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# Stability analysis of uncertain fractional-order neutral-type delay systems with actuator saturation

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**Key words:** Fractional-order system; Stability; Neutral delay; Robust; Saturation

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# Motivation

1. Fractional-order (FO) differential equations (FODEs) are valuable tools for modeling complex dynamics of physical systems in different fields. Accordingly, the stability of dynamic FO systems has become a key problem when considering real-world processes that are modeled with FODEs.

2. Uncertainties in the dynamical models can strongly compromise the controller design procedure. Consequently, robust stability of the control systems, including FO systems, is of key importance.

3. The existence of time delay in the control loop brings fundamental problems, making the analysis and design of the control system complicated, and leading to performance degradation or even system instability. Delayed systems of neutral type involving delays both in the states and in their derivatives can be found in a number of distinct areas.

4. The input saturation constraint is also a key problem in the controller design. Control input saturation is a major issue in practice because actuator capabilities are limited.

5. It is still challenging to study neutral delay in FO dynamic systems together with input saturation and uncertainty.

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# Main idea

1. In this paper we analyze the problem of robust stability of FO delay systems of neutral type under actuator saturation.

2. A Lyapunov–Krasovskii (LK) function is constructed and conditions of the asymptotic robust stability of such systems are given, formulated by linear matrix inequalities (LMIs).

3. An algorithm is introduced to compute the state feedback controller gain.

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# Method

1. Using the LK function, we derive robust stability criteria expressed by LMIs that are sufficient for achieving an appropriate state feedback control strategy.

2. We also tackle the problem of finding a controller gain that can extend the domain of attraction.

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# Method

An uncertain FO neutral-type (FONT) delay system under input saturation is described in the dynamic equation:

$$\begin{aligned}&{}_0^{\mathrm{RL}}\mathfrak{D}_t^{\alpha}\boldsymbol{\xi}(t)-(\boldsymbol{C}+\Delta\boldsymbol{C}(t))\,{}_0^{\mathrm{RL}}\mathfrak{D}_t^{\alpha}\boldsymbol{\xi}(t-h(t))\\=&(\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{\xi}(t)+(\boldsymbol{A}_0+\Delta\boldsymbol{A}_0(t))\boldsymbol{\xi}(t-h(t))\\&+\boldsymbol{B}\boldsymbol{u}(t),\end{aligned}$$

The uncertainty terms are considered as

$$[\Delta\boldsymbol{A}(t)\ \ \Delta\boldsymbol{A}_0(t)\ \ \Delta\boldsymbol{C}(t)]=\boldsymbol{H}\boldsymbol{\mathcal{G}}(t)[\boldsymbol{F}_0\ \ \boldsymbol{F}_1\ \ \boldsymbol{F}_2],$$

Furthermore, $h(t)$ denotes the unknown but bounded, continuous, and differentiable delay:

$$0\le h(t)\le\bar{h},\ \ \dot{h}(t)\le d<1.$$

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# Method

The control signal is obtained as

$$\boldsymbol{u}(t) = \mathrm{sat}(\boldsymbol{K}\boldsymbol{\xi}(t))$$

We define

$$\boldsymbol{\psi}(\boldsymbol{K}\boldsymbol{\xi}(t)) = \boldsymbol{K}\boldsymbol{\xi}(t) - \mathrm{sat}(\boldsymbol{K}\boldsymbol{\xi}(t)),$$

Then, the overall FO system is expressed as

$$\begin{aligned} &{}_{0}^{\mathrm{RL}}\mathfrak{D}_{t}^{\alpha}\boldsymbol{\xi}(t) - (\boldsymbol{C} + \Delta\boldsymbol{C}(t))\,{}_{0}^{\mathrm{RL}}\mathfrak{D}_{t}^{\alpha}\boldsymbol{\xi}(t - h(t)) \\ &= (\boldsymbol{A} + \Delta\boldsymbol{A}(t))\boldsymbol{\xi}(t) + (\boldsymbol{A}_0 + \Delta\boldsymbol{A}_0(t))\boldsymbol{\xi}(t - h(t)) \\ &\quad + \boldsymbol{B}\mathrm{sat}(\boldsymbol{K}\boldsymbol{\xi}(t)). \end{aligned}$$

