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Affine formation tracking control of unmanned aerial vehicles

Key words: Affine formation; Fixed-wing unmanned aerial vehicles; Multi-agent system

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Motivation

1. For fixed-wing unmanned aerial vehicles (UAVs), it is imperative to improve the trajectory tracking ability in dynamic environments.

2. It is difficult to control the distances and relative position among agents in a multi-agent system without increasing the complexity of control protocols.

3. The affine formation control scheme can drive the multi-agent system to track time-varying affine formation shape transformations along diverse trajectories.

Main idea

1. The affine formation control strategy has higher flexibility and scalability, and can preserve the distances among agents and the orientation of the formation and deal with geometric distortions, such as translation, rotation, scaling, shear, and their combinations.

2. A saturated control strategy is effective in meeting the asymmetrical speed constraints of fixed-wing UAVs, which can be verified by numerical simulations.

Method

1. The leader-follower strategy is applied to solve the affine formation tracking control problem.

2. An original distributed control law based on stress matrix is proposed to achieve the desired time-varying formation pattern and to track affine transformations along diverse trajectories.

3. Theoretical analysis and simulation results are provided to illustrate the effectiveness of the proposed control strategy.

Method

Affine transformation preserves straightness and parallelism

Nominal configuration:

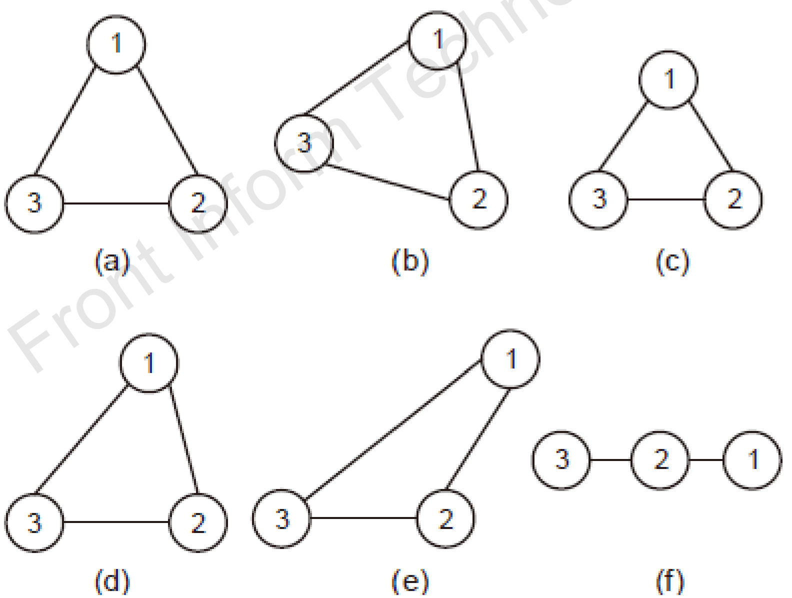
$$\boldsymbol{r} = [\boldsymbol{r}_1^{\mathrm{T}}, \boldsymbol{r}_2^{\mathrm{T}}, \ldots, \boldsymbol{r}_n^{\mathrm{T}}]^{\mathrm{T}} = [\boldsymbol{r}_1^{\mathrm{T}}, \boldsymbol{r}_f^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{dn}$$

Target configuration:

$$\boldsymbol{p}^*(t) = (\boldsymbol{I}_n \otimes \boldsymbol{A}(t))\boldsymbol{r} + \mathbf{1}_n \otimes \boldsymbol{b}(t),$$

where the desired position of the i^{th} UAV is

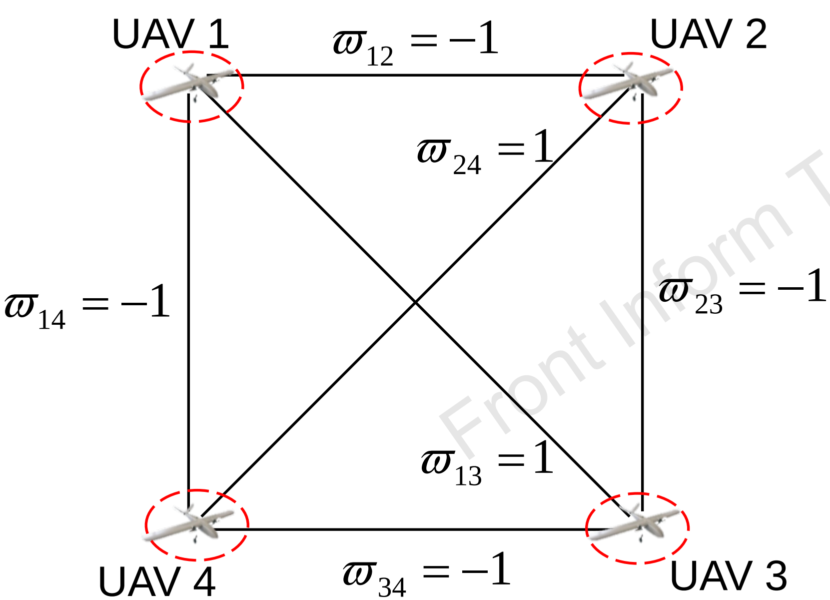
$$\boldsymbol{p}_i^*(t) = \boldsymbol{A}(t)\boldsymbol{r}_i + \boldsymbol{b}(t).$$



