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# Constant-gain nonlinear adaptive observers revisited: an application to chemostat systems

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**Key words:** Nonlinear observers; Adaptive observers; Coordinate change; Chemostat; Pollutants observation

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# Motivation

WHY CONSTANT-GAIN ADAPTIVE OBSERVERS?

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- *It provides the* ***most affordable adaptive observer solutions*** *in terms of the additional required observer dynamics.*
- *It represents a* ***theoretical challenge****. Existing solutions are not feasible, persistent excitation or the strictly positive realness of some signals is not a trivial requirement.*
- *It provide* ***easily implementing observers*** *solutions with just one gain parameter to be tuned.*

# Main idea

Nonlinear affine systems

$$\dot{x} = f(x) + g(x)(u + \delta)$$
$$y = h(x) = x_1$$

Nonlinear systems, but
*p*-constant input-vector

$$\dot{x} = f(x) + p(u + \delta)$$
$$y = cx$$

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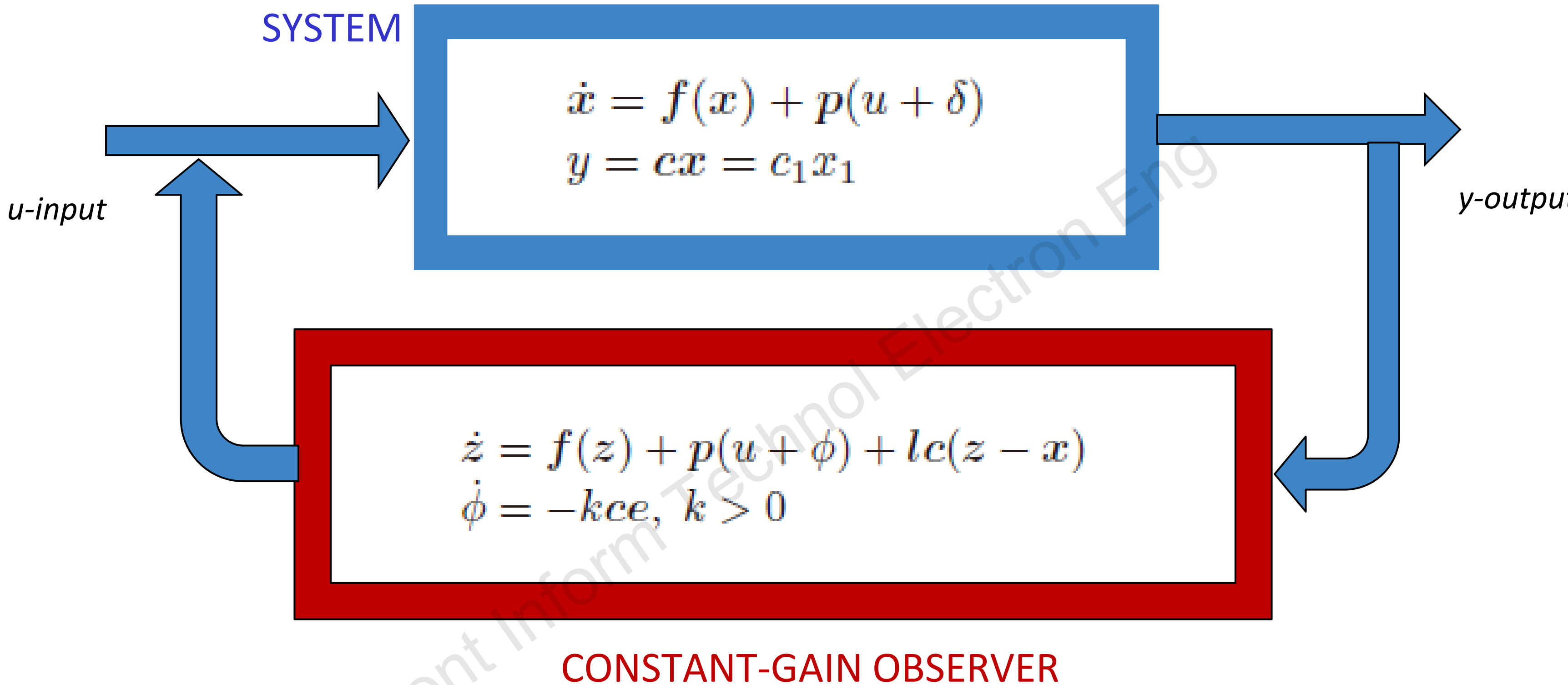
# Theoretical findings