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# Major results

**Theorem 1** Let us consider any appropriate dimension matrices $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, symmetric positive definite matrices $\boldsymbol{P}$, $\boldsymbol{M}$, $\boldsymbol{N}$, $\boldsymbol{Z}$, and a positive definite diagonal matrix $\boldsymbol{\Lambda}$ satisfying the following LMI, which verifies the convergence of the nominal FONT trajectories asymptotic to the origin.

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_{11} & * & * & * & * & * \\ \boldsymbol{\Gamma}_{21} & \boldsymbol{\Gamma}_{22} & * & * & * & * \\ \boldsymbol{\Gamma}_{31} & \boldsymbol{\Gamma}_{32} & \boldsymbol{\Gamma}_{33} & * & * & * \\ \boldsymbol{\Gamma}_{41} & \boldsymbol{\Gamma}_{42} & \boldsymbol{\Gamma}_{43} & \boldsymbol{\Gamma}_{44} & * & * \\ \boldsymbol{\Gamma}_{51} & \boldsymbol{\Gamma}_{52} & \boldsymbol{\Gamma}_{53} & \boldsymbol{\Gamma}_{54} & \boldsymbol{\Gamma}_{55} & * \\ \boldsymbol{\Gamma}_{61} & \boldsymbol{\Gamma}_{62} & \boldsymbol{\Gamma}_{63} & \boldsymbol{\Gamma}_{64} & \boldsymbol{\Gamma}_{65} & \boldsymbol{\Gamma}_{66} \end{bmatrix} < 0,$$

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# Major results

**Theorem 2**  Consider the positive scalars $q$, $\beta$, $\rho$, a diagonal matrix $\boldsymbol{R}$ with appropriate dimension, symmetric positive-definite matrices $\bar{\boldsymbol{M}}$, $\bar{\boldsymbol{N}}$, $\bar{\boldsymbol{P}}$, $\bar{\boldsymbol{Z}}$, matrices $\boldsymbol{X}$, $\boldsymbol{U}$, $\boldsymbol{L}$ with compatible dimensions, and a real scalar $Y$ satisfying the conditions. Then, applying the gain of controller $\boldsymbol{K}=\boldsymbol{U}\boldsymbol{X}^{\mathrm{T}}$, the FONT system is asymptotically robustly stable.

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$$
\begin{bmatrix}
\boldsymbol{\Phi} & * & * & * & * & * & * \\
\boldsymbol{\vartheta} & \boldsymbol{\Theta} & * & * & * & * & * \\
\boldsymbol{\Xi}_{31} & \boldsymbol{\Xi}_{32} & \boldsymbol{\Xi}_{33} & * & * & * & * \\
\boldsymbol{\Xi}_{41} & \boldsymbol{\Xi}_{42} & \boldsymbol{\Xi}_{43} & \boldsymbol{\Xi}_{44} & * & * & * \\
\boldsymbol{\Xi}_{51} & \boldsymbol{\Xi}_{52} & \boldsymbol{\Xi}_{53} & \boldsymbol{\Xi}_{54} & \boldsymbol{\Xi}_{55} & * & * \\
\boldsymbol{\Xi}_{61} & \boldsymbol{\Xi}_{62} & \boldsymbol{\Xi}_{63} & \boldsymbol{\Xi}_{64} & \boldsymbol{\Xi}_{65} & \boldsymbol{\Xi}_{66} & * \\
\boldsymbol{F}_0\boldsymbol{X}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{F}_1\boldsymbol{X}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{F}_2\boldsymbol{X}^{\mathrm{T}} & \boldsymbol{0} & -q\boldsymbol{I}
\end{bmatrix}
< 0
$$