An illustration of affine transformations of a nominal configuration: (a) nominal; (b) rotation; (c) scaling; (d–f) shear

Method

For a graph $G = (\mathcal{V}, \mathcal{E})$, a stress is a set of scalars that are assigned to all the edges, that is, ϖ_{ij} for $(i, j) \in \mathcal{E}$.



- Equilibrium stress

$$\sum_{j \in \mathcal{N}_i} \varpi_{ij} (\boldsymbol{p}_j - \boldsymbol{p}_i) = \boldsymbol{0},$$

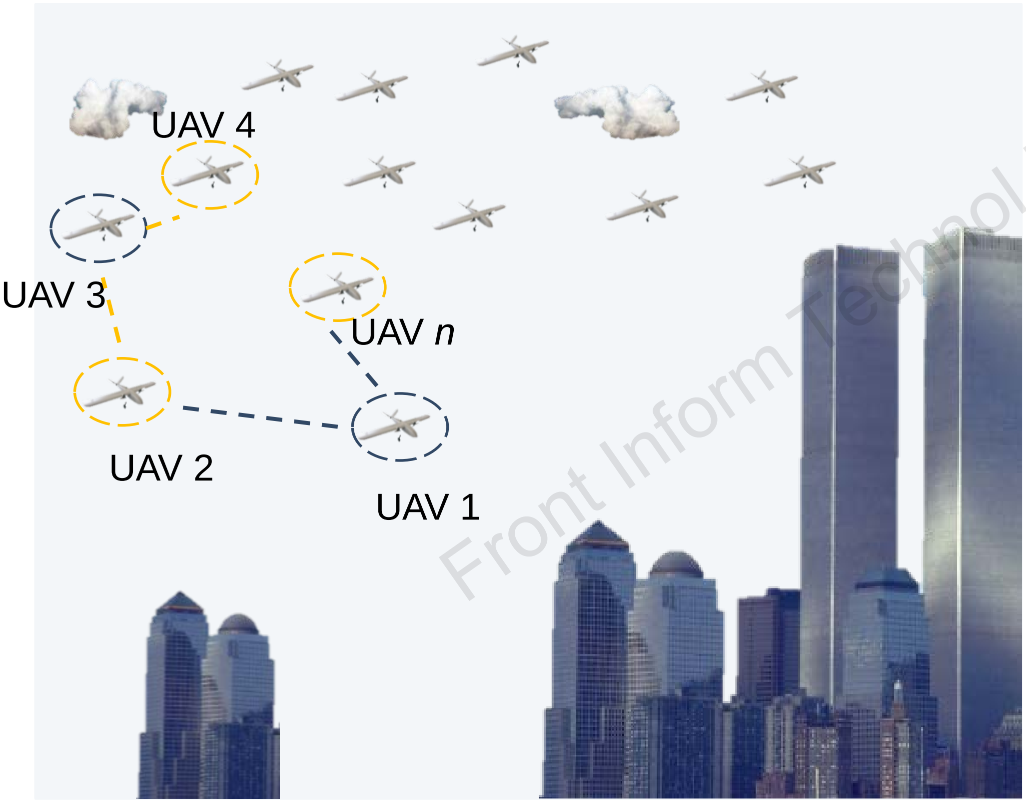
$$\Rightarrow \quad (\boldsymbol{\Omega} \otimes \boldsymbol{I}_d)\boldsymbol{p} = \boldsymbol{0},$$

where the stress matrix is defined as

$$[\Omega]_{ij} = \begin{cases} -\varpi_{ij}, & i \neq j, \quad j \in \mathcal{N}_i, \\ 0, & i \neq j, \quad j \notin \mathcal{N}_i, \\ \sum_{k \in \mathcal{N}_i} \varpi_{ik}, & i = j. \end{cases}$$

Major results

- Given a group of n fixed-wing UAVs:



- Fixed-wing UAVs：

$$\begin{cases} \dot{x}_i = v_i \cos\theta_i, \\ \dot{y}_i = v_i \sin\theta_i, \\ \dot{\theta}_i = \omega_i, \end{cases}$$

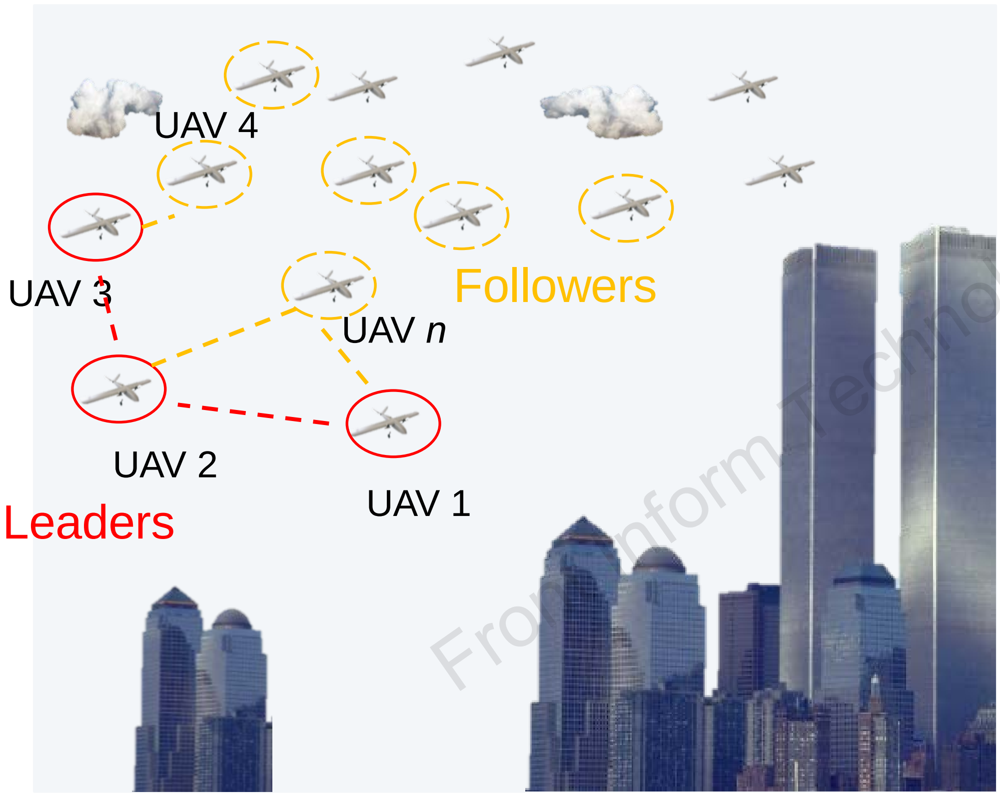
$$0 < v_{\min} \leq v_i \leq v_{\max}, \quad |\, \omega_i \,| \leq \omega_{\max}$$

- Communication topology:

$$G = (\mathcal{V}, \mathcal{E})$$

- Neighbor UAVs: N_i

Major results



Target configuration:

$$p^*(t) = [I_n \otimes A(t)]r + 1_n \otimes b(t)$$

$$p_f^* = -\overline{\Omega}_{ff}^{-1}\overline{\Omega}_{fl}\, p_l^*$$

$$h_i^*(t) = [\cos\theta_i^*, \sin\theta_i^*]^T$$

$$\Rightarrow \quad \lim_{t\to\infty} p(t) = p^*(t), \qquad \lim_{t\to\infty} h(t) = h^*(t)$$

If $p_l(t) = p_l^*(t)$ and $h_l(t) = h_l^*(t)$, then define the tracking error as

$$\delta_{p_f}(t) = p_f(t) - p_f^*(t)$$
$$\delta_{h_f}(t) = h_f(t) - h_f^*(t)$$

$$\Leftarrow \quad \lim_{t\to\infty} \delta_{p_f}(t) = 0, \qquad \lim_{t\to\infty} \delta_{h_f}(t) = 0$$

Major results

- Control law:

$$v_i = \boldsymbol{h}_i^{\mathrm{T}} \left[-\sum_{j \in \mathcal{N}_i} \varpi_{ij} (\boldsymbol{p}_i - \boldsymbol{p}_j) + \boldsymbol{k}_i \bar{\boldsymbol{v}}_1 + \omega_{\mathrm{c}} \boldsymbol{h}_i^{\perp} \right],$$

$$\omega_i = (\boldsymbol{h}_i^{\perp})^{\mathrm{T}} \left[-\sum_{j \in \mathcal{N}_i} \varpi_{ij} (\boldsymbol{p}_i - \boldsymbol{p}_j) + \boldsymbol{k}_i \bar{\boldsymbol{v}}_1 + \omega_{\mathrm{c}} \boldsymbol{h}_i^{\perp} \right]$$

- Theorem

The control law designed for the fixed-wing UAVs holds the following results: the followers' tracking errors $\delta_{\mathrm{p_f}}$ and $\delta_{\mathrm{h_f}}$ can converge to zero when $v_i>0$ for $i \in \mathrm{V}_f$.

