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# Joint tracking and classification of extended targets with complex shapes

**Key words:** Extended target; Fourier descriptors; Joint tracking and classification; Random hypersurface model; Bernoulli filter

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# Motivation

1. The process of joint tracking and classification (JTC) can make full use of information on both target estimation and decision, as well as improve the accuracy of both.
2. The JTC-RMM (random matrix model) method provides a novel mean for the JTC of extended targets. The drawback is that the performance of this method degrades when targets have similar sizes.
3. To overcome the drawback of the JTC-RMM method, and to use the extent state as the classification feature (both shape and size information) more effectively, we propose a JTC method for extended targets with a complex shape.

# Main idea

1. To use the extent state as the classification feature (both shape and size information) more effectively, we adopt the star-convex random hypersurface model (RHM) to model the extent state, and use the extent state as a classification feature.
2. The target state is modeled by two vectors to alleviate the influence of the high-dimensional state space and the severely nonlinear observation model on target state estimation, while the Euclidean distance metric of the normalized Fourier descriptors is applied to obtain the analytical solution of the updated class probability, called “JTC-RHM.”
3. The proposed JTC-RHM is integrated into a Bernoulli filter framework to solve the JTC of a single extended target in the presence of detection uncertainty and clutter, resulting in a JTC-RHM-Ber filter.

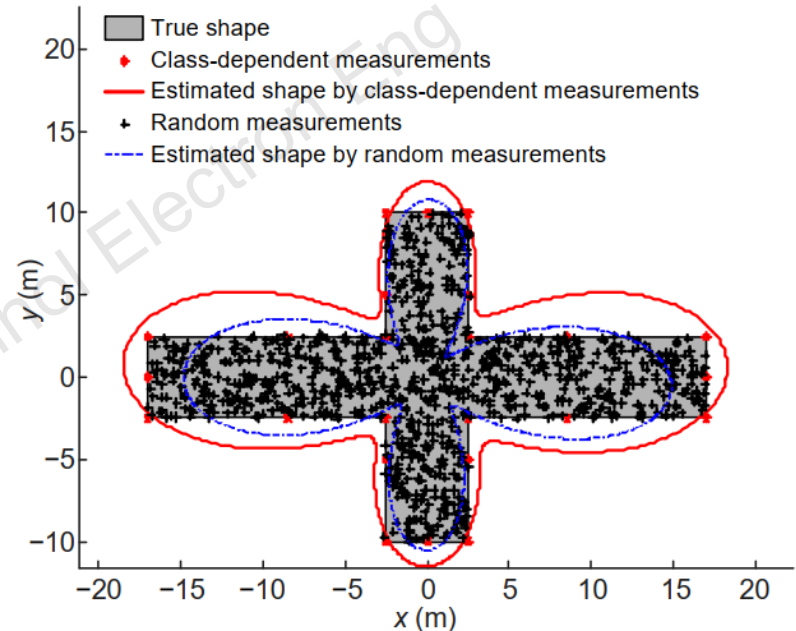
# Method

## 1. Prior class-dependent measurement information

The JTC method uses not only the measurements received through sensors, but also the prior class-dependent information.

For a known class of an extended target, the positions of the feature points on the target contour can be acquired in advance.

These feature points can be used as the prior information of the target class, represented as  $\underline{Z}_k \triangleq \{Z_k, \{Z_k^c\}_{c=1}^{n_c}\}$ .



**Fig. 1 Estimation results of the extent state using two kinds of measurements (References to color refer to the online version of this figure)**

# Method

## 2. Calculation of the updated class probability

For target classification, the class probability of the proposed JTC-RHM method is calculated by

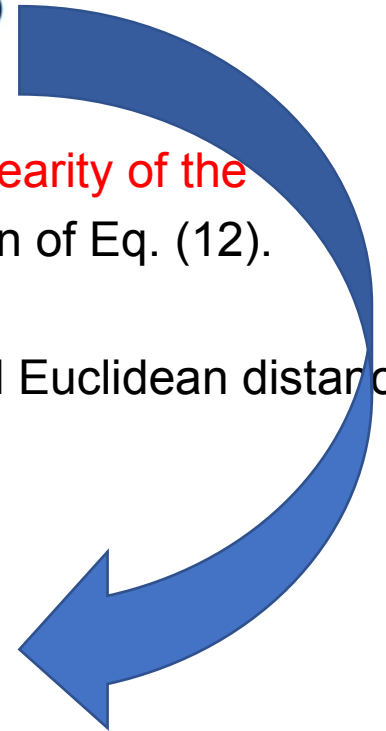
$$\tilde{\mu}_k^c \triangleq p(c | \underline{Z}^k) = \frac{p(\underline{Z}_k, \underline{Z}_k^c | c, \underline{Z}^{k-1}) p(c | \underline{Z}^{k-1})}{p(\underline{Z}_k | \underline{Z}^{k-1})}, \quad (12)$$

Due to the **high dimensionality of state vectors** and the **nonlinearity of the observation model**, it is difficult to derive the analytical solution of Eq. (12).