# Major results

**Extension of the domain of attraction:** The extension of the domain of attraction is accomplished while taking into account conditions of the eigenvalues $\tilde{\boldsymbol{X}}\tilde{\boldsymbol{M}}\tilde{\boldsymbol{X}}^{\mathrm{T}}$, $\tilde{\boldsymbol{X}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{X}}^{\mathrm{T}}$, $\tilde{\boldsymbol{X}}\tilde{\boldsymbol{N}}\tilde{\boldsymbol{X}}^{\mathrm{T}}$, and $\tilde{\boldsymbol{X}}\tilde{\boldsymbol{Z}}\tilde{\boldsymbol{X}}^{\mathrm{T}}$.

$$
\begin{aligned}
&\min \operatorname{tr}\{\bar{\boldsymbol{M}}\tilde{\boldsymbol{M}}+\bar{\boldsymbol{P}}\tilde{\boldsymbol{P}}+\bar{\boldsymbol{N}}\tilde{\boldsymbol{N}}+\bar{\boldsymbol{Z}}\tilde{\boldsymbol{Z}}\\
&\qquad +(\boldsymbol{X}+\boldsymbol{X}^{\mathrm{T}})(\tilde{\boldsymbol{X}}+\tilde{\boldsymbol{X}}^{\mathrm{T}})\},\\
&\text{s.t.}\begin{bmatrix}\bar{\boldsymbol{M}} & *\\ \boldsymbol{I} & \tilde{\boldsymbol{M}}\end{bmatrix}\geq 0,\begin{bmatrix}\bar{\boldsymbol{P}} & *\\ \boldsymbol{I} & \tilde{\boldsymbol{P}}\end{bmatrix}\geq 0,\begin{bmatrix}\bar{\boldsymbol{N}} & *\\ \boldsymbol{I} & \tilde{\boldsymbol{N}}\end{bmatrix}\geq 0,\\
&\begin{bmatrix}\bar{\boldsymbol{Z}} & *\\ \boldsymbol{I} & \tilde{\boldsymbol{Z}}\end{bmatrix}\geq 0,\begin{bmatrix}\boldsymbol{X}+\boldsymbol{X}^{\mathrm{T}} & *\\ \boldsymbol{I} & \tilde{\boldsymbol{X}}+\tilde{\boldsymbol{X}}^{\mathrm{T}}\end{bmatrix}\geq 0.
\end{aligned}
$$