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Provided the LMI

$$\begin{cases} A^{\mathrm{T}}P + PA + c^{\mathrm{T}}Y^{\mathrm{T}} + Yc < -2Pk_{\mathrm{L}} - Q < 0 \\ Pp = c^{\mathrm{T}} \end{cases}$$

# Main practical motivation

## Are the observers truly useful?

CHEMOSTAT

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- Complex bio-process widely used (food, chemical, ..., industries)
- Monitoring and control ➡ ROBUST OBSERVERS

# A theoretical useful transformation

Standard **chemostat** systems:

$$\dot{\xi} = f(\xi) + g(\xi)(u + \delta)$$
$$y = h(\xi) = \xi_1$$

It can be transformed

$$x_i = \alpha(\xi_i(t))$$

$$\dot{x} = f(x) + p(u + \delta)$$
$$y = cx$$

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# Main results

Numerical simulation: wastewater treatment system

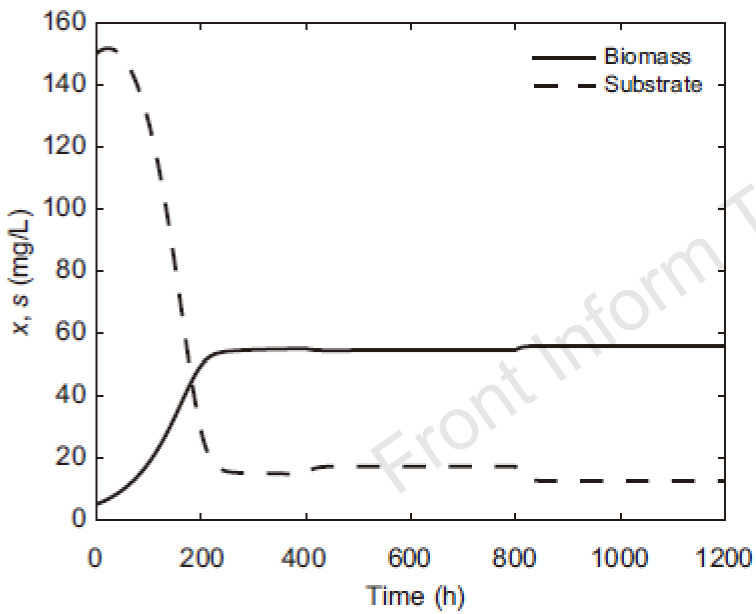


**Fig. 1** Bioreactor state responses for input/ disturbance stimulus

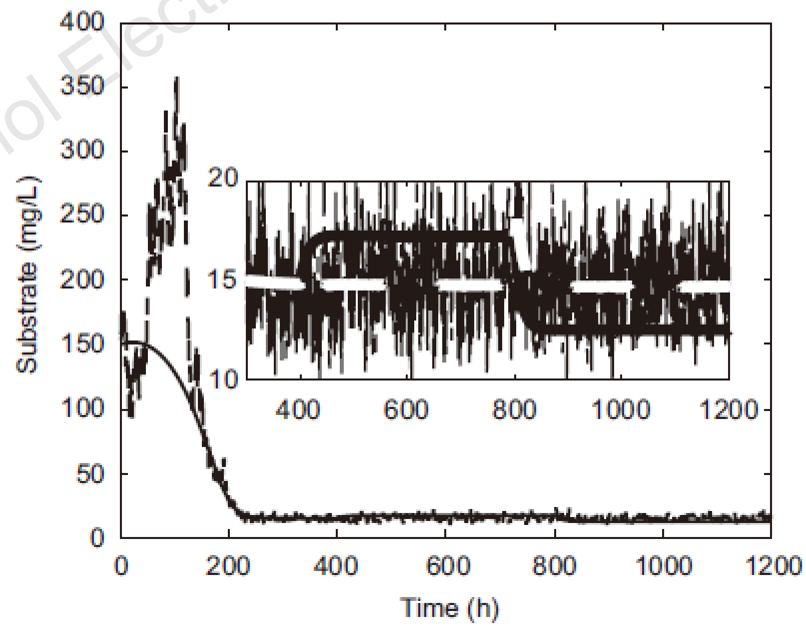


**Fig. 3** High-gain substrate estimation. The full line is the real state, the black dashed line is the high-gain state with output noise, and the white dashed line is the high-gain state without output noise