The calculation of  $p(\underline{Z}_k, \underline{Z}_k^c | c, \underline{Z}^{k-1})$  is substituted by the modified Euclidean distance of normalized **Fourier descriptors**:

$$\tilde{\eta}_k^c = \exp \left( -\frac{1}{\sigma} \sqrt{\sum_{\tilde{m}=1}^{n_D-1} \|\tilde{f}_{\tilde{m},0}^c - \tilde{f}_{\tilde{m},k}^c\|^2} \right), \quad (16)$$

$$\tilde{\mu}_k^c = \frac{\tilde{\eta}_k^c \tilde{\mu}_{k-1}^c}{\sum_{c=1}^{n_c} \tilde{\eta}_k^c \tilde{\mu}_{k-1}^c}. \quad (17)$$



# Method

## 3. Main steps of the JTC-RHM method

$$\begin{aligned} \mathbf{z}_{k,l} &= S_{k,l} \mathbf{r}_{\phi_{k,l}} \mathbf{x}_k^e \mathbf{e}_{\phi_{k,l}} + \mathbf{x}_k^p + \mathbf{v}_{k,l} \\ &= S_{k,l} \mathbf{r}_{\phi_{k,l}} \mathbf{x}_k^e \mathbf{e}_{\phi_{k,l}} + \mathbf{H} \mathbf{x}_k^k + \mathbf{v}_{k,l} \\ &= \mathbf{H} \mathbf{x}_k^k + \tilde{\mathbf{v}}_{k,l}, \end{aligned} \quad (18)$$

The target state is modeled as  $\underline{\mathbf{x}}_k \triangleq (\mathbf{x}_k^k, \mathbf{x}_k^e)$

The Bayesian estimator for target tracking in the proposed JTC-RHM method is

$$\begin{aligned} p(\underline{\mathbf{x}}_k | c, \underline{\mathcal{Z}}^k) \\ &= \frac{p(\mathcal{Z}_k, \mathcal{Z}_k^c | \underline{\mathbf{x}}_k, c, \underline{\mathcal{Z}}^{k-1}) p(\mathbf{x}_k^e | \mathbf{x}_k^k, c, \underline{\mathcal{Z}}^{k-1})}{p(\mathcal{Z}_k, \mathcal{Z}_k^c | \mathbf{x}_k^k, c, \underline{\mathcal{Z}}^{k-1})} \quad (21) \\ &= \frac{p(\mathcal{Z}_k | \mathbf{x}_k^k) p(\mathbf{x}_k^k | \mathcal{Z}^{k-1})}{p(\mathcal{Z}_k | \mathcal{Z}^{k-1})}, \end{aligned}$$

# Method

## 3. Main steps of the JTC-RHM method

The recursive process of the JTC-RHM method includes two steps: prediction and update.

### Prediction

$$\begin{aligned}
 p(\underline{\mathbf{x}}_{k-1} | c, \underline{\mathcal{Z}}^{k-1}) \\
 &= p(\mathbf{x}_{k-1}^e | \mathbf{x}_{k-1}^k, c, \underline{\mathcal{Z}}^{k-1}) p(\mathbf{x}_{k-1}^k | \mathcal{Z}^{k-1}) \quad (24) \\
 &= \mathcal{N}(\mathbf{x}_{k-1}^e; \mathbf{m}_{k-1}^{e,c}, \mathbf{P}_{k-1}^{e,c}) \mathcal{N}(\mathbf{x}_{k-1}^k; \mathbf{m}_{k-1}^k, \mathbf{P}_{k-1}^k),
 \end{aligned}$$



$$\begin{aligned}
 p(\underline{\mathbf{x}}_k | c, \underline{\mathcal{Z}}^{k-1}) \\
 &= \mathcal{N}(\mathbf{x}_k^e; \mathbf{m}_{k|k-1}^{e,c}, \mathbf{P}_{k|k-1}^{e,c}) \mathcal{N}(\mathbf{x}_k^k; \mathbf{m}_{k|k-1}^k, \mathbf{P}_{k|k-1}^k). \quad (25)
 \end{aligned}$$