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# Major results

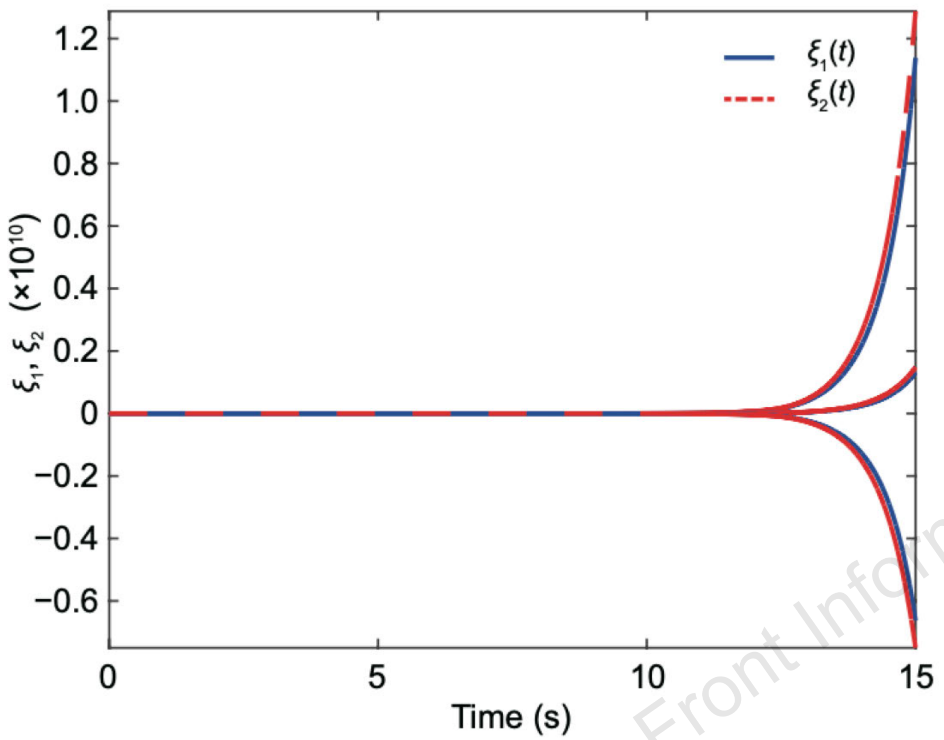


Fig. 1 The open-loop system states of example 1 with $\alpha$=0.9

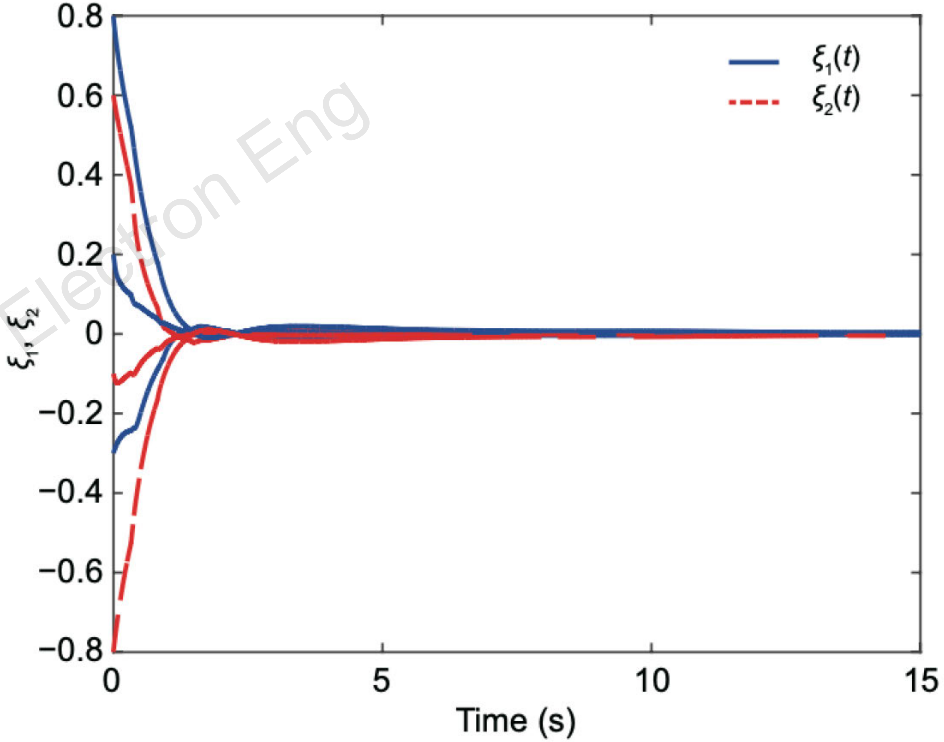


Fig. 2 The closed-loop system states of example 1 with $\alpha$=0.9

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# Major results

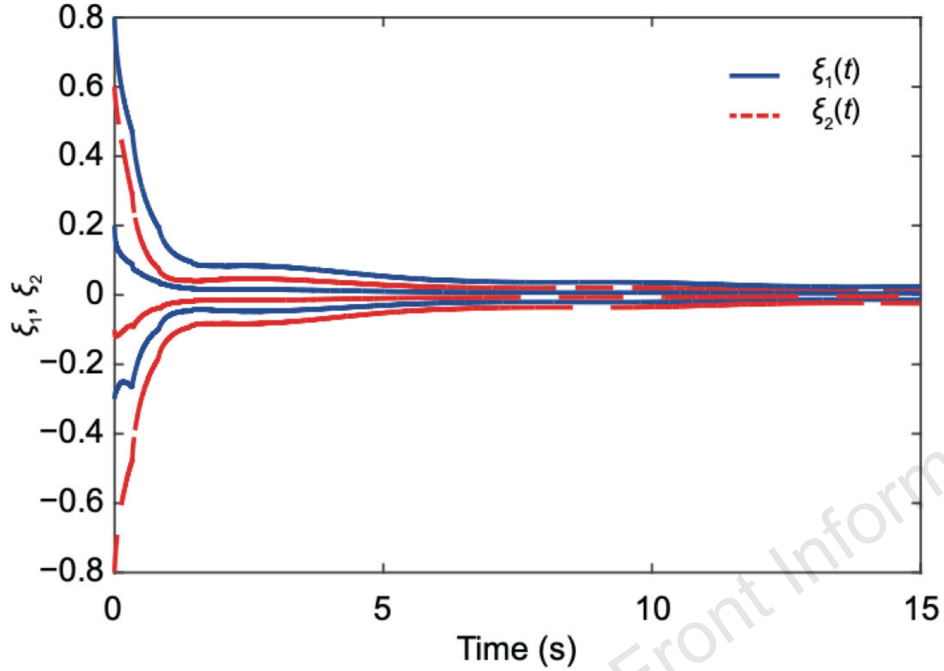


**Fig. 3 The states of example 1 with *α*=0.7 for different initial conditions**

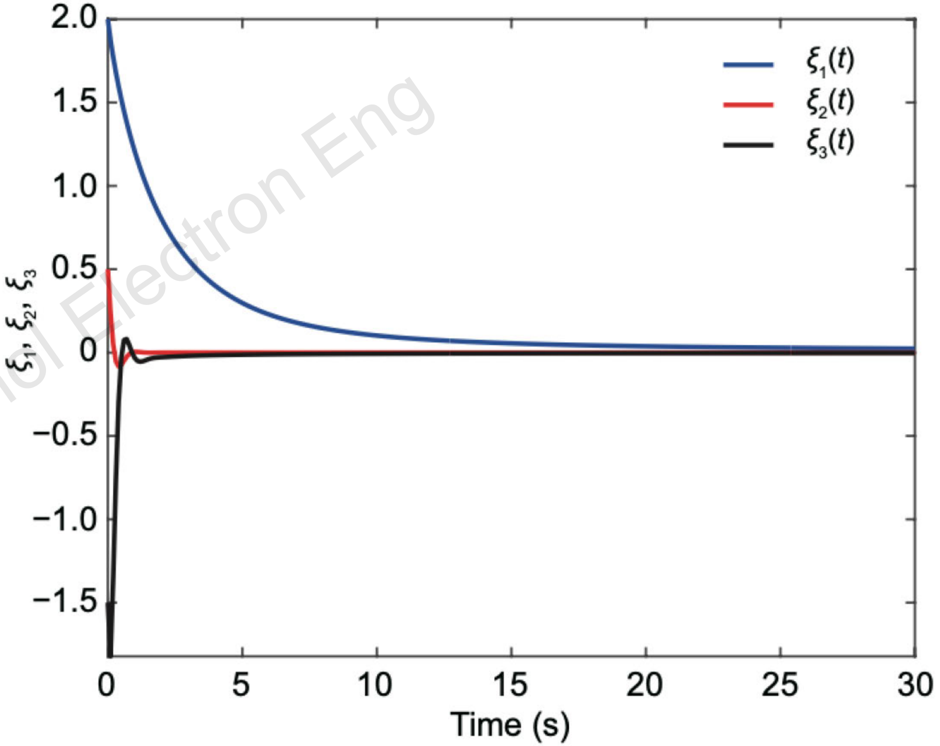


**Fig. 4 The states of example 2 with *α*=0.9**

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# Conclusions

This paper studied the robust stability of uncertain neutral-type fractional-order systems with actuator saturation. The LK function was constructed to determine the criteria with the help of LMIs. A state feedback controller was designed to stabilize this system, and an algorithm was proposed via the CCL method to compute the stabilizing gains. Numerical examples demonstrated the applicability of the new method.

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