Luda ZHAO, Bin WANG, Xiaoping JIANG, Yicheng LU, Yihua HU, 2022. DIP-MOEA: a double-grid interactive preference based multi-objective evolutionary algorithm for formalizing preferences of decision makers. *Frontiers of Information Technology & Electronic Engineering*, 23(11):1714-1732. https://doi.org/10.1631/FITEE.2100508

DIP-MOEA: a double-grid interactive preference based multi-objective evolutionary algorithm for formalizing preferences of decision makers

Key words: Multi-objective evolutionary algorithm (MOEA); Formalizing preference of decision makers; Population renewal strategy; Preference interaction

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Motivation

1. President preference based multi-objective evolutionary algorithms (MOEAs) do not consider decision maker (DM) preference setting from the actual preference point of view.
2. In the real-world preference based MOEA problems, DM may provide preference information in different forms. However, no MOEA can effectively deal with formal DM preferences, which makes it impossible to solve practical problems flexibly.

Main idea

1. The formal DM preference in practical multi-objective optimization problems (MOPs) is fuzzily processed, the processed objective preference is mapped to the optimization objective in the optimization model, and the corresponding relationship between the DM's formal preference and model preference is established.

2. To avoid performance degradation caused by traditional MOEAs in solving preference-based MOPs, we consider to reset the individual dominance relationship in the objective space and set the individual updating strategy in the grid to keep the distribution of population and the accuracy of the preference area in the final solution set.

3. In practical problems, DM preference may change, which requires that the DM's preference interaction be considered when solving MOPs. Therefore, based on setting the initial grid, DM preference can be adjusted in real time by adjusting the grid.

Method

Preference degree and preference error

DM preference and preference transformation in MOPs

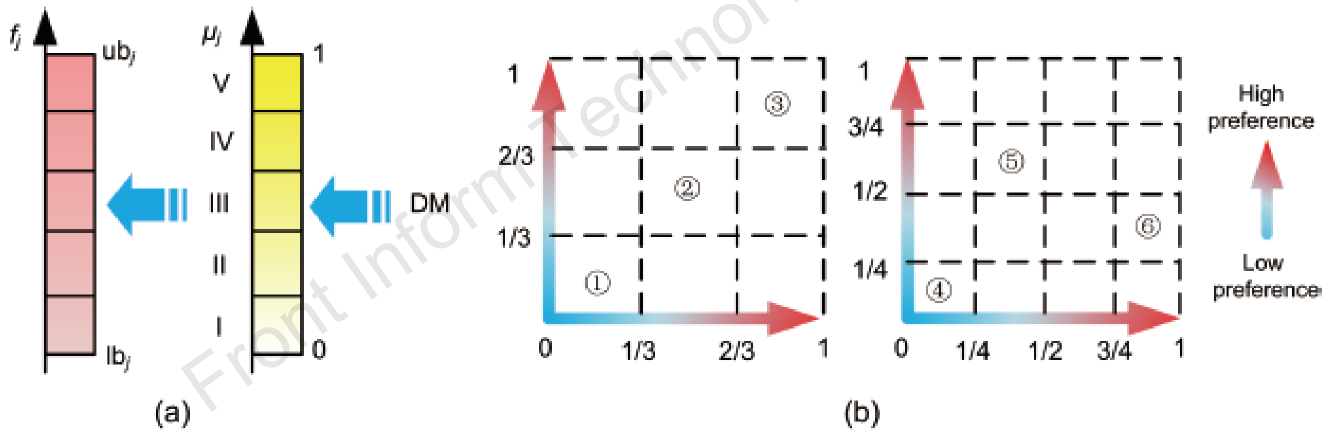

Fig. 1 An illustration of the transition between actual DM preference and preference in MOPs, where (a) shows the corresponding relationship between the decision preference of the j^{th} goal and the fuzzy membership function, and (b) indicates that when the DM preference has three or four levels in the two-dimensional problem, the preference is converted into a prior preference two-dimensional hypercube

Method (Cont’d)