$$\mathbf{m}_{k|k-1}^k = \mathbf{F}_k^k \mathbf{m}_{k-1}^k, \quad (26)$$

$$\tilde{\mathbf{P}}_{k|k-1}^k = \tilde{\mathbf{F}}_k^k \tilde{\mathbf{P}}_{k-1}^k (\tilde{\mathbf{F}}_k^k)^\top + \tilde{\mathbf{Q}}_k^k, \quad (27)$$

$$\mathbf{m}_{k|k-1}^{e,c} = \mathbf{m}_0^{e,c}, \quad \mathbf{P}_{k|k-1}^{e,c} = \mathbf{P}_0^{e,c}, \quad (28)$$

### Update

$$\begin{aligned}
 p(\underline{\mathbf{x}}_k | c, \underline{\mathcal{Z}}^{k-1}) \\
 &= \mathcal{N}(\mathbf{x}_k^e; \mathbf{m}_{k|k-1}^{e,c}, \mathbf{P}_{k|k-1}^{e,c}) \mathcal{N}(\mathbf{x}_k^k; \mathbf{m}_{k|k-1}^k, \mathbf{P}_{k|k-1}^k). \quad (25)
 \end{aligned}$$



$$p(\underline{\mathbf{x}}_k | c, \underline{\mathcal{Z}}^k) = \mathcal{N}(\mathbf{x}_k^e; \mathbf{m}_k^{e,c}, \mathbf{P}_k^{e,c}) \mathcal{N}(\mathbf{x}_k^k; \mathbf{m}_k^k, \mathbf{P}_k^k). \quad (29)$$

$$\mathbf{m}_k^k = \mathbf{m}_{k|k-1}^k + \mathbf{K}_{k|k-1}^k (\bar{\mathbf{z}}_k - \mathbf{H} \mathbf{m}_{k|k-1}^k), \quad (30)$$

$$\tilde{\mathbf{P}}_k^k = \tilde{\mathbf{P}}_{k|k-1}^k - \tilde{\mathbf{K}}_{k|k-1}^k \tilde{\mathbf{S}}_{k|k-1}^k (\tilde{\mathbf{K}}_{k|k-1}^k)^\top, \quad (31)$$

$$\text{UKF} \left\{ \begin{aligned} \mathbf{m}_k^{\text{pe},c} &= \mathbf{m}_{k|k-1}^{\text{pe},c} + \mathbf{K}_k (0 - \hat{\mathbf{z}}_{k|k-1}), \quad (42) \\ \mathbf{P}_k^{\text{pe},c} &= \mathbf{P}_{k|k-1}^{\text{pe},c} - \mathbf{K}_k \mathbf{P}_{y_k y_k} (\mathbf{K}_k^{\text{pe}})^\top, \quad (43) \end{aligned} \right.$$

# Method

## 4. The proposed JTC-RHM-Ber filter

For the proposed JTC-RHM-Ber filter, an extended target at time  $k-1$  is modeled as  $\mathcal{X}_{k-1} = \emptyset$  or  $\mathcal{X}_{k-1} = \{\xi_{k-1}\}$ . The existence probability is  $q_{k-1}$  and  $\xi_{k-1} \triangleq (\underline{\mathbf{x}}_{k-1}, c)$ .

$$\begin{aligned} \mathcal{S}_{k-1}(\xi) &= \mathcal{S}_{k-1}(\underline{\mathbf{x}} | c) p(c) \\ &= \sum_{j=1}^{J_{k-1}} \left[ w_{k-1}^{(j)} \mathcal{N}(\mathbf{x}^e; \mathbf{m}_{k-1}^{e,c,(j)}, \mathbf{P}_{k-1}^{e,c,(j)}) \right. \\ &\quad \left. \cdot \mathcal{N}(\mathbf{x}^k; \mathbf{m}_{k-1}^{k,(j)}, \mathbf{P}_{k-1}^{k,(j)}) \tilde{\mu}_{k-1}^{c,(j)} \right], \quad (49) \end{aligned}$$

