivative and SF.
3. The base layers are fused through IFS, which aims at dealing with fuzzy and uncertain things and quantifies them, making full use of the richness of gray values of images.

Method

1. Average filtering is used to decompose the original images into two-scale representations. The base layer is obtained as follows:

$$B_n = u_n * Z,\ n = 1, 2,$$

where $u_n\ (n = 1, 2)$ is the original image, and Z is average filtering, whose size is 30×30. Based on the base layer, the detail layer can be obtained easily: $D_n = u_n - B_n,\ n = 1, 2.$



Two-scale representations of the original images

Method (Cont'd)

2. To integrate the base information effectively, IFS is used to fuse the base layers. On the basis of IFS, fuzzy entropy which represents the richness of the information contained in the image is calculated. The fusion rule for the base layers is as follows (ENT: fuzzy entropy):

$$B(i,j) = \begin{cases} B_1(i,j), & \mathrm{ENT}_1(i,j) \geq \mathrm{ENT}_2(i,j), \\ B_2(i,j), & \text{otherwise}. \end{cases} \tag{21}$$

Method (Cont'd)

3. With fractional-order derivative, FSF is more sensitive to different features of an image than SF. FSF is defined as

$$\mathrm{FSF} = \sqrt{\mathrm{FRF}^2 + \mathrm{FCF}^2} = \| D^{\alpha} \boldsymbol{f} \|_{\mathrm{F}}, \quad (11)$$

$$\mathrm{FRF} = \| \boldsymbol{M} \boldsymbol{f} \|_{\mathrm{F}}, \quad (12)$$

$$\mathrm{FCF} = \| \boldsymbol{f} \boldsymbol{M} \|_{\mathrm{F}}, \quad (13)$$

where *M* is the coefficient matrix of the fractional-order derivative, and $\| \cdot \|_{\mathrm{F}}$ denotes the Frobenius norm.

Then, the detail layers are fused by FSF:

$$D(i,j) = \begin{cases} D_1(i,j), & \mathrm{FSF}_1(i,j) \geq \mathrm{FSF}_2(i,j), \\ D_2(i,j), & \text{otherwise.} \end{cases} \quad (22)$$

Method (Cont'd)

4. Finally, the two layers of the source image are added to obtain the complete fusion image: $u = B + D$.

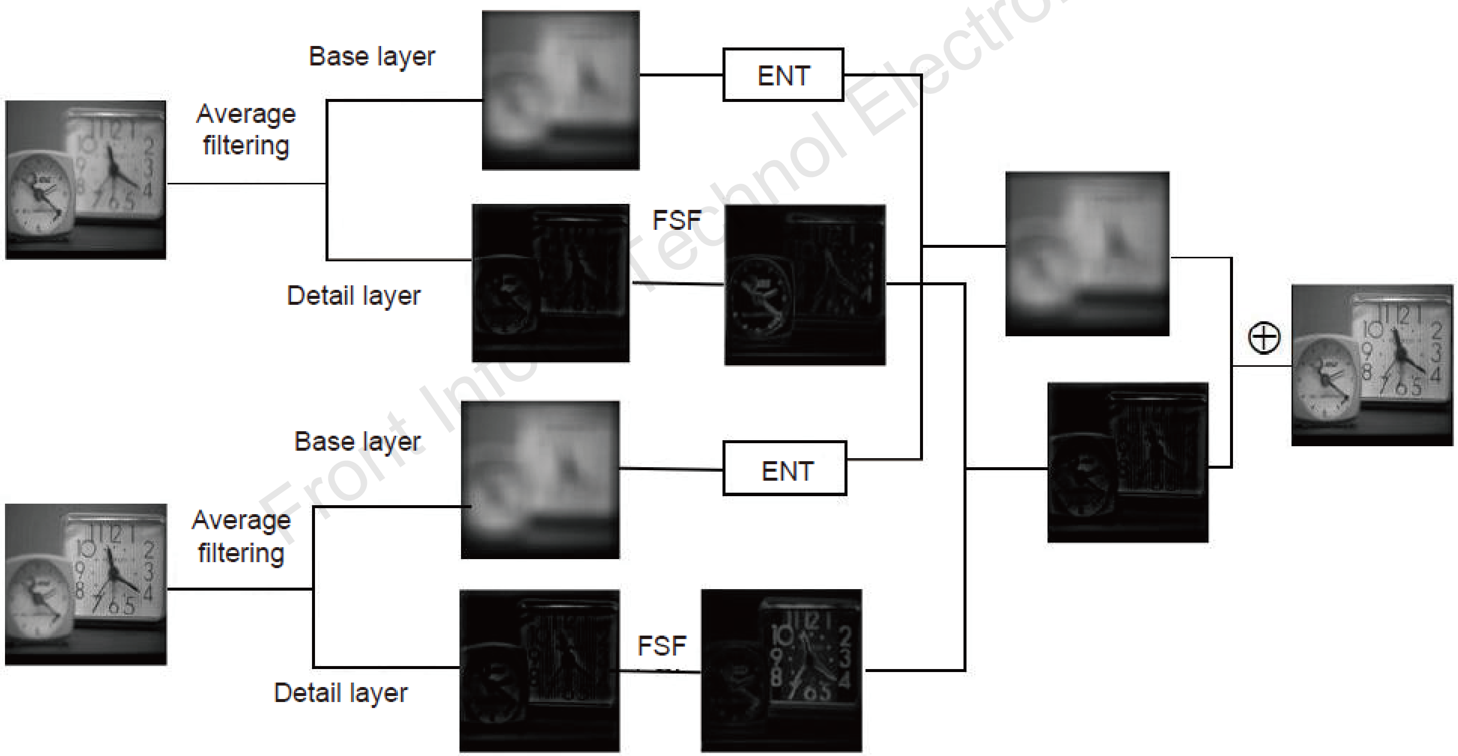


Fig. 3 Flow diagram of the proposed multi-focus image (ENT: fuzzy entropy)

