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# Convergence of time-varying networks and its applications

**Key words:** Time-varying networks; Unknown control directions; Nussbaum-type function; Cut-balance condition

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# Motivation

1. The cooperative control of undirected/directed networks has received much attention in the last several decades, and the design of distributed control algorithms is a fundamental task because of their considerable applications.
2. Consensus seeking is one of the fundamental distributed control algorithms in multiagent systems (MASs) such that the agents can cooperate to achieve some common limits (values).
3. The design of distributed control algorithms for consensus of MASs with unknown control directions (UCDs) has attracted a lot of research interest since some practical systems do not possess the prior knowledge of control directions.

# Main idea

1. We apply the convergence property to cooperative control of nonlinear MASs with UCDs, and illustrate a new kind of Nussbaum-type function based control algorithms.
2. It is proven that if the time-varying networks are cut-balance, the convergence of nonlinear MASs with nonidentical UCDs is achieved using the presented algorithms.
3. A critical feature of this application is that the designed algorithms can deal with nonidentical UCDs by employing conventional Nussbaum-type functions.

# Method

1. The convergence property is derived for time-varying networks, and results are generalized to time-varying networks with disturbances.
2. We apply the convergence property to the cooperative control of nonlinear MASs with nonidentical UCDs under time-varying networks satisfying the cut-balance condition. The communication network in this study is the mild condition for nonlinear MASs with nonidentical UCDs. Furthermore, one critical feature is that conventional Nussbaum-type functions are sufficient to deal with nonidentical UCDs.

# Major results

**Theorem 1** Let Assumption 1 hold, and assume that  $\mathcal{G}(t)$  is the unbounded interactions graph. The time-varying network (1) can achieve the convergence objective (2). Furthermore, we have: (1)  $\xi_i(t)$  ( $i = 1, 2, \dots, N$ ) is bounded, and for every  $k$  and  $i$ ,  $\int_0^\infty a_{ik}(\tau) |\xi_k(\tau) - \xi_i(\tau)| d\tau < \infty$ ; (2) If  $i$  and  $k$  are in the same connected component,  $\lim_{t \rightarrow \infty} \xi_i(t) = \xi_i^* = \lim_{t \rightarrow \infty} \xi_k(t) = \xi_k^*$ .

# Major results (Case 1)

Case 1:

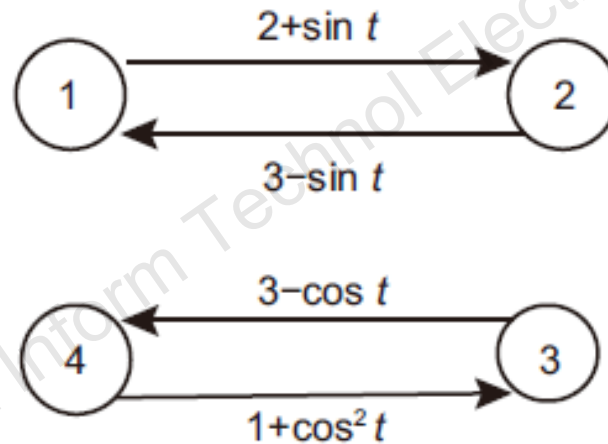


Fig. 1 Graph  $\mathcal{G}(t)$  satisfying the cut-balance condition for case 1

# Major results (Case 1)

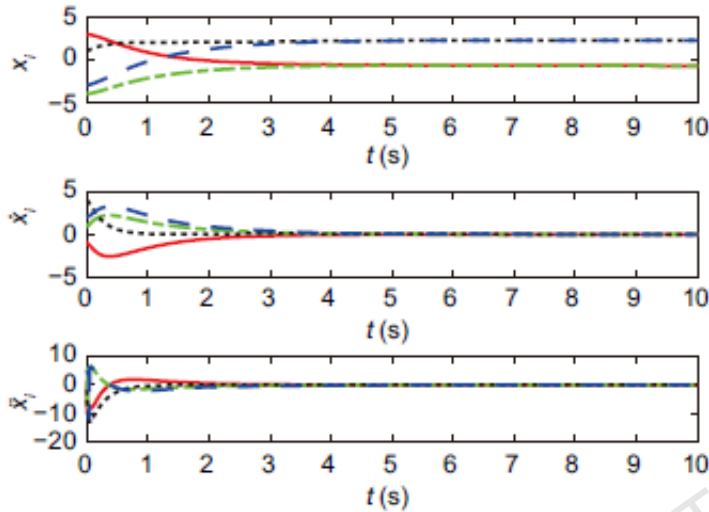


Fig. 3 State trajectories of agents under time-varying networks for case 1

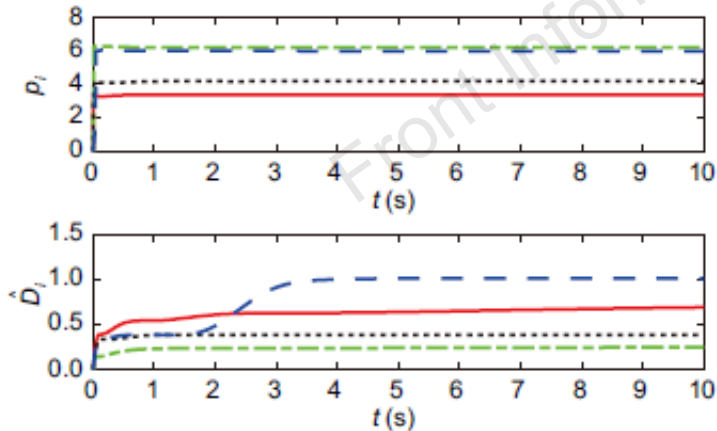


Fig. 4  $p_i(t)$  and  $\hat{D}_i(t)$  of agents under time-varying networks for case 1

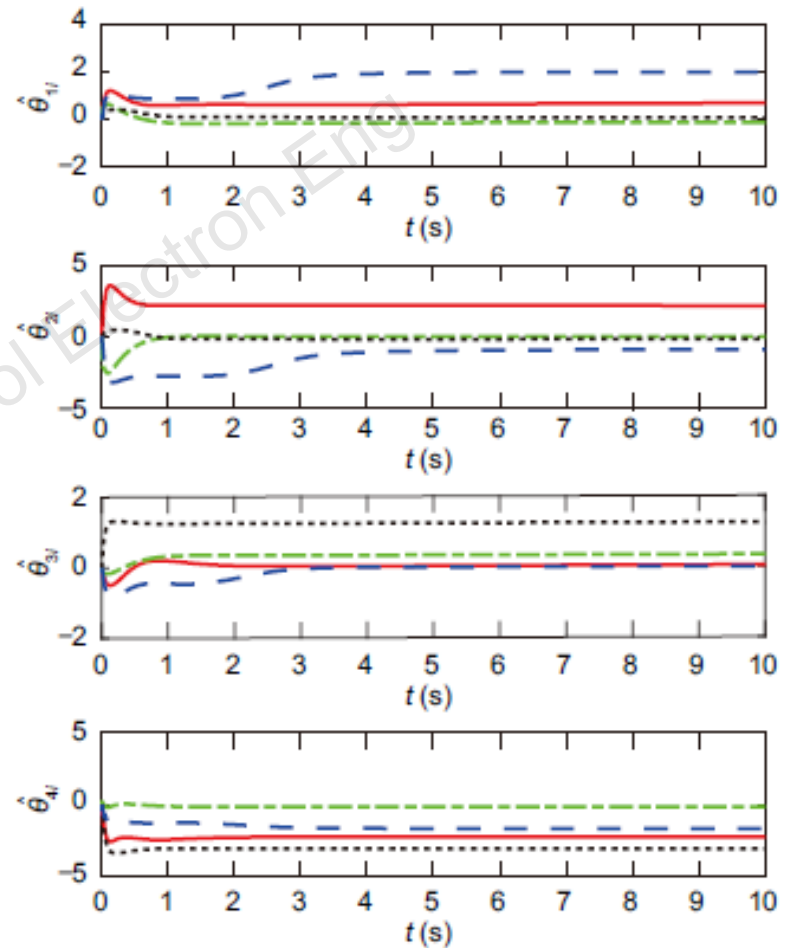


Fig. 5 Estimated  $\hat{\theta}_i(t) = [\hat{\theta}_{1i}(t), \hat{\theta}_{2i}(t), \hat{\theta}_{3i}(t), \hat{\theta}_{4i}(t)]^T$  of agents under time-varying networks for case 1

