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# Firefly algorithm with division of roles for complex optimal scheduling

**Key words:** Firefly algorithm (FA); Division of roles; Cauchy mutation; Elite neighborhood search; Optimal scheduling

Corresponding author: Renbin XIAO

E-mail: [rbxiao@hust.edu.cn](mailto:rbxiao@hust.edu.cn)

 ORCID: <https://orcid.org/0000-0003-0951-2734>

# Motivation

1. The step factor of the firefly algorithm (FA) is fixed, which is not conducive to the search of the solution space.
2. The learning mechanism of FA has high time complexity, and firefly movement is easy to oscillate, which wastes computing resources.
3. The single learning strategy of FA makes the population diversity easy to lose and the algorithm is easy to fall into local optima.

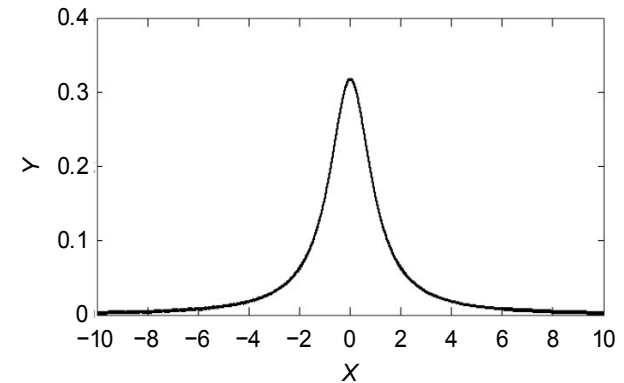
# Main idea

1. Using the idea of division of labor and cooperation, the firefly population is divided into leaders, developers, and followers, with each role undertaking different search tasks.
2. Different learning strategies are assigned to each role: the leader carries out greedy Cauchy mutation; the developer randomly selects two leaders for local development using the elite neighborhood search strategy; the follower randomly selects two excellent particles for global exploration.
3. A new method of changing step size, step-down step size, was proposed to meet the needs of firefly movement in different stages of evolution.

# Method

1. Leader: The probability density function of the standard Cauchy distribution is shown in Fig. 2. Some have shown that the long tail of the Cauchy distribution can help the trapped firefly jump to a better position, and the greedy strategy can save the dominant information of the population. The equation of the greedy mutation strategy is as follows:

$$\mathbf{x}_i(t+1) = \begin{cases} \mathbf{x}_i(t) + \mathbf{cauchy}(), & f(\mathbf{x}_i(t) + \mathbf{cauchy}()) < f(\mathbf{x}_i(t)), \\ \mathbf{x}_i(t), & f(\mathbf{x}_i(t) + \mathbf{cauchy}()) \geq f(\mathbf{x}_i(t)), \end{cases}$$



**Fig. 2** The standard Cauchy distribution probability density function

# Method (Cont'd)

2. Developer: The developer undertakes the local development of the algorithm. To increase the convergence speed and development accuracy of the algorithm, we propose an elite neighborhood search strategy. The neighborhood search mechanism is to assign a neighborhood structure to the current individual to be optimized.

$$\mathbf{x}_i(t) = r_1 \mathbf{x}_i(t) + r_2 \mathbf{gbest} + r_3 (\mathbf{x}_j(t) - \mathbf{x}_k(t)) + \alpha(t) \mathbf{S}\boldsymbol{\varepsilon},$$

# Method (Cont'd)

3. Follower: The task of the follower is to explore more unknown space in the process of following the excellent particles, that is, to undertake the exploration work of the algorithm.

$$\begin{aligned} \mathbf{x}_i(t+1) = & \mathbf{x}_i(t) + r_4\beta_1(\mathbf{x}_j(t) - \mathbf{x}_i(t)) \\ & + r_5\beta_2(\mathbf{x}_k(t) - \mathbf{x}_i(t)) + \alpha(t)\mathcal{S}\boldsymbol{\varepsilon}, \end{aligned}$$

