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Caustics of developable surfaces

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Key words: Caustics; Developable surface; Reflected light rays; Curve of regression

Corresponding author: Miklós HOFFMANN
E-mail: hoffmann.miklos@uni-eszterhazy.hu
ORCID: https://orcid.org/0000-0001-8846-232X

Motivation

1. Developable surfaces are commonly used in architecture. These surfaces often have some reflectivity property, and as a result, certain areas may be exposed to different lighting.
2. Several articles on caustic surfaces are available, but their results are based on numerical calculations.
3. A closed form of the caustic surfaces can help the designer detect proplematic areas.

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Main idea

1. In case of classical parametric surfaces, writing its caustic surface in a closed form is practically impossible. The construction of the developable surfaces makes it possible to calculate the envelope of the reflected light.
2. For any generator of the developable surface, incoming light rays meeting the mirror surface along this generator are coplanar, and due to the fixed tangent plane along the generator, the reflected light rays are also coplanar. We calculate the envelope of these planes.

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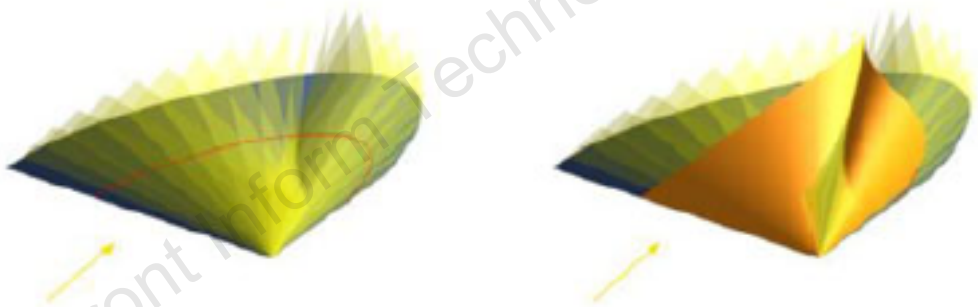
Method

1. At first we compute the plane of reflected light rays, the envelope of which is the caustic surface.
2. We present the general computations to three different types of developable surfaces (cone, cylinder, and tangent surface).

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Major results

1. The caustic surfaces of all of these three types of developable surfaces are described in a closed form. This provides an opportunity to display the caustic surfaces with computer software.



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Fig. 1 The family of planes of reflected rays (in yellow) and their envelope caustic surface (in orange) in the case of a conic mirror (in blue) defined by a cubic Bézier curve (in red)

The direction of the incoming light rays (in yellow) is shown.

Major results

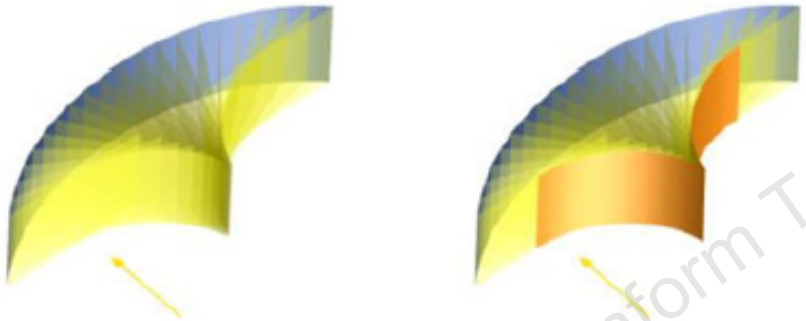


Fig. 2 The family of planes of reflected rays (in yellow) and their envelope caustic surface (in orange) in the case of a cylindric mirror (in blue)

The direction of the incoming light rays (in yellow) is shown.



Fig. 3 The family of planes of reflected rays (in yellow) and their envelope caustic surface (in orange) in the case of a tangent surface of a spatial curve as a mirror (in blue)

The direction of the incoming light rays (in yellow) is shown.

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Major results

2. In case of the tangent surface, its caustic surface is a developable surface. Specifically, it is a tangent developable surface of the curve of regression.

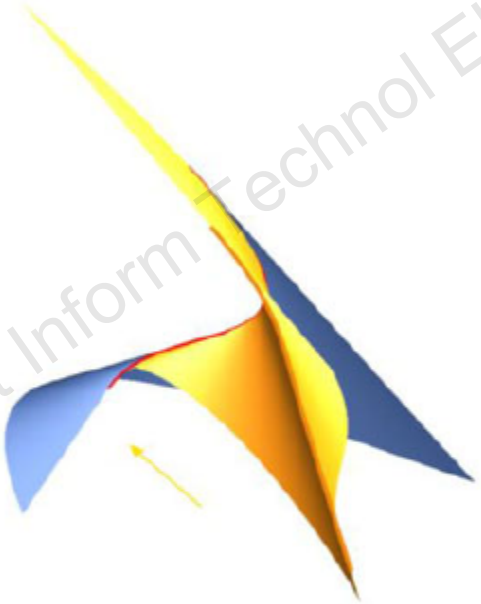


Fig. 4 The caustic surface (in orange) along with its regression curve in the case of a tangent surface of a spatial curve as a mirror (in blue) defined by a cubic Bézier curve (in red)

The direction of the incoming light rays (in yellow) is shown.

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Conclusions

1. The caustic surfaces of each of the three different types of developable surfaces have been expressed in an exact closed form.
2. In case of the tangent surface of a spatial curve, the curve of regression of the caustic surface has also been computed. This curve has been applied in a practical computation of finding the cusp of the intersection curve of a caustic surface and a plane.

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Miklos HOFFMANN is a full professor at the Eszterházy Károly Catholic University, Hungary, and a full professor at the University of Debrecen, Hungary. He received his PhD degree in 1998, from Kossuth Lajos University of Sciences. He has been supported by the project “EFOP-3.6.3-VEKOP-16-2017-00002.” His research interests include geometric modelling, computer aided geometric design, and the relationship of art and science (especially geometry).



Imre JUHÁSZ is a full professor at the University of Miskolc, Miskolc, Hungary. He received his PhD degree in 1998, from the Kossuth Lajos University of Sciences. His research interests include computer aided geometric design.

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Ede TROLL is an assistant lecturer at the Eszterházy Károly Catholic University, Eger. He received his MS degree from the Eszterházy Károly University, Hungary, in 2014. He has been supported by the project “EFOP-3.6.3-VEKOP-16-2017-00002.” His research interests include computer aided geometric design.