Major results

Multi-focus image fusion

Original image

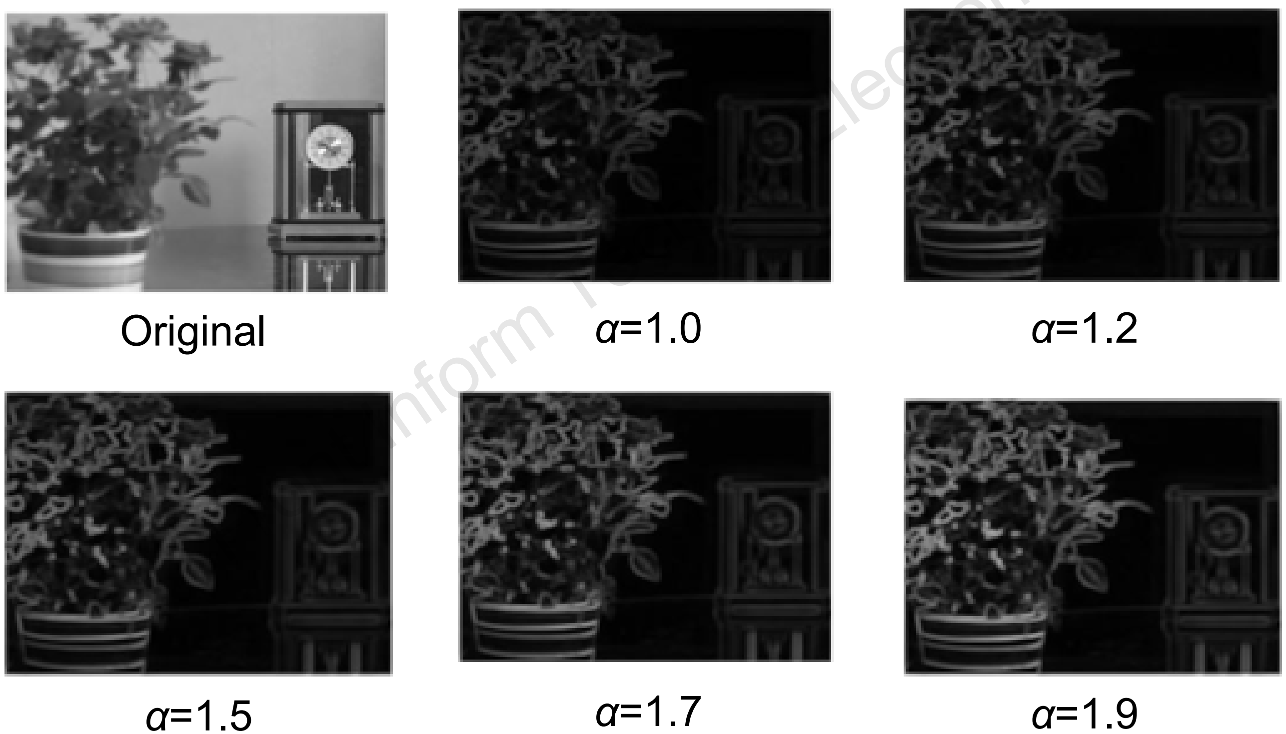
Original image

Proposed

Fusion results of our method

Major results (Cont'd)

FSF images of the detail layer with different orders



FSF images of the detail layer

Conclusions

1. This new approach is based on two-scale image decomposition, by which an image is decomposed into two layers by average filtering.

2. The base layers are fused in IFS, which can solve the uncertainty problem effectively.

3. The detail layers are fused in FSF, which combines SF and a fractional-order derivative to indicate the clarity of the image.

Xue-feng ZHANG is currently with the School of Sciences, Northeastern University, Shenyang, China. He received the B.Sc. degree in Applied Mathematics in 1989, the M.S. degree in Control Theory and Control Engineering in 2004, and the Ph.D. degree in Control Theory and Control Engineering in 2008 from Northeastern University, China. He is an Associate Editor of *IEEE Access* and also a committee member of the Technical Committee on Fractional and Control and a member of the Chinese Association of Automation. His research interests include fractional- order control systems and singular systems.

Hui YAN is currently pursuing her Master's degree at the School of Sciences, Northeastern University, Shenyang, China. Her current research interest is the algorithm of image processing based on fractional differential.

Hao HE is currently pursuing his Master's degree at the School of Sciences, Northeastern University, Shenyang, China. His current research interest is the fractional-order control system.