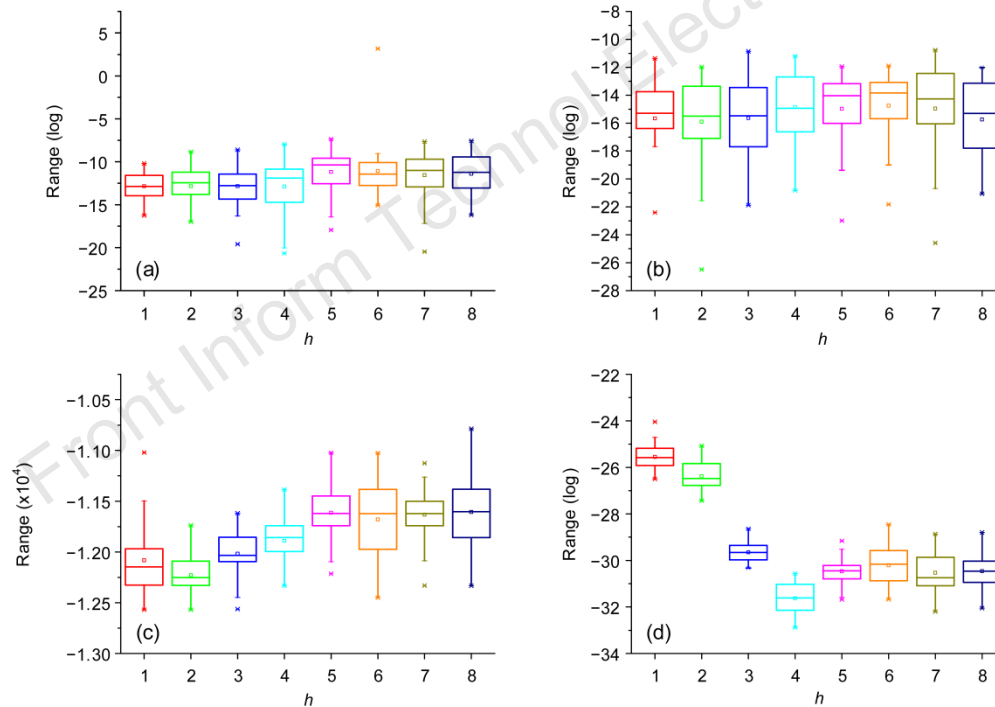
# Method (Cont'd)

4. Stepped down step: To meet the requirements of the algorithm for the search step length in different periods, the FA search step length decreases step by step with the operation of the algorithm.

$$\alpha(t+1) = \begin{cases} \alpha(t)/\tau(i), & \text{mod}(t, c) = 0, \\ \alpha(t), & \text{otherwise,} \end{cases}$$

# Strategy analysis

1. Analysis of role matching in firefly population: Leaders, developers, and followers in the firefly population are divided according to the proportion of 1:1: $h$ .



**Fig. 3** Box diagram of the optimization results of the DRFA algorithm with different role proportions: (a) Rosenbrock ( $f_5$ ); (b) quartic with noise ( $f_7$ ); (c) Schwefel 2.26 ( $f_8$ ); (d) penalized ( $f_{12}$ )

# Strategy analysis (Cont'd)

2. Effect of different roles: DRFA allocates various strategies to leaders, developers, and followers in the way of division of labor and cooperation. To explore the influence of roles and their division of labor on the algorithm, we combine the roles and their division of labor.