Major results

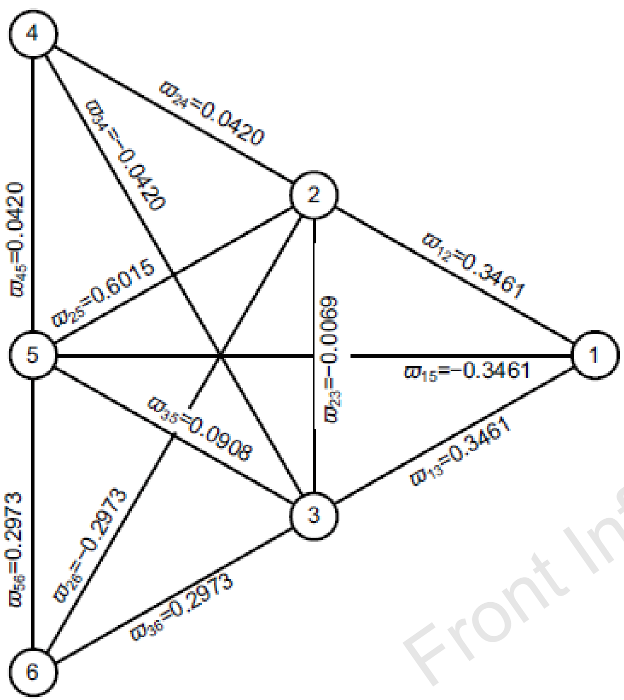
To meet the speed constraints of fixed-wing UAVs, define a saturation function as

$$\mathrm{sat}(x,a,b)=\begin{cases} a, & x<a,\\ x, & a\le x\le b,\\ b, & x>b. \end{cases}$$

➡ Control law:

$$v_i=\mathrm{sat}\left(\boldsymbol{h}_i^{\mathrm{T}}\left(-\sum_{j\in\mathcal{N}_i}\varpi_{ij}(\boldsymbol{p}_i-\boldsymbol{p}_j)+\boldsymbol{k}_i\bar{v}_{\mathrm{l}}+\omega_{\mathrm{c}}\boldsymbol{h}_i^{\perp}\right),\right.$$
$$\left. v_{\min}, v_{\max}\right),$$
$$\omega_i=\mathrm{sat}\left(\boldsymbol{h}_i^{\perp\mathrm{T}}\left(-\sum_{j\in\mathcal{N}_i}\varpi_{ij}(\boldsymbol{p}_i-\boldsymbol{p}_j)+\boldsymbol{k}_i\bar{v}_{\mathrm{l}}+\omega_{\mathrm{c}}\boldsymbol{h}_i^{\perp}\right),\right.$$
$$\left. -\omega_{\max}, \omega_{\max}\right).$$

Major results



Nominal formation of six fixed-wing UAVs

- Nominal configuration:

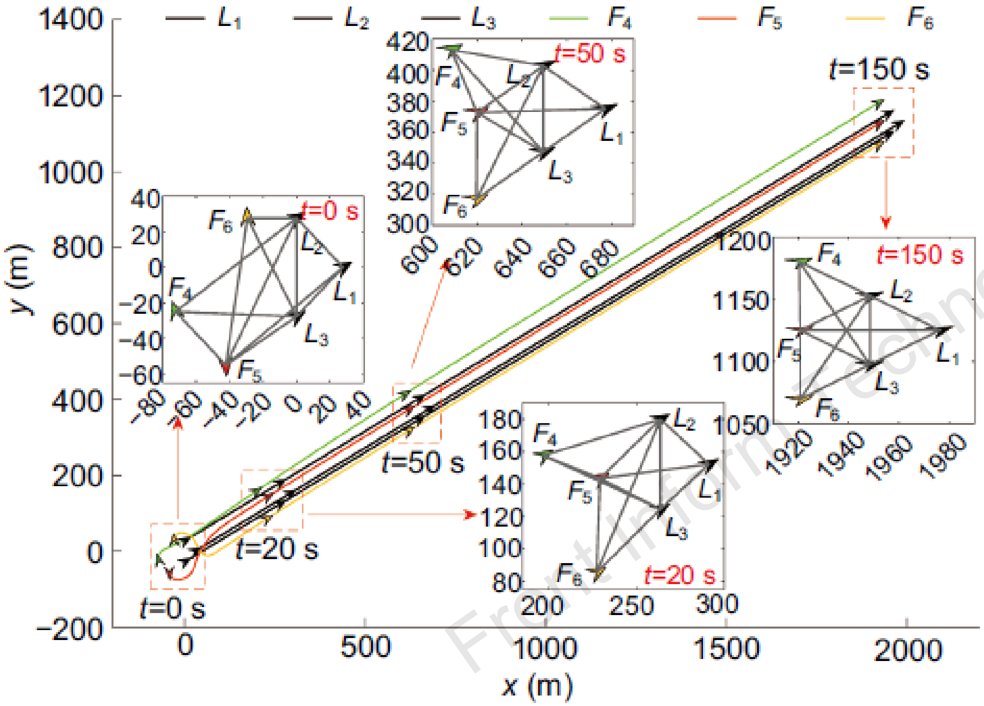
$$r_1 = 8\left[\sqrt{3}, 0, 0, 1, 0, -1\right]^{\mathrm{T}}$$

