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Robust distributed model predictive consensus of discrete-time multi-agent systems: a self-triggered approach

Key words: Consensus; Self-triggered control; Distributed model predictive control

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Motivation

1. The consensus of multi-agent system (MAS) using the distributed model predictive control (DMPC) scheme is a hot research area. However, no matter whether it is for linear or nonlinear systems, few studies considered the uncertainties in the practical environment.

2. Self-triggered mechanism can efficiently reduce the communication load and avoid the disadvantage of the high-frequency sampling problem of the event-triggered mechanism. However, few studies combined the self-triggered mechanism and DMPC algorithm to solve the consensus problem of nonlinear uncertain MAS.

Main idea

1. Considering bounded additive disturbances, for each discrete-time nonlinear agent, we use a min–max robust DMPC, which explicitly includes uncertainty realizations as optimized decision variables in the entire optimization control problem.

2. A self-triggered strategy via optimization is introduced to relieve the heavy communication burden.

3. Sufficient conditions are presented to guarantee the feasibility of the optimization algorithm and the consensus over the considered MAS. Moreover, we provide a specific form of compatibility constraints and consensus error terminal regions.

Method

1.In order to make the considered MAS tend to be consensus, the key is to design a distributed control protocol based on network topology and system dynamics. For each agent $x_i(t+1) = f\big(x_i(t), u_i(t), w_i(t)\big), i \in \{1,2,\ldots,M\}$, a local minimum–maximum optimization problem at the triggering instant can be defined as

$$V_i^{H_k^i}(e_i(t_k^i),T) \triangleq \min_{\boldsymbol{u_i}(t_k^i)} \max_{\boldsymbol{w_i}(t_k^i)} J_i^{H_k^i}(x_i(t_k^i), \hat{\boldsymbol{x}}_{\boldsymbol{-i}}(t_k^i), \boldsymbol{u_i}(t_k^i), \boldsymbol{w_i}(t_k^i), T)$$

subject to

$$u_i(t_k^i + l \mid t_k^i) \in U_i, w_i(t_k^i + l \mid t_k^i) \in W_i,$$

$$x_i(t_k^i + l + 1 \mid t_k^i) = f(x_i(t_k^i + l \mid t_k^i), u_i(t_k^i + l \mid t_k^i), w_i(t_k^i + l \mid t_k^i)),$$

$$\left\| x_i(t_k^i + l \mid t_k^i) - \hat{x}_i(t_k^i + l \mid t_k^i) \right\| \le \frac{a}{(T - H_k^i)} \min_{j \in N_i} \left\| x_j(\Gamma_j(t_k^i)) - \hat{x}_{-j}(\Gamma_j(t_k^i) \mid \Gamma_j(t_k^i)) \right\|,$$

$$x_i(t_k^i + T \mid t_k^i) - \hat{x}_{-i}(t_k^i + T \mid t_k^i) \in E_i^f$$

Method (Cont'd)

2. The self-triggered mechanism is used to effectively ease the communication load, which the optimal triggering interval $(H_k^i)^*$ can be obtained by checking whether the optimal cost is dropping and by choosing a satisfied maximum triggering interval. The triggering instant is defined as follows:

$$t_{k+1}^i = t_k^i + (H_k^i)^*$$

$$(H_k^i)^* \triangleq \max\{H_k^i \mid H_k^i \in \mathbb{N}_{[1,H_{\max}]}, V_i^{H_k^i}(e_i(t_k^i),T) \le V_i^1(e_i(t_k^i),T)\}$$

Method (Cont'd)

3. To guarantee the feasibility of the optimization algorithm and the consensus over the considered MAS, we employed the invariant set theory to implement the input-to-state practical stability (ISpS) framework with respect to the consensus error.

Definition 1 (Robust positively invariant (RPI) set -- For the established system model, a set E_i is called an "RPI set" if for all $e_i(t) \in E_i$, $e_i(t+l) \in E_i$ exists for all $w_i \in W_i$.

Definition 2 (Regional ISpS) -- If there exists an RPI set E_i including the origin, a KL function β_i, a K function α_i, and a constant d_i satisfying

$$\|e_i(t)\| \leq \beta_i(e_i(0), t) + \alpha_i\left(\sup_{\tau \in [0,t-1]} \|w_i(\tau)\|\right) + d_i,$$

where each $e_i(0) \in E_i$ and $w_i(t) \in W_i$, then the state consensus error dynamics of the system is said to be ISpS in E_i with respect to w_i.