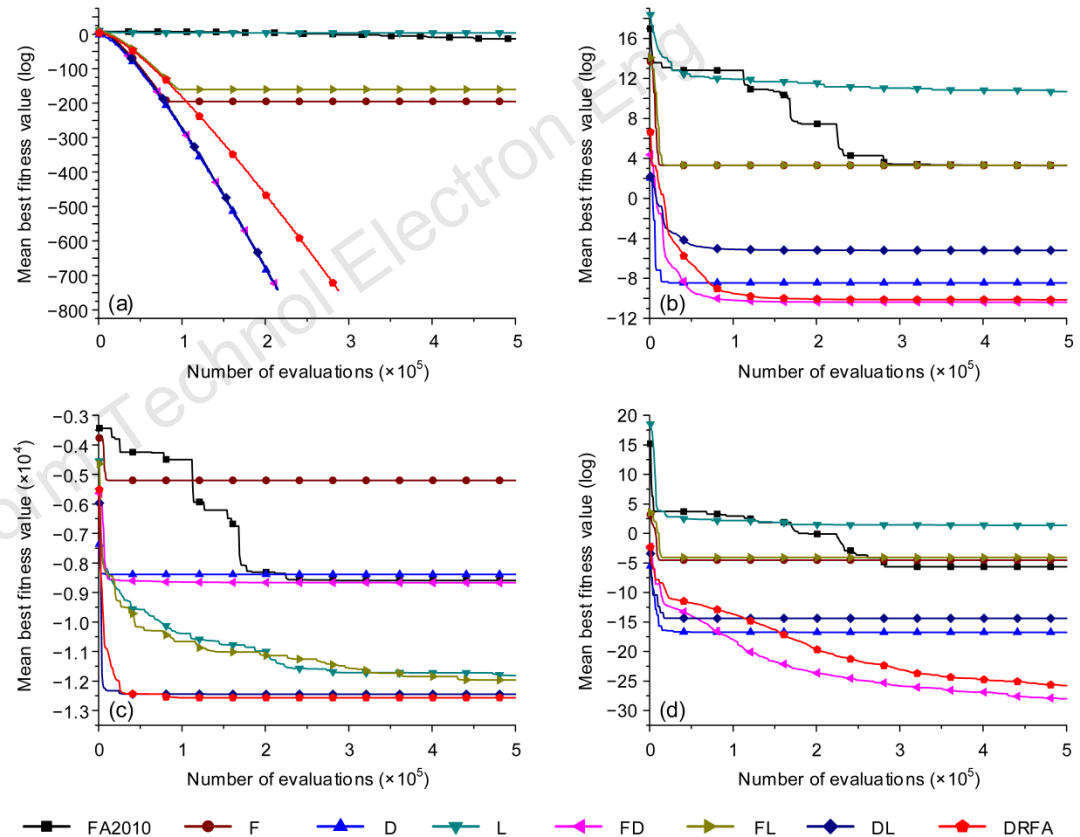


Fig. 4 Convergence curve of the FA algorithm with different combinations of division of roles: (a) sphere ( $f_1$ ); (b) Rosenbrock ( $f_3$ ); (c) Schwefel 2.26 ( $f_8$ ); (d) penalized ( $f_{12}$ )