$$r_{\mathrm{f}} = 8\left[-\sqrt{3}, 2, -\sqrt{3}, 0, -\sqrt{3}, -2\right]^{\mathrm{T}}$$

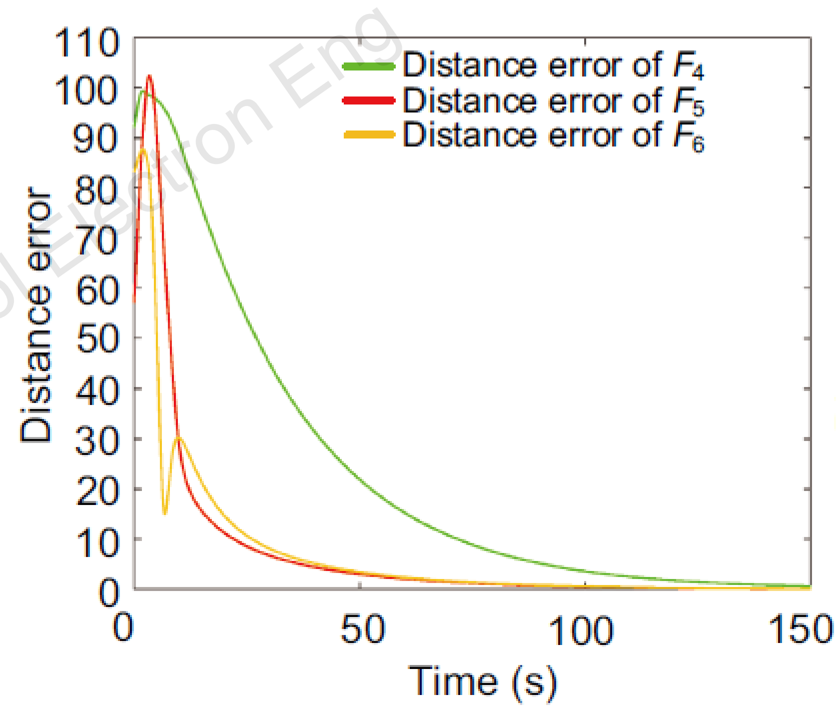
- Equilibrium stress matrix:

$$\Omega = \begin{bmatrix} 0.3461 & -0.3461 & -0.3461 & 0 & 0.3461 & 0 \\ -0.3461 & 0.6854 & 0.0069 & -0.0420 & -0.6015 & 0.2973 \\ -0.3461 & 0.0069 & 0.6854 & 0.0420 & -0.0908 & -0.2973 \\ 0 & -0.0420 & 0.0420 & 0.0420 & -0.0420 & 0 \\ 0.3461 & -0.6015 & -0.0908 & -0.0420 & 0.6854 & -0.2973 \\ 0 & 0.2973 & -0.2973 & 0 & -0.2973 & 0.2973 \end{bmatrix}$$