Prediction



$$\begin{aligned} \mathcal{S}_{k|k-1}(\xi) &= \mathcal{S}_{k|k-1}(\xi | c) p(c) = \frac{p_B(1 - q_{k-1}) \mathcal{S}_{k,B}(\xi)}{q_{k|k-1}} \\ &+ \frac{p_S q_{k-1}}{q_{k|k-1}} \sum_{j=1}^{J_{k-1}} \left[ w_{k-1}^{(j)} \mathcal{N}(\mathbf{x}^e; \mathbf{m}_{k|k-1}^{e,c,(j)}, \mathbf{P}_{k|k-1}^{e,c,(j)}) \right. \\ &\quad \left. \cdot \mathcal{N}(\mathbf{x}^k; \mathbf{m}_{k|k-1}^{k,(j)}, \mathbf{P}_{k|k-1}^{k,(j)}) \tilde{\mu}_{k-1}^{c,(j)} \right], \end{aligned} \quad (51)$$

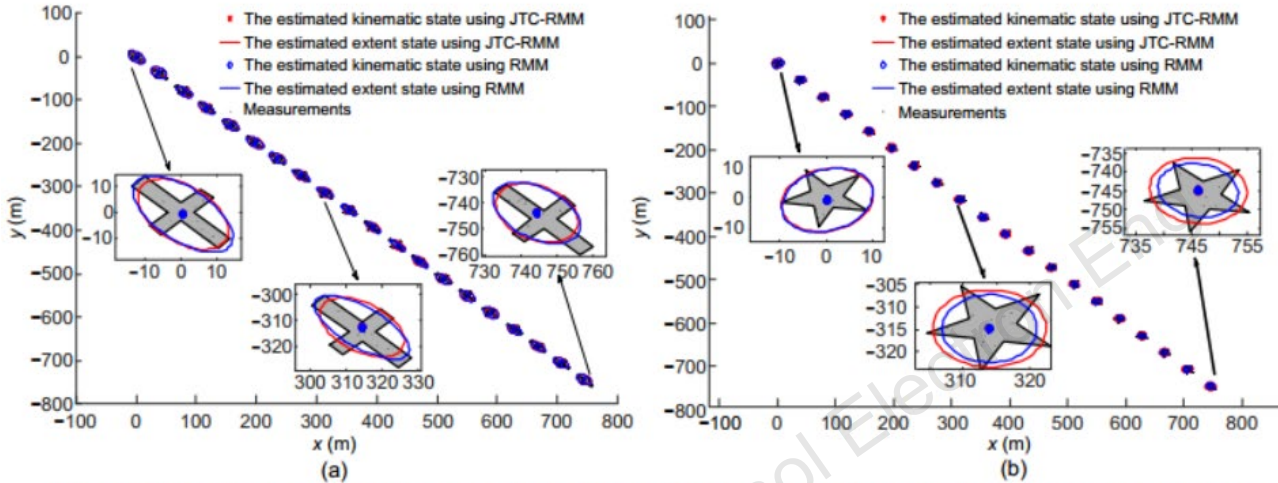
Update



$$q_{k|k-1} = p_B(1 - q_{k-1}) + p_S q_{k-1}, \quad (52)$$

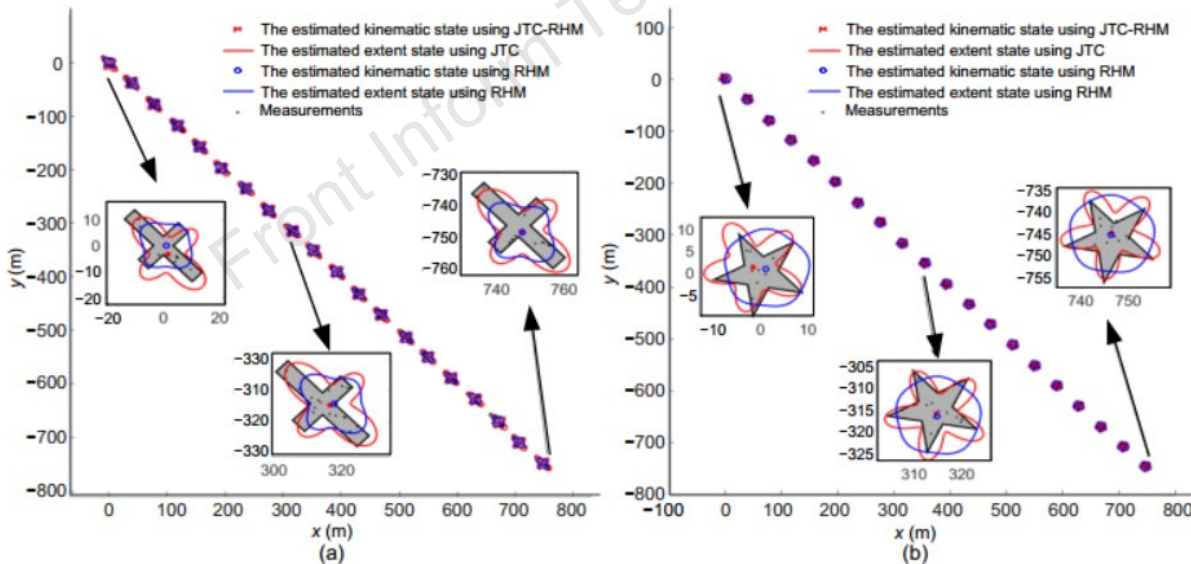
$$\begin{aligned} \mathcal{S}_{k|k-1}(\xi) &= \mathcal{S}_{k|k-1}(\xi | c) p(c) \\ &= \sum_{j=1}^{J_{k|k-1}} \left[ w_{k|k-1}^{(j)} \mathcal{N}(\mathbf{x}^e; \mathbf{m}_{k|k-1}^{e,c,(j)}, \mathbf{P}_{k|k-1}^{e,c,(j)}) \right. \\ &\quad \left. \cdot \mathcal{N}(\mathbf{x}^k; \mathbf{m}_{k|k-1}^{k,(j)}, \mathbf{P}_{k|k-1}^{k,(j)}) \tilde{\mu}_{k-1}^{c,(j)} \right], \end{aligned} \quad (65)$$