# Major results

## 1. Optimization results of 10 algorithms on 12 test functions

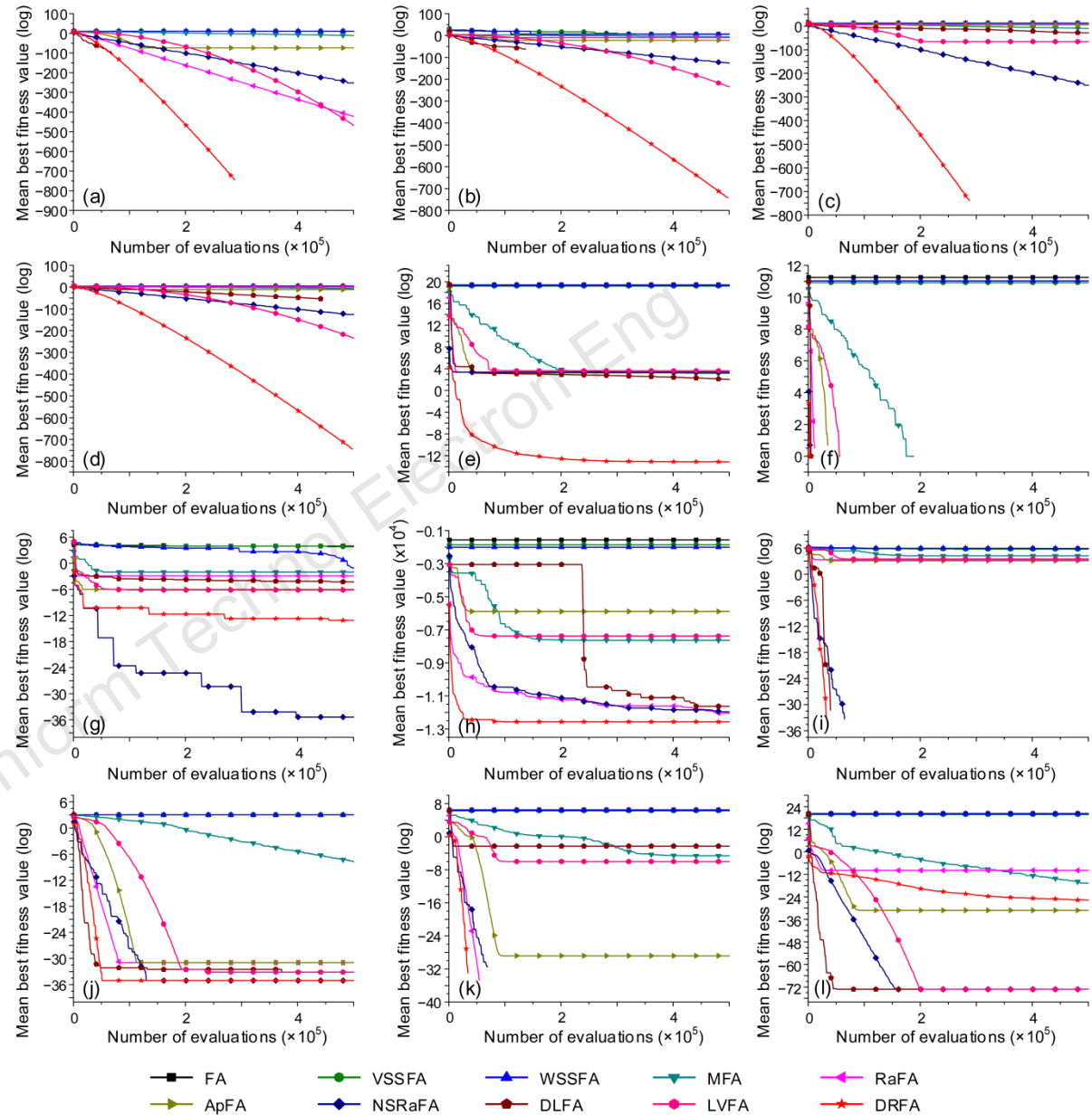
Table 5 Optimization results of 10 algorithms on 12 test functions