Major results

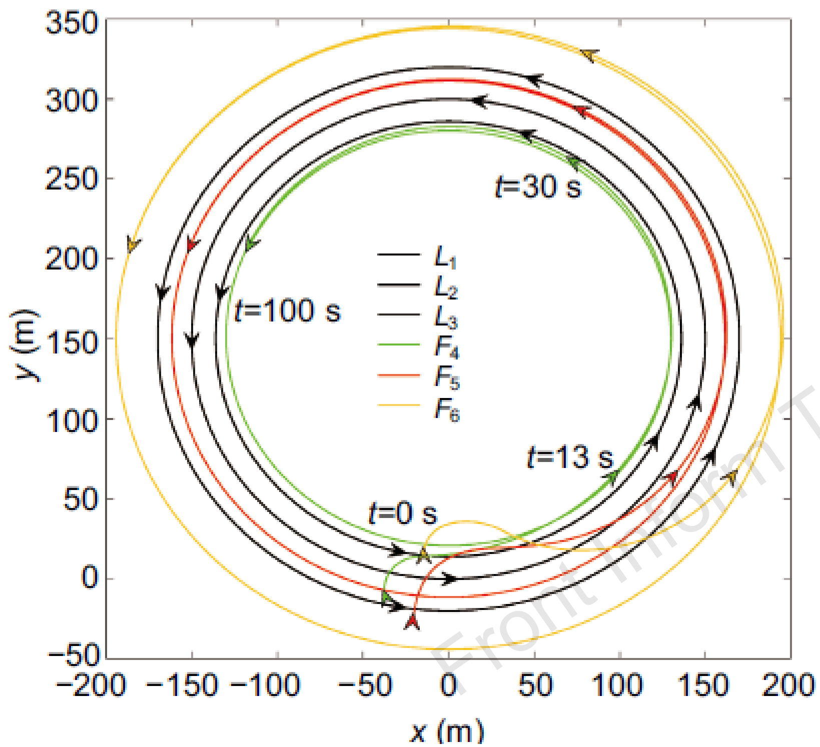


The affine formation moving along a straight line

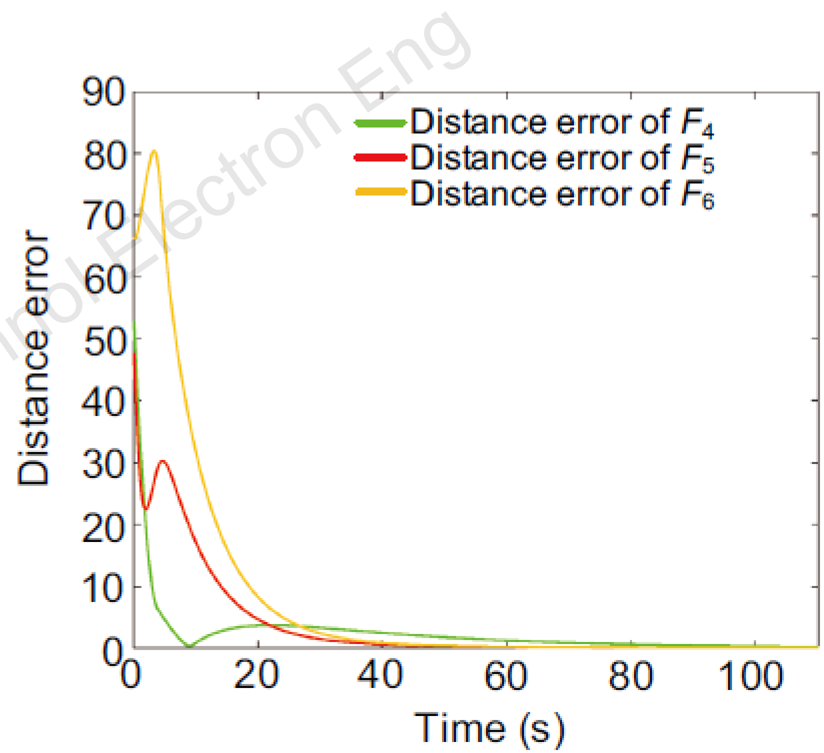


Tracking errors of three following fixed-wing UAVs moving along a straight line

Major results

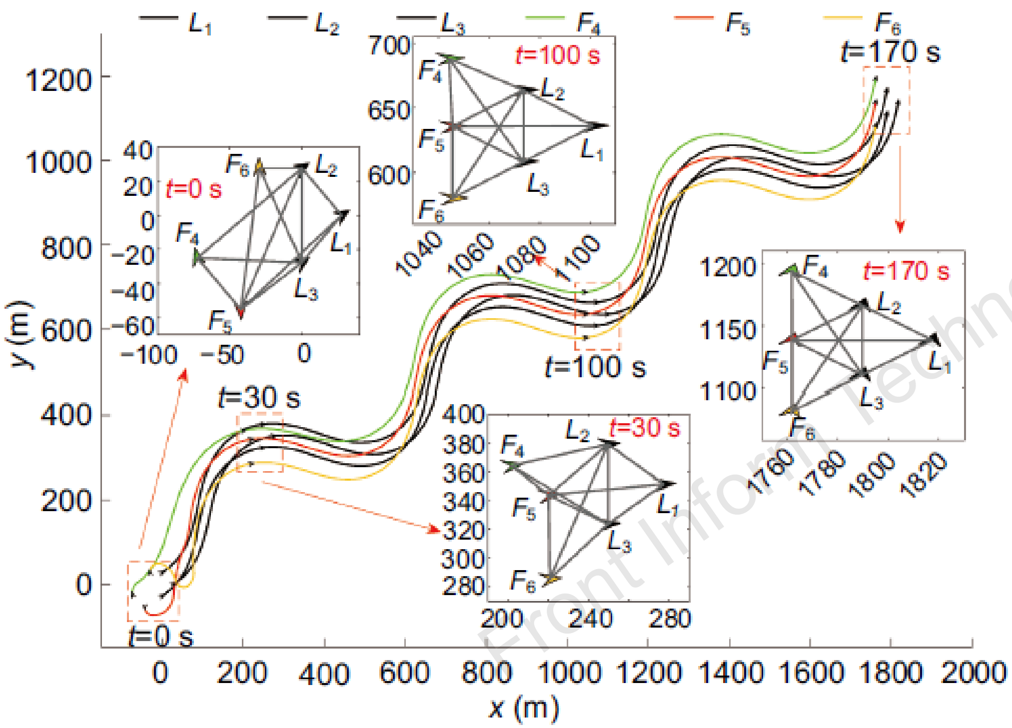


The affine formation moving along a circle

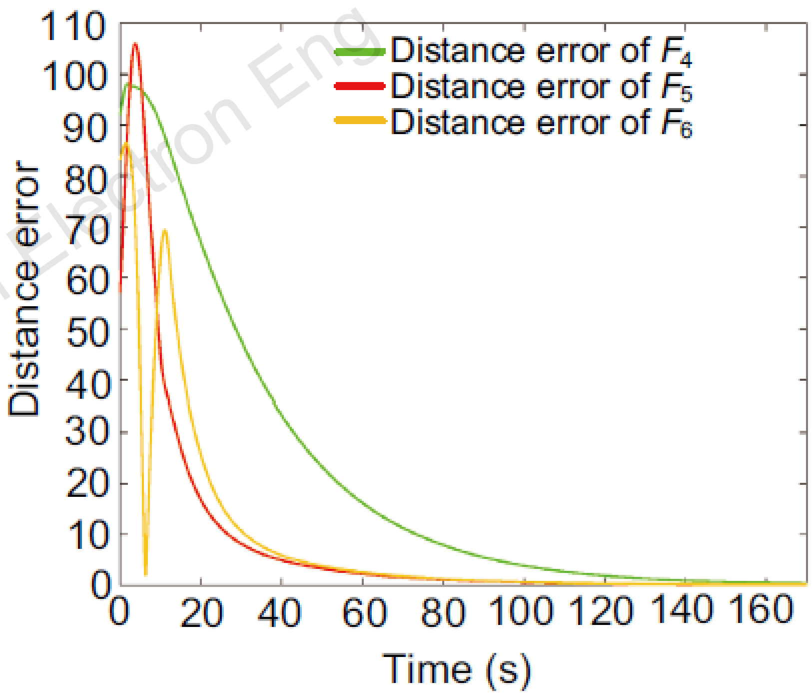


Tracking errors of three following fixed-wing UAVs moving along a circle

Major results

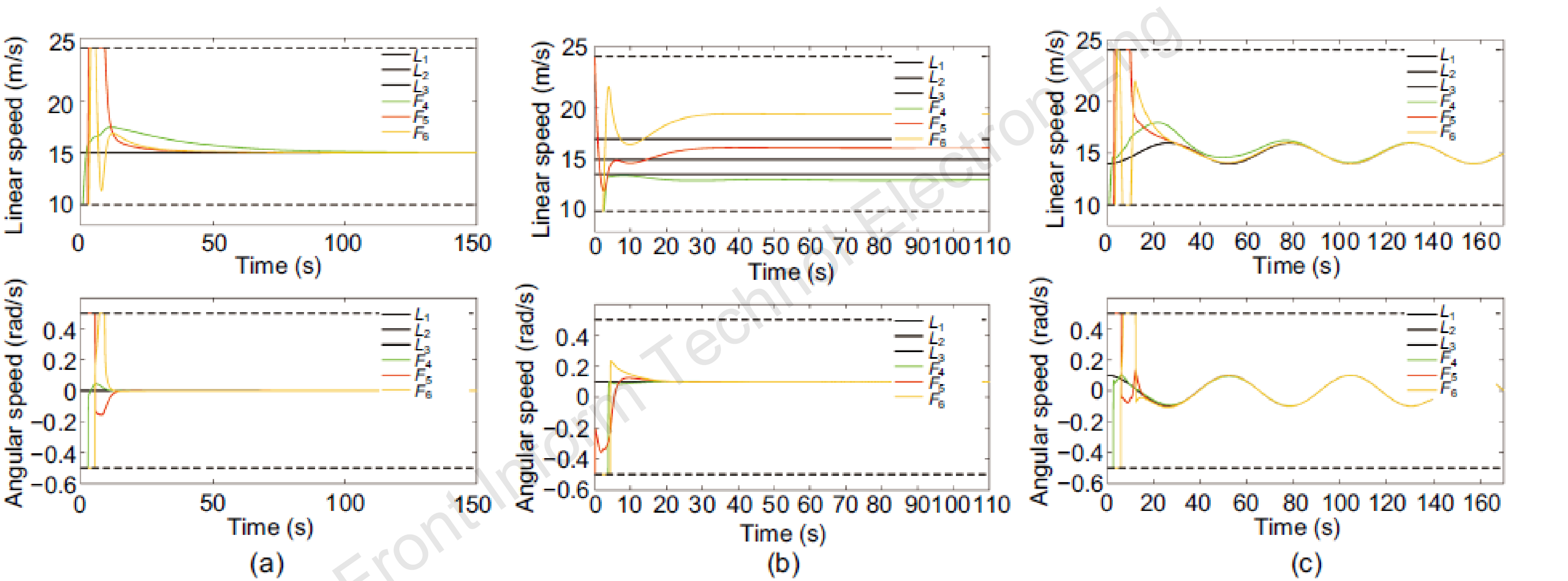


The affine formation moving along a sine curve