Method (Cont'd)

4. The self-triggered robust DMPC consensus algorithm for the whole system is summarized in Algorithm 1.

Algorithm 1 Self-triggered robust DMPC consensus

Off-line:

Require: $\lambda, \psi, \mu, \gamma, \alpha, \beta_i, k_i, T, H_{\max}$

On-line:

for each agent $i \in \{1, 2, \ldots, M\}$ **do**
  set $t_0^i = 0$ as the first triggering instant with k=0;
  transmit its state sequence $\hat{x}_i(\tau \mid 0)$ to its neighbors and receive $\hat{x}_j(\tau \mid 0)$ from every number $j \in N_i$, $\tau \in [0, T)$;
  solve problem SP_i to obtain $(H_k^i)^*$ and $u_i^*(0)$;
end for // Initialization
while each $e_i(t_k^i + l \mid t_k^i) \notin E_i^f$ **do**
  while t_k^i $(k \geq 1)$ is not triggered **do**
    apply $u_i^{\mathrm{ST}}(t_{k-1}^i)$ at $t \in [t_{k-1}^i, t_{k-1}^i + T)$;
    obtain $\hat{x}_i(\tau \mid t_k^i)$ $(\tau \in [t_k^i, t_{k+1}^i])$ based on Eqs. (3) and (4);
    store $u_i^*(t_k^i \mid t_{k-1}^i)$ and $x_i^*(t_k^i \mid t_{k-1}^i)$;
  end while
  measure the current state $x(t_k^i)$;
  solve SP_i in problem (5) and DPMC mechanism (8) to obtain $u_i^*(\tau \mid t_k^i)$ and $H_i^*(t_k^i)$ $\left(\tau \in [t_k^i, t_k^i + T)\right)$;
  set k=k+1;
end while
apply the auxiliary feedback control law $\bar{u}_i = k_i(x_i, \hat{x}_{-i})$ to the corresponding subsystems;

Major results

Comparison of two configurations of γ

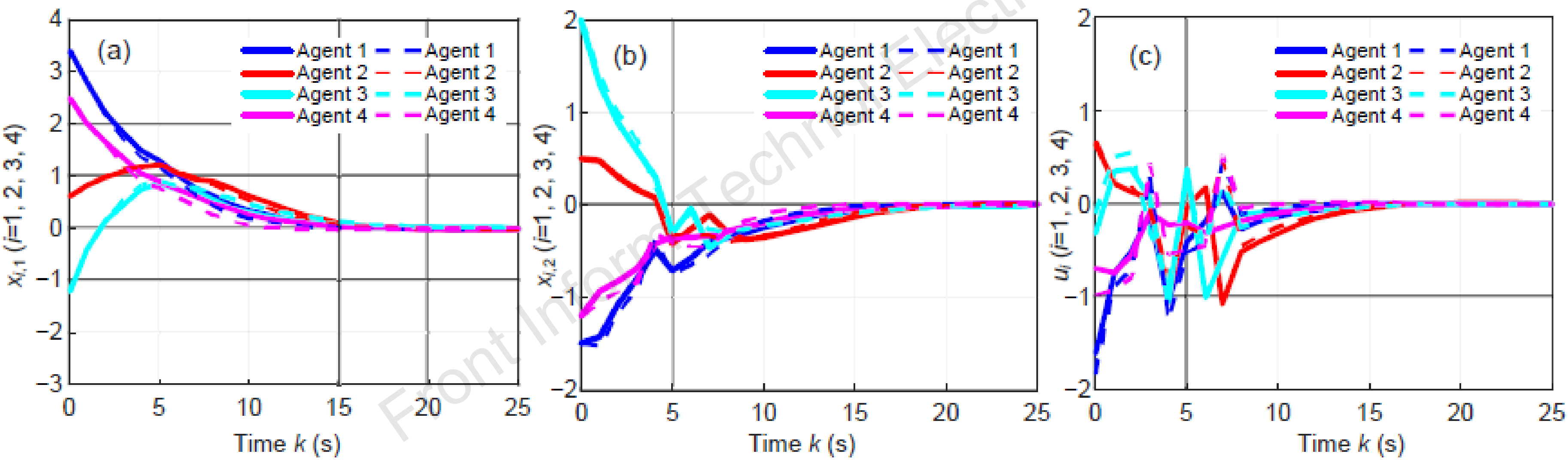

Fig. 1 Trajectories of system states $x_{i,1}$ (a), $x_{i,2}$ (b), and u_i (c) with γ=0.85 (solid lines) and γ=0.5 (dashed lines)

Major results (Cont'd)