# Major results



**Fig. 4 Simulation results of target tracking using the JTC-RMM method for targets A (a) and B (b) in scenario 1**  
 JTC: joint tracking and classification; RMM: random matrix model. References to color refer to the online version of this figure

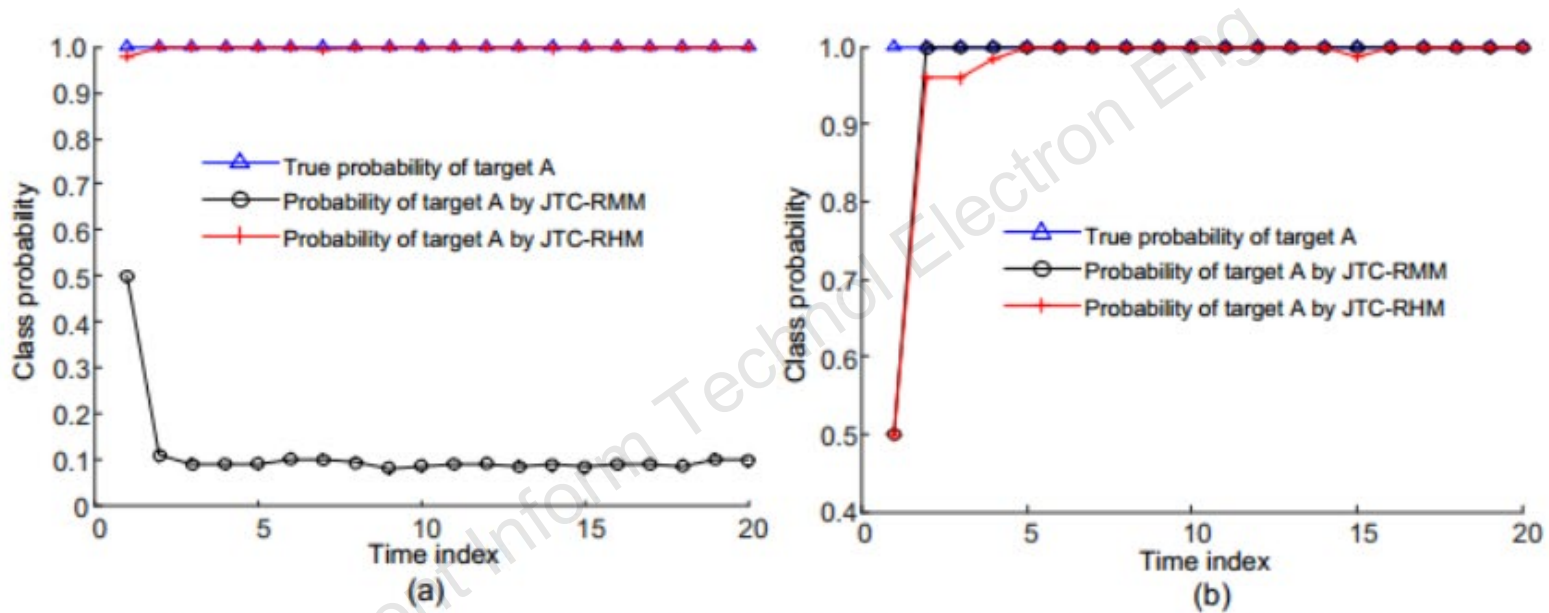
JTC-RMM  
 vs.  
 JTC-RHM  
 in tracking