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# Main experimental results

## Wastewater treatment system

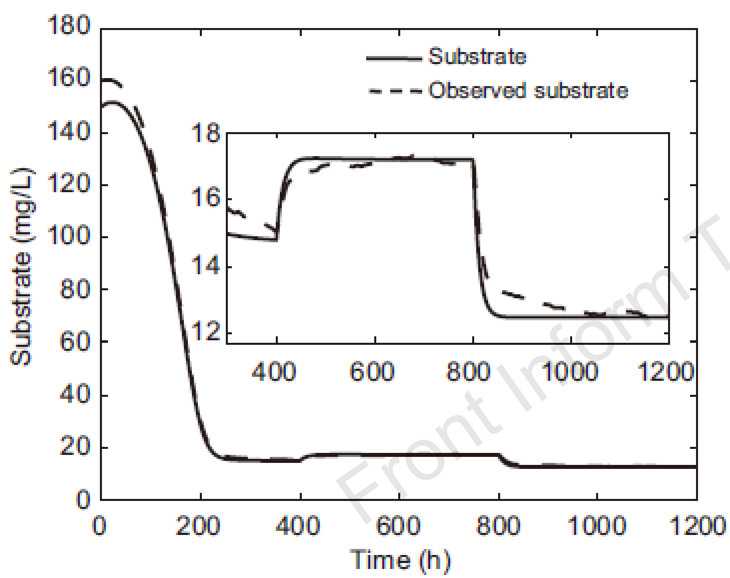


Fig. 2 Dynamics of the observed substrate by the constant-gain adaptive observer

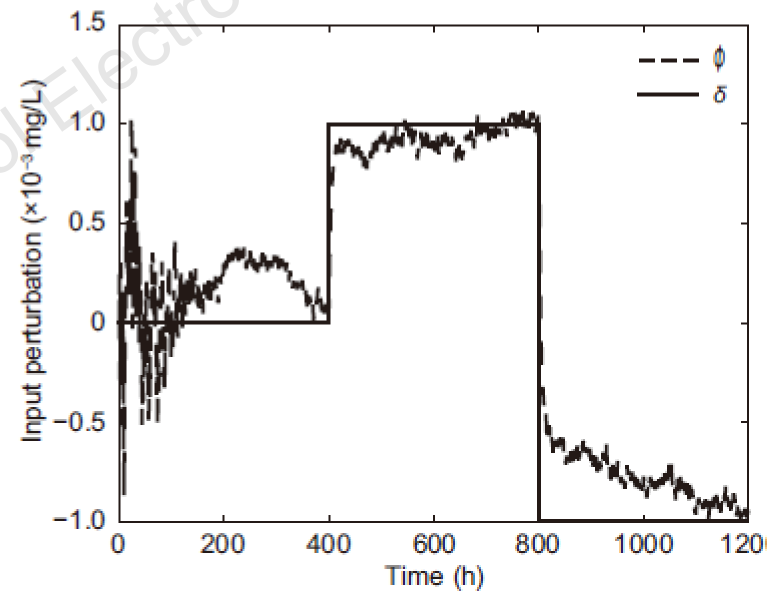


Fig. 4 Estimation of the input perturbation $\delta$ by the constant-gain adaptive observer

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# Main results (Cont'd)