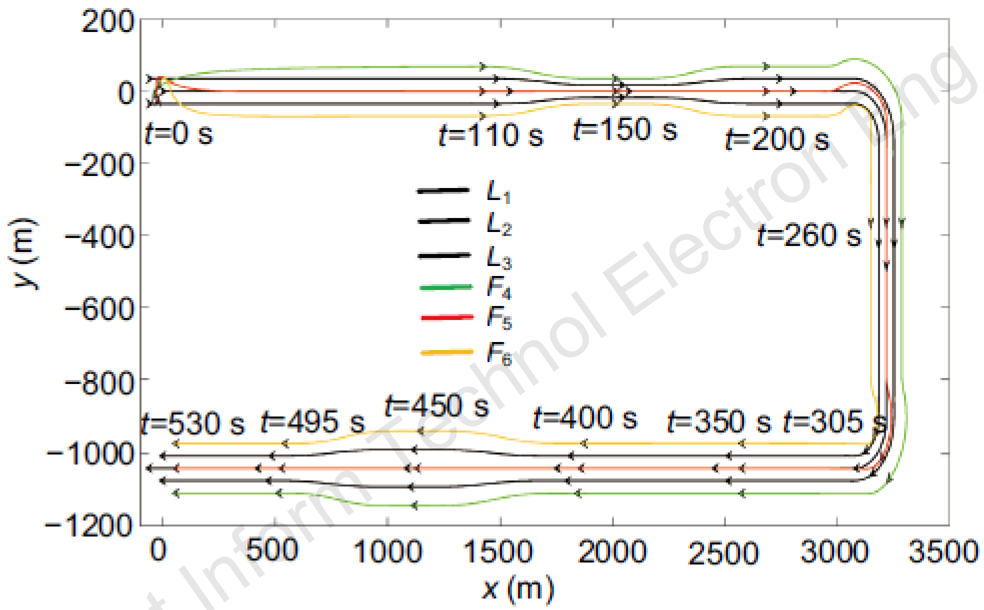
Tracking errors of three following fixed-wing UAVs moving along a sine curve

Major results



Constrained linear and angular speeds of the fixed-wing UAVs in the affine formation. The leaders move along different trajectories: (a) straight line; (b) circle; (c) sine curve (L: leader; F: follower)

Major results



A simulation example to illustrate the affine transformations of the six fixed-wing UAVs, including translation, zooming in, zooming out, and shearing

Conclusions

1. We proposed a distributed formation tracking control strategy based on the stress matrix to achieve the desired time-varying formation pattern and to track affine transformations along diverse trajectories, which improves the maneuverability of fixed-wing UAV formation in dynamic environments.

2. The convergence of the distributed control law was analyzed for unicycle-type agents. In addition, a saturated control strategy was proposed to meet the asymmetrical speed constraints of fixed-wing UAVs, and the effectiveness was verified by numerical simulations.

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