**Fig. 5 Simulation results of target tracking using the proposed JTC-RHM method for targets A (a) and B (b) in scenario 1**  
 JTC: joint tracking and classification; RHM: random hypersurface model. References to color refer to the online version of this figure

# Major results

## JTC-RMM vs. JTC-RHM in classification

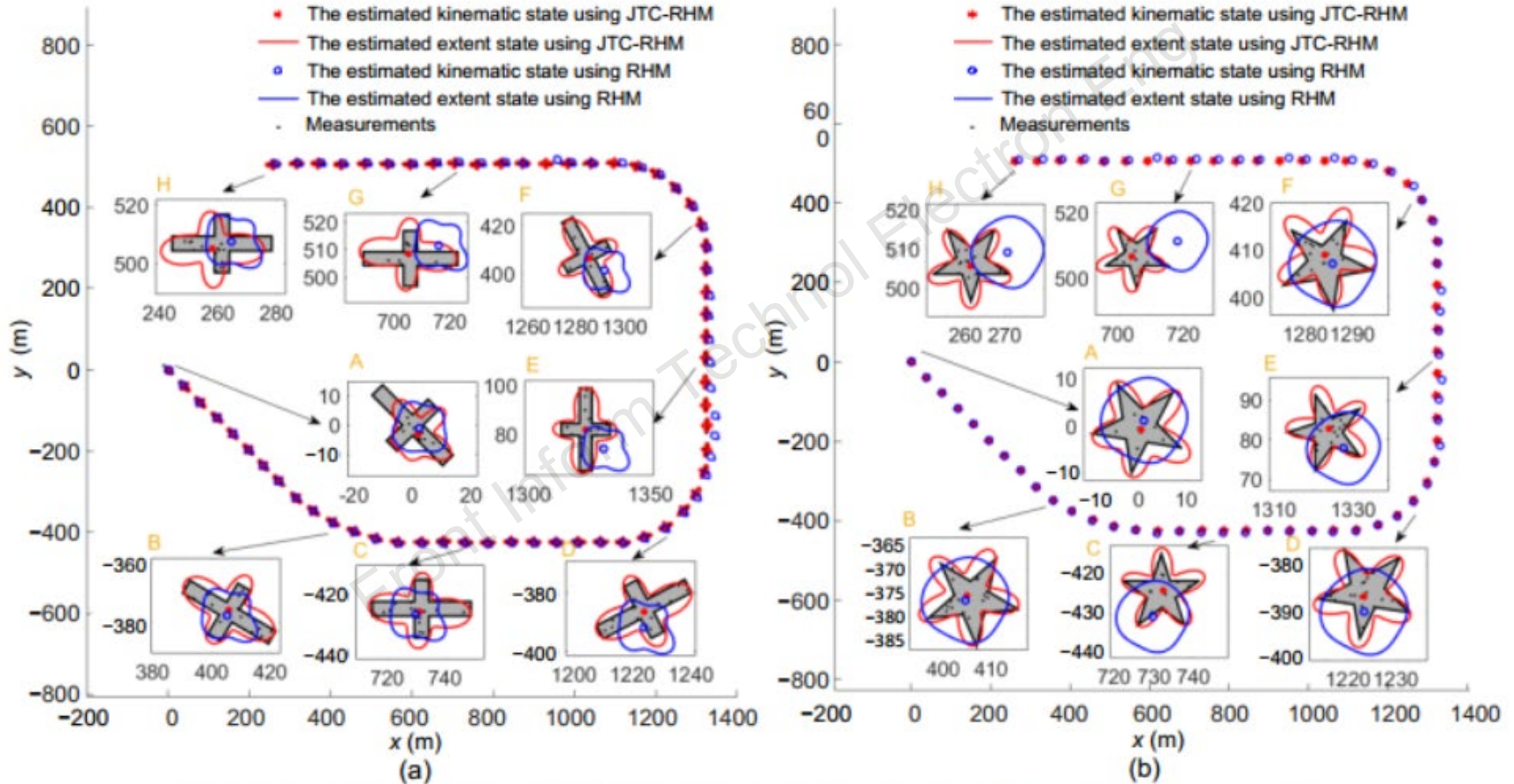


**Fig. 6 Simulation results of target classification for targets A (a) and B (b) in scenario 1**

JTC: joint tracking and classification; RMM: random matrix model; RHM: random hypersurface model. References to color refer to the online version of this figure