Numerical simulation: wastewater treatment system

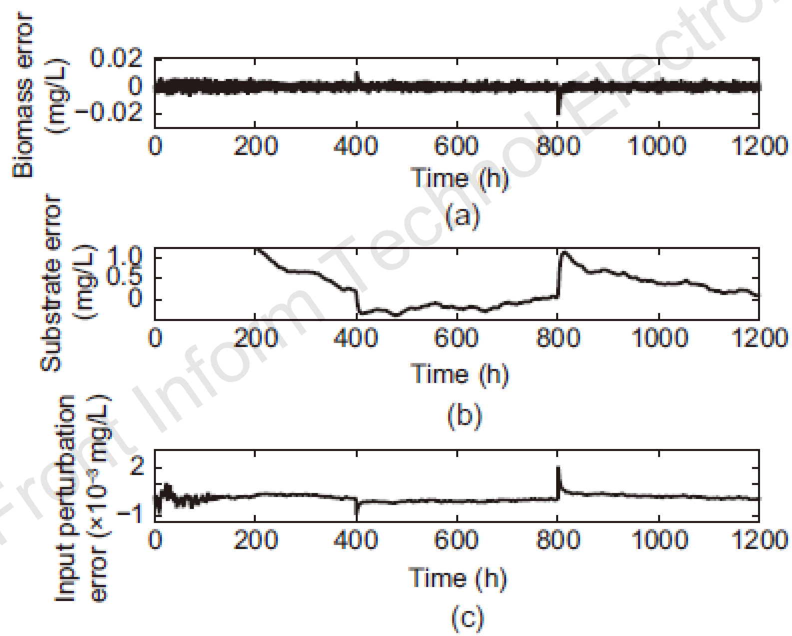


**Fig. 5 Estimation errors by the constant-gain adaptive observer: (a) biomass error; (b) substrate error; (c) input perturbation error**

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# Final remarks

- A feasible solution of the constant-gain nonlinear adaptive observer for a class of nonlinear systems was given.
- The chemostat process was considered for the proof of concept. A diffeomorphic state space transformation was proposed.
- Observer performances were evaluated numerically on a wastewater removal nonlinear system model.
- The easy design observer is a good option for real-time applications.
- Future work points out to general nonlinear affine systems.

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# IT WORTHS CONSTANT GAIN ADAPTATION !

$$\dot{x} = f(x) + g(x)(u + \delta),$$
$$y = h(x) = x_1,$$

## DYNAMIC GAIN

**OBSERVER**

$$\dot{\hat{x}}(t) = A\hat{x} + \bar{g}(u,\hat{x}) + \bar{\Psi}(u,\hat{x})\hat{\rho}(t) - \theta\Delta_\theta^{-1}\left(S^{-1} + \Upsilon(t)P(t)\Upsilon^{\mathrm{T}}(t)\right)C^{\mathrm{T}}K(C\tilde{x})$$

**ADAPTATION LAW**

$$\dot{\hat{\rho}}(t) = -\theta P(t)\Upsilon^{\mathrm{T}}(t)C^{\mathrm{T}}K(C\tilde{x})$$
$$\dot{\Upsilon}(t) = \theta(A - S^{-1}C^{\mathrm{T}}C)\Upsilon(t) + \Delta_\theta\bar{\Psi}(u(t),\hat{x}(t)) \quad \text{with } \Upsilon(0) = 0$$
$$\dot{P}(t) = -\theta P(t)\Upsilon^{\mathrm{T}}(t)C^{\mathrm{T}}C\Upsilon(t)P(t) + \theta P(t) \quad \text{with } P(0) = P^{\mathrm{T}}(0) > 0$$

## CONSTANT GAIN

**OBSERVER**

$$\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}) + \boldsymbol{p}(u + \phi) + \boldsymbol{lc}(\boldsymbol{z} - \boldsymbol{x})$$

**ADAPTATION LAW**

$$\dot{\phi} = -k\boldsymbol{ce}, \quad k > 0$$

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Jorge A. TORRES was born in Mexico City, on May 13, 1960. He received his PhD degree in automatic control from LAG, INPG, France, in 1990. He joined the Department of Electrical Engineering at the CINVESTAV, Mexico, in 1990. His research interest lies in the structural approach of linear systems, stability of multivariate polynomials, and the control of bioprocess for waste water treatment and the control of mini-submarines.

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