Function	Type	FA	WSSFA	VSSFA	MFA	RaFA	ApFA	NSRaFA	DLFA	LVFA	DRFA
$f_1$	Mean	6.67E+04	6.34E+04	5.84E+04	1.56E-05	5.36E-184	2.02E-44	4.11E-110	<b>0.00E+00</b>	7.65E-204	<b>0.00E+00</b>
	Std.	1.83E+04	4.91E+04	1.17E+04	2.31E-05	6.82E-184	2.85E-44	9.05E-110	<b>0.00E+00</b>	0.00E+00	<b>0.00E+00</b>
$f_2$	Mean	5.19E+02	1.35E+02	1.13E+02	1.85E-03	8.76E-05	1.83E-12	1.35E-55	<b>0.00E+00</b>	1.24E-102	<b>0.00E+00</b>
	Std.	1.42E+02	5.66E+02	3.93E+01	3.57E-03	7.58E-05	4.01E-12	6.22E-56	<b>0.00E+00</b>	6.24E-103	<b>0.00E+00</b>
$f_3$	Mean	2.43E+05	1.10E+05	1.16E+05	5.89E-05	4.91E+02	1.01E+01	1.59E-109	1.58E-09	1.53E-21	<b>0.00E+00</b>
	Std.	4.85E+04	4.60E+05	3.64E+04	4.52E-05	1.06E+02	5.92E+00	4.95E-109	1.10E-08	4.51E-20	<b>0.00E+00</b>
$f_4$	Mean	8.35E+01	7.59E+01	8.18E+01	1.73E-03	2.43E+00	1.30E-07	1.88E-55	<b>0.00E+00</b>	1.00E-96	<b>0.00E+00</b>
	Std.	3.16E+01	1.88E+01	2.32E+01	3.86E-03	1.87E+00	8.85E-08	5.87E-56	<b>0.00E+00</b>	2.70E-95	<b>0.00E+00</b>
$f_5$	Mean	2.69E+08	2.49E+08	2.16E+08	2.29E+01	2.92E+01	2.81E+01	2.85E+01	1.99E+00	3.82E+01	<b>1.26E-05</b>
	Std.	6.21E+07	2.76E+08	5.79E+07	3.25E+00	4.19E+00	3.98E-01	2.03E-02	1.62E+01	3.61E+02	<b>1.50E-04</b>
$f_6$	Mean	7.69E+04	6.18E+04	5.48E+04	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>
	Std.	3.38E+03	5.12E+04	2.16E+03	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>	<b>0.00E+00</b>
$f_7$	Mean	5.16E+01	3.24E-01	4.43E+01	1.30E-01	5.47E-02	2.76E-03	<b>4.41E-16</b>	1.26E-02	1.96E-03	9.09E-07
	Std.	2.46E+01	4.80E+01	1.72E+01	2.36E-01	4.65E-02	1.10E-02	<b>1.25E-16</b>	1.86E-02	4.54E-03	7.44E-06
$f_8$	Mean	-1.56E+03	-2.01E+03	-1.85E+03	-7.63E+03	-1.21E+04	-6.25E+03	-1.20E+04	-8.86E+03	-7.38E+03	<b>-1.22E-04</b>
	Std.	3.77E+03	3.23E+03	8.56E+02	8.72E+02	3.61E+02	1.26E+02	2.25E+02	2.04E+04	4.48E+03	<b>1.09E+03</b>
$f_9$	Mean	3.33E+02	3.61E+02	3.12E+02	6.47E+02	2.69E+01	1.21E+01	<b>0.00E+00</b>	<b>0.00E+00</b>	3.19E+01	<b>0.00E+00</b>
	Std.	6.28E+01	8.56E+01	4.18E+01	2.53E+01	1.52E+01	2.77E+00	<b>0.00E+00</b>	<b>0.00E+00</b>	5.07E+01	<b>0.00E+00</b>
$f_{10}$	Mean	2.03E+01	2.05E+01	2.03E+01	4.23E-04	3.61E-14	2.55E-14	<b>5.89E-16</b>	5.68E-15	5.09E-15	<b>5.89E-16</b>
	Std.	2.23E-01	5.56E-01	2.46E-01	3.35E-04	5.98E-14	6.48E-15	<b>0.00E+00</b>	9.64E-15	8.61E-15	<b>0.00E+00</b>
$f_{11}$	Mean	6.54E+02	6.09E+02	5.47E+02	9.86E-03	<b>0.00E+00</b>	3.33E-16	<b>0.00E+00</b>	2.99E-02	2.38E-03	<b>0.00E+00</b>
	Std.	1.69E+02	4.19E+02	1.29E+02	6.81E-03	<b>0.00E+00</b>	2.22E-16	<b>0.00E+00</b>	1.52E-01	2.45E-02	<b>0.00E+00</b>
$f_{12}$	Mean	7.16E+08	6.18E+08	3.99E+08	5.04E-08	4.50E-05	1.23E-16	<b>1.57E-32</b>	1.57E-32	1.57E-32	4.10E-12
	Std.	1.82E+08	8.38E+08	1.05E+08	3.27E-08	6.28E-04	1.59E-16	<b>0.00E+00</b>	1.50E-47	1.49E-47	1.38E-11
$p$		<b>0.002</b>	<b>0.002</b>	<b>0.002</b>	<b>0.003</b>	<b>0.005</b>	<b>0.013</b>	0.327	<b>0.043</b>	<b>0.016</b>	
$w/t/l$		12/0/0	12/0/0	12/0/0	11/1/0	10/2/0	10/1/1	6/4/2	6/5/1	10/1/1	