# Major results (Case 2)

Case 2:

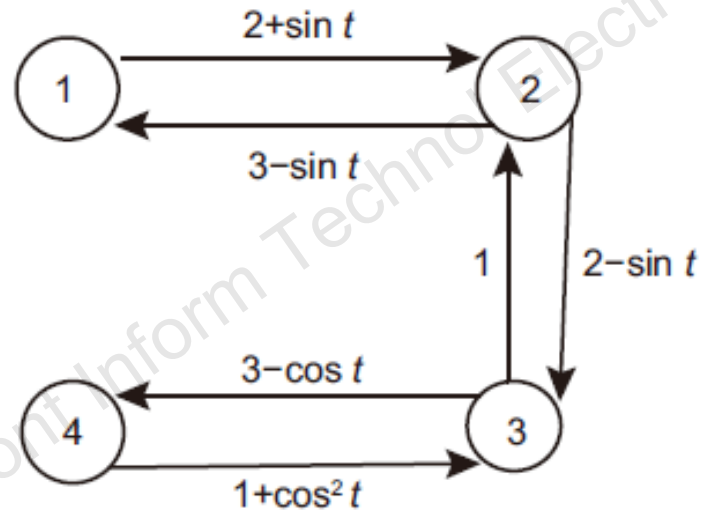


Fig. 2 Graph  $\mathcal{G}(t)$  satisfying the cut-balance condition for case 2

# Major results (Case 2)

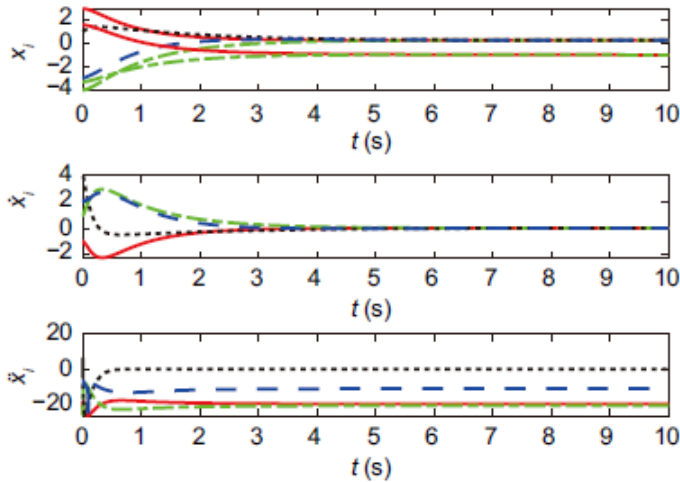


Fig. 6 State trajectories of agents under time-varying networks for case 2

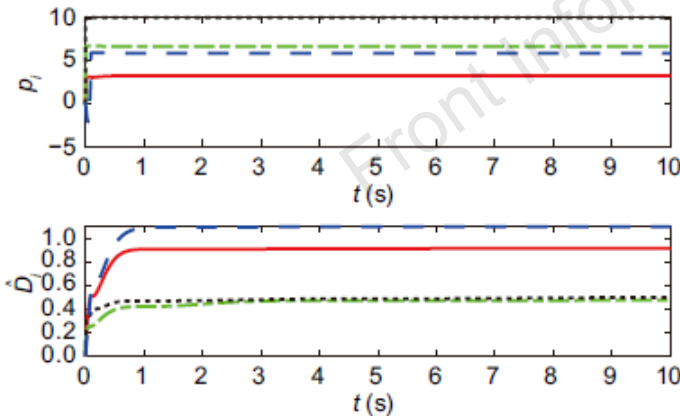


Fig. 7  $p_i(t)$  and  $\hat{D}_i(t)$  of agents under time-varying networks for case 2