# Major results

## JTC-RHM vs. RHM

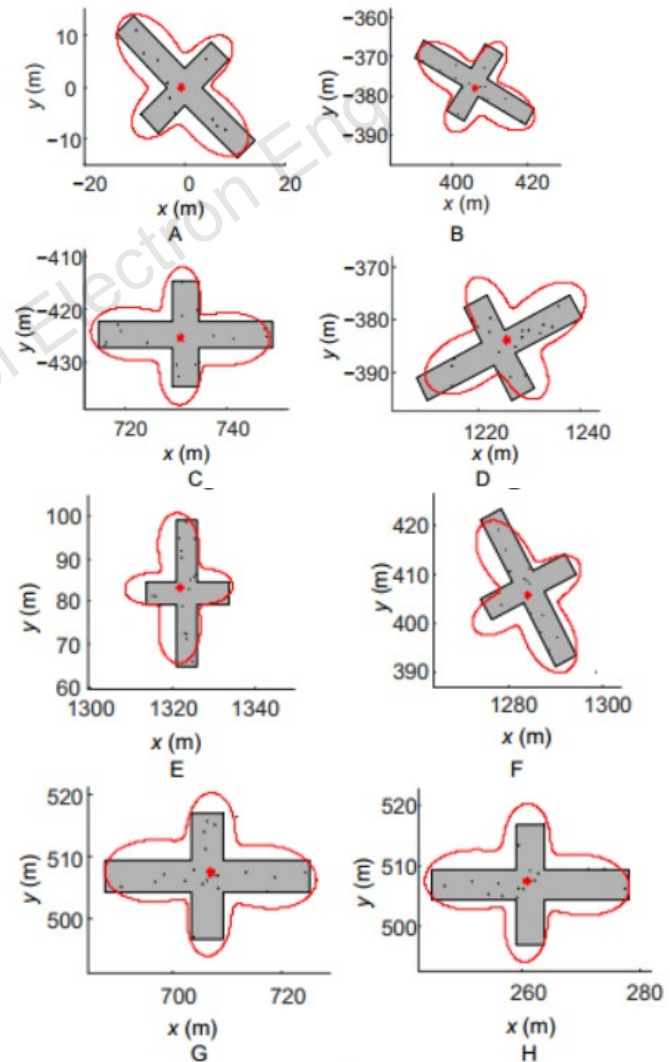
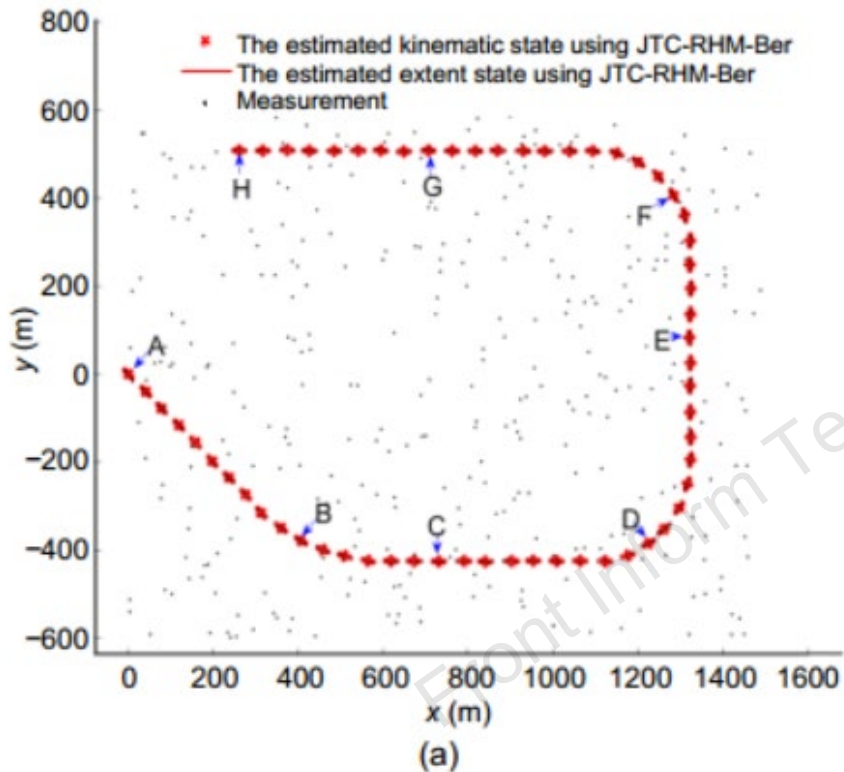