Triggering instants of each agent

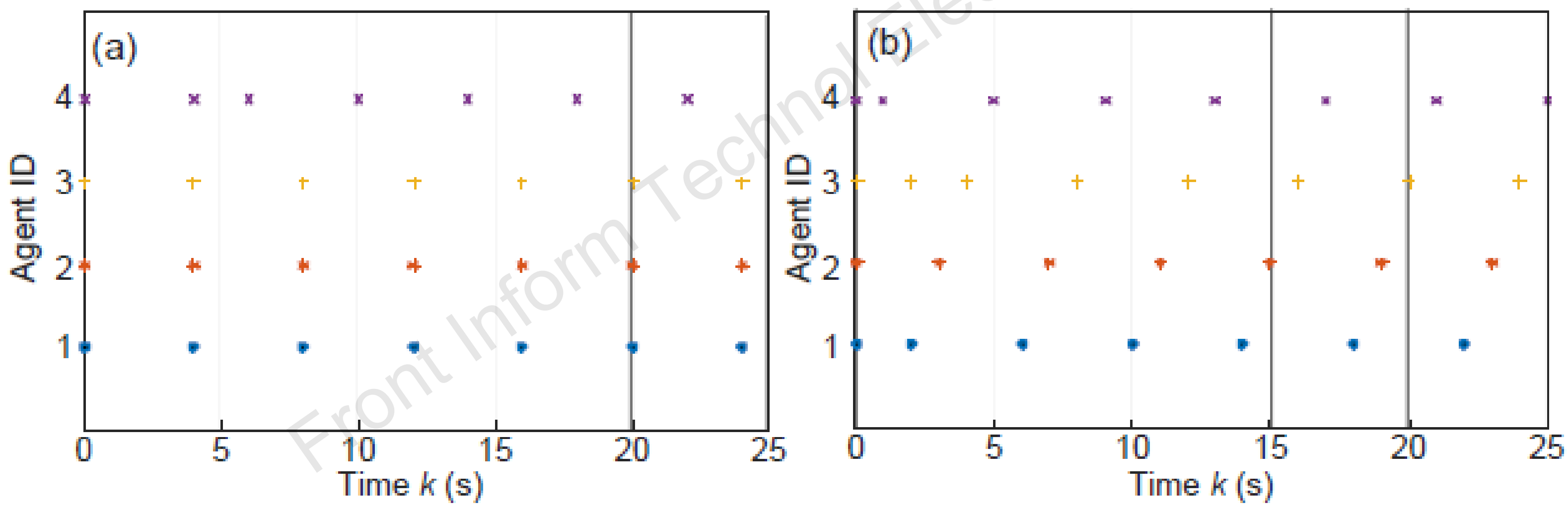

Fig. 2 Triggering instants with γ=0.5 (a) and γ=0.85 (b)

Major results (Cont'd)

Comparison of self-triggered and time-driven control

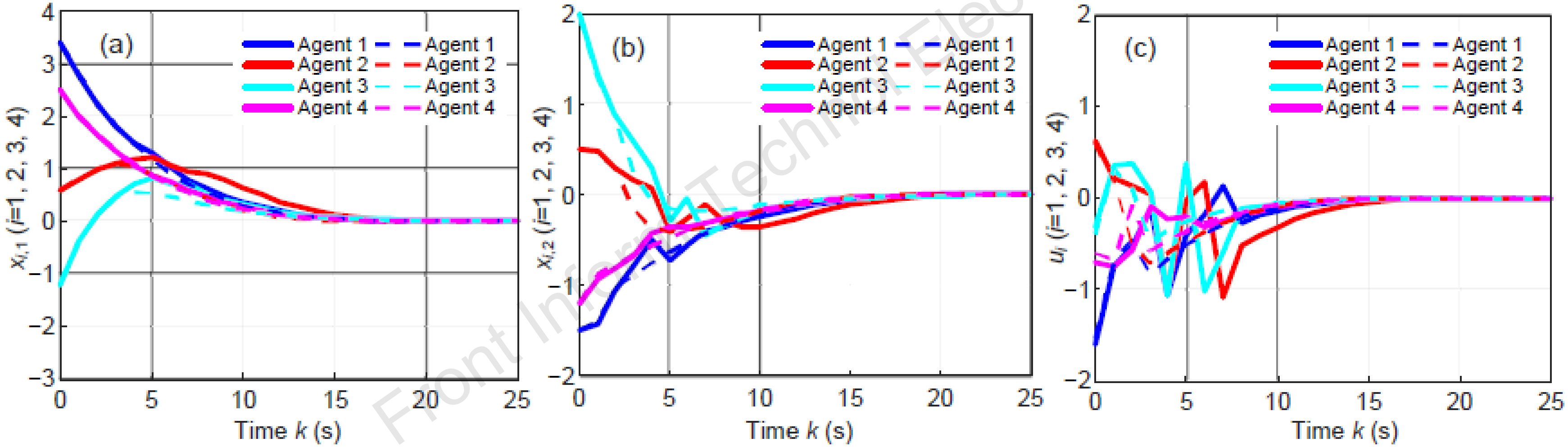

Fig. 3 State trajectories $x_{i,1}$ (a), $x_{i,2}$ (b), and u_i (c) using self-triggered (solid lines) and time-driven (dashed lines) control

Conclusions

1. A robust DMPC method has been used to study the consensus problem of discrete nonlinear MAS with additive disturbances.

2. A self-triggered control scheduler based on a min–max optimization problem to determine the control inputs and maximize the triggering interval was proposed, which can significantly reduce communication costs.

3. The conditions that guarantee the feasibility of the algorithm and the consensus over the perturbed nonlinear MAS are sufficient and practicable. Simulation examples have proved the effectiveness of Algorithm 1.