The best optimization results are in bold. The  $p$  values are obtained by the Wilcoxon test (García et al., 2009) for DRFA and other FAs. The  $p$  value less than 0.05 is in bold, indicating that DRFA is clearly superior to the compared algorithm.  $w/t/l$  indicates that compared with the algorithm, the performance of DRFA is better on  $w$  functions, equivalent on  $t$  functions, and worse on  $l$  functions

# Major results (Cont'd)

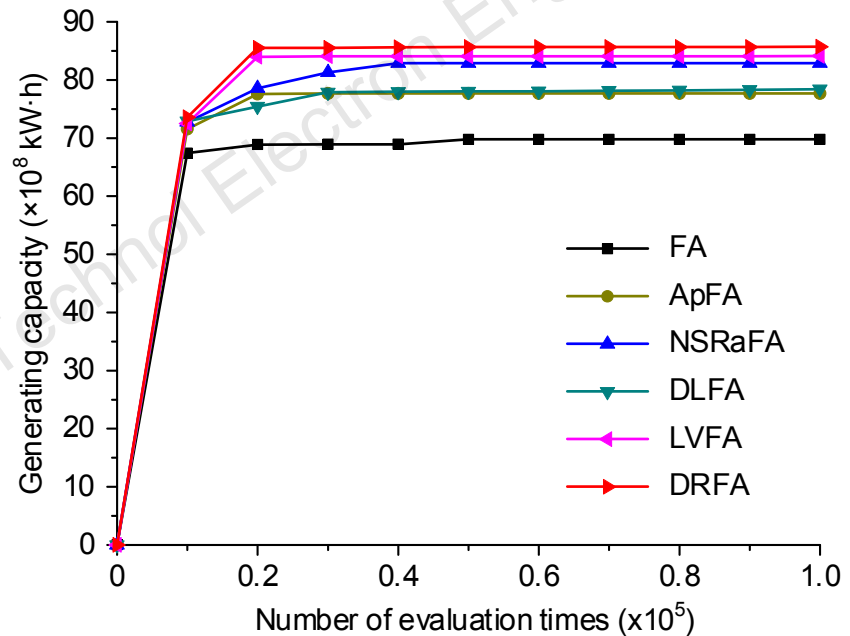
## 2. Convergence process curves of 10 algorithms on 12 test functions



**Fig. 5** Convergence process curves of 10 algorithms on 12 test functions: (a) sphere ( $f_1$ ); (b) Schwefel 2.22 ( $f_2$ ); (c) Schwefel 1.2 ( $f_3$ ); (d) Schwefel 2.21 ( $f_4$ ); (e) Rosenbrock ( $f_5$ ); (f) step ( $f_6$ ); (g) quartic with noise ( $f_7$ ); (h) Schwefel 2.26 ( $f_8$ ); (i) Rastrigin ( $f_9$ ); (j) Ackley ( $f_{10}$ ); (k) Griewank ( $f_{11}$ ); (l) penalized ( $f_{12}$ )

# Major results (Cont'd)

## 3. Optimal scheduling of cascade reservoirs



**Fig. 6 Convergence diagram of power generation and the number of evaluation times of six algorithms**

# Conclusions

1. Taking the scheduling of cascade reservoirs as an example, we determine that the current optimal operation problem is a complex problem featuring multi-objective, multi-constraint, multi-stage, and strong coupling.
2. Given that FA cannot solve the complex optimal scheduling problem, DRFA is proposed by integrating various learning strategies into FA with the idea of division of roles, which makes up for the deficiency of FA using a single learning strategy.