**Fig. 7 Simulation results of the JTC-RHM method for targets A (a) and B (b) in scenario 2**

JTC: joint tracking and classification; RHM: random hypersurface model. References to color refer to the online version of this figure

# Major results

## JTC-RHM-Ber in tracking



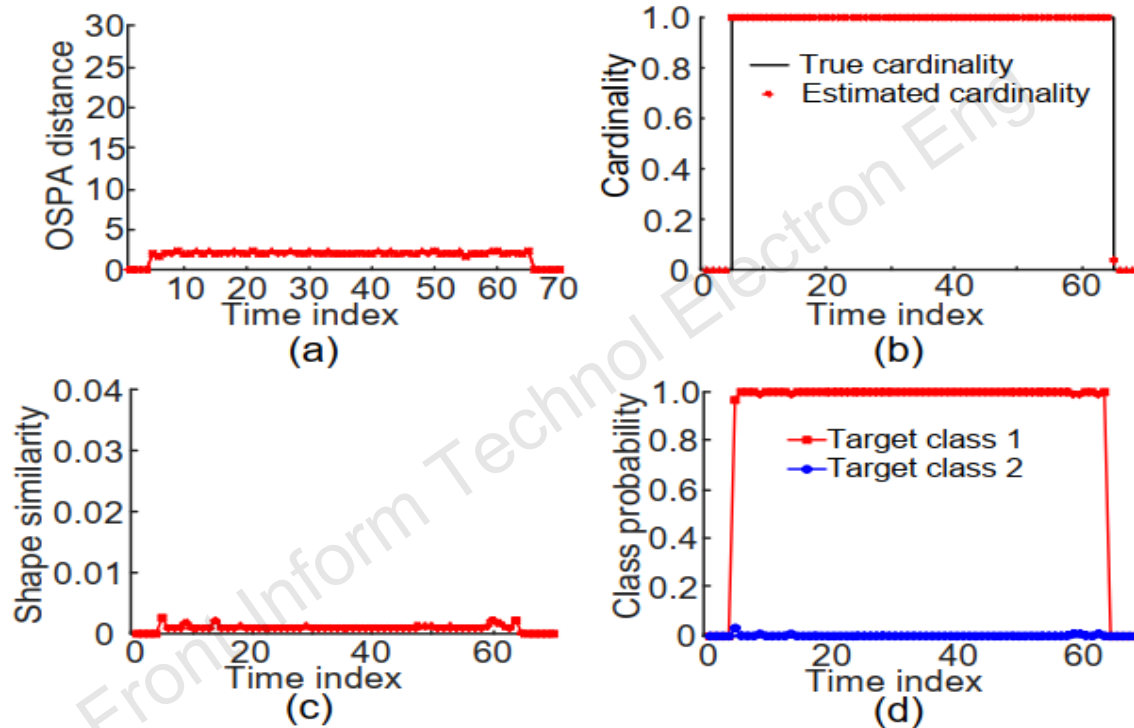
**Fig. 10** Simulation results of the JTC-RHM-Ber filter for target A in scenario 3: (a) total result for a single run; (b) partially enlarged subfigures of (a)

JTC: joint tracking and classification; RHM: random hyper-surface model; Ber: Bernoulli. References to color refer to the online version of this figure

(b)

# Major results

## JTC-RHM-Ber in classification



**Fig. 12 Simulation results of the JTC-RHM-Ber filter for target A in scenario 3 over 100 Monte Carlo trials: (a) OSPA distance; (b) cardinality estimation; (c) shape similarity; (d) class probability**

JTC: joint tracking and classification; RHM: random hyper-surface model; Ber: Bernoulli; OSPA: optimal subpattern assignment. References to color refer to the online version of this figure

# Conclusions

- The proposed JTC-RHM method can classify the targets with complex shapes and similar sizes more correctly, compared with the JTC method based on the random matrix model.
- The proposed method performs better in target state estimation than the star-convex RHM based extended target tracking method.
- The proposed JTC-RHM-Ber filter has promising performance in state detection and estimation, and can achieve correct target classification.