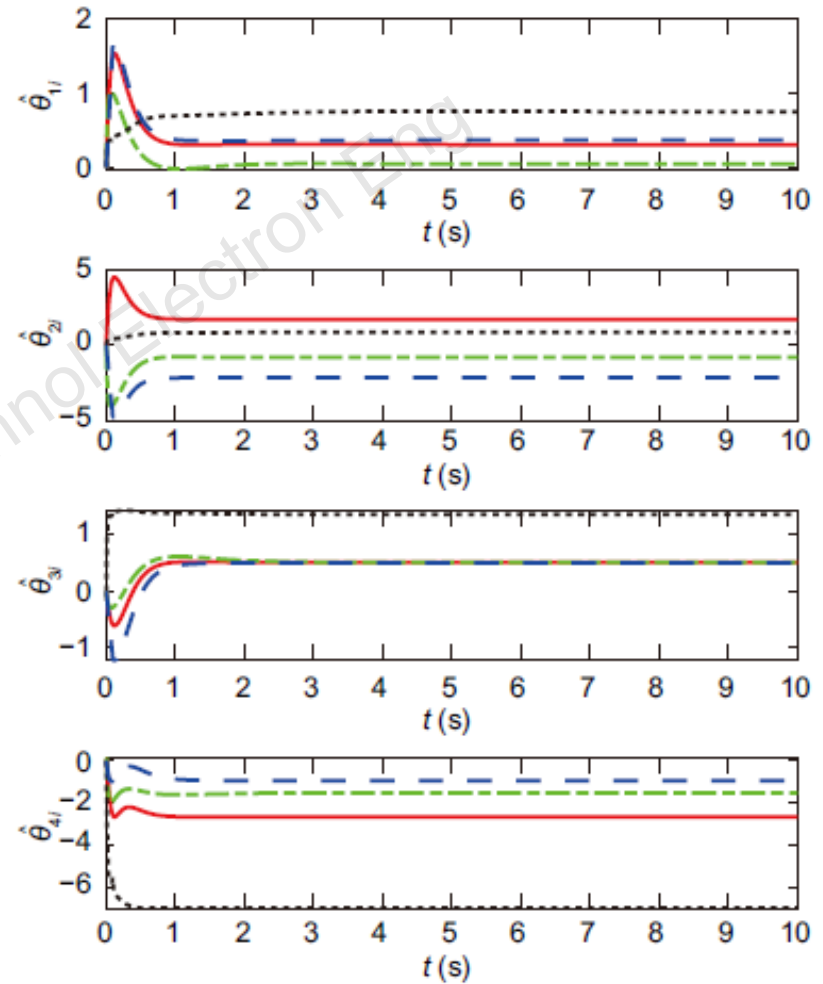
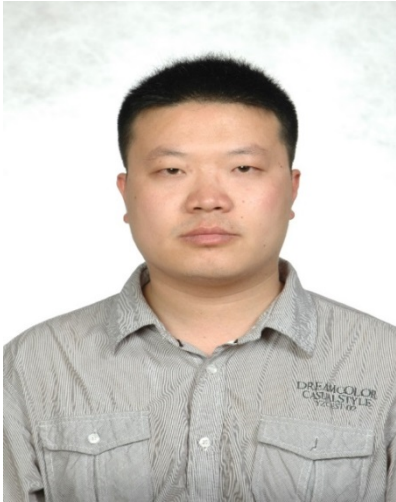


Fig. 8 Estimated  $\hat{\theta}_i(t) = [\hat{\theta}_{1i}(t), \hat{\theta}_{2i}(t), \hat{\theta}_{3i}(t), \hat{\theta}_{4i}(t)]^T$  of agents under time-varying networks for case 2

# Conclusions

1. We addressed the convergence property of time-varying networks, and then investigated the cooperative control of nonlinear MASs with UCDs under time-varying networks, where the control algorithms with classical Nussbaum-type functions were presented.
2. It was shown that if the time-varying networks are cut-balance, the convergence of nonlinear MASs can be achieved with the proposed algorithms of nonlinear MASs subject to non-identical UCDs.
3. Further research may focus on robust convergence of more general time-varying networks and cooperative control of nonlinear agents with complex dynamics.



Qingling WANG is an associate professor with School of Automation, Southeast University, Nanjing, China. He received his PhD degree in control science and engineering from Harbin Institute of Technology, China, in 2014. He was a visiting student with the Australia National University, Canberra, ACT, Australia, from 2012 to 2014, and was a visiting scholar at the Technical University of Berlin, Germany, in 2016. His current research interests include constrained control, adaptive control, and cooperative control of multi-agent systems.