Preference degree and preference error

Generation of the preference degree grid and preference error grid

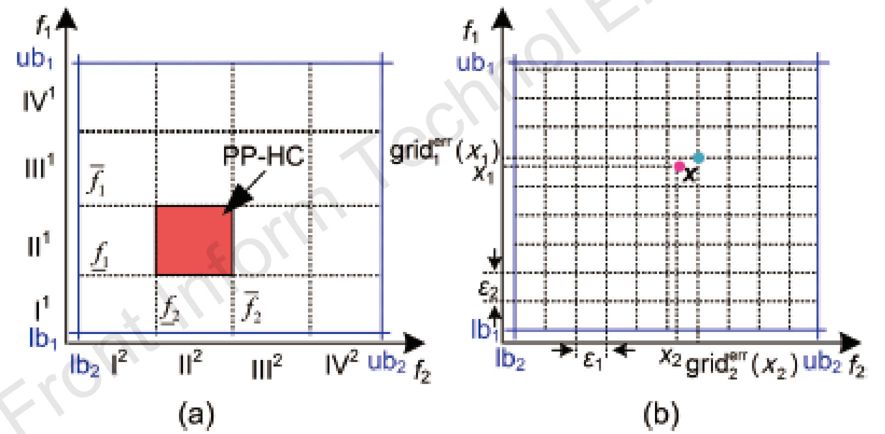

Fig. 2 An illustration used to determine the preference degree grid and preference error grid in a two-dimensional optimization problem, where (a) represents the DM preference degree grid and the PP-HC when the decision preference has four levels, and (b) represents the error grid determined by the single x (pink point) and error grid vertex (blue point)

Method (Cont'd)

Population renewal strategy

Preference degree dominance and preference error dominance

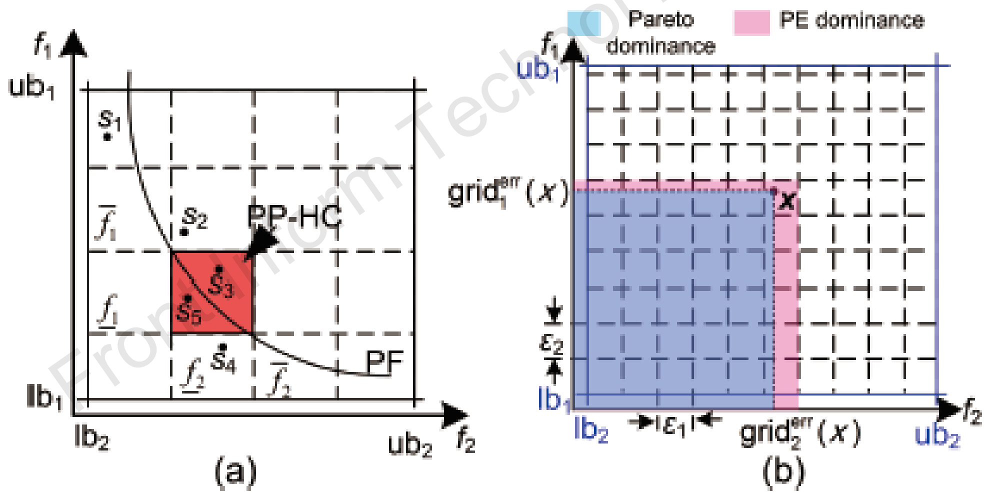

Fig. 3 Illustration of the preference degree (PD) dominance strategy (a) and preference error (PE) dominance strategy (b) in two-dimensional optimization problems

Method (Cont'd)

Population renewal strategy

Population updating in the grid

Algorithm 1 update(N_P, N_Q, η, T): updating for the PE Pareto set and PD Pareto set

Input: population size in preference space N_P, population size in preference error space N_Q, SBX operator parameter η, and number of population renewals T
Output: preference space population $P(T)$ and preference error space population $Q(T)$

$t = 0$
while $t \leq T$ **do**
  initialize $P(t)$ at random
  for $k = 1$ to N_P **do**
    for $n = 1$ to $0.5(N_P + N_Q)$ **do**
      for $m = 1$ to N_Q **do**
        for p_k at $P(t)$ **do**
          if $p_k \prec_{\mathrm{PE}} p_{k+1} \eta$ **then**
            $q_k \Leftarrow p_k$
            $Q(t) \Leftarrow q_k$
          end if
        end for
        for q_m at $Q(t)$ **do**
          if $(q_m \succ_{\mathrm{PE}} q_{m+1}) \vee [(q_m \nsucc_{\mathrm{PE}} q_{m+1}) \wedge (q_{m+1} \nsucc_{\mathrm{PE}} q_m)]$ **then**
            $Q(t) \Leftarrow q_m$
          end if
        end for
        $c' \Leftarrow \mathrm{SBX}\,(q_k, q_m, \eta)$
        $C(t) \Leftarrow c'(\mathrm{randperm}(2))$ /* randomly select one of the two individuals obtained by SBX into the set $C(t)$ */
        for c_n at $C(t)$ **do**
          if $\exists p_k \in P(t), c_n \succ_{\mathrm{PD}} p_k$ **then**
            $p_k \Leftarrow c_n$
          else if $\forall p_k \in P(t), (c_n \nsucc_{\mathrm{PD}} p_k) \wedge (q_k \nsucc_{\mathrm{PD}} c_n)$ **then**
            $p_k \Leftarrow c_n$
          end if
        end for
        if $\exists q_m \in Q(t), c_n \succ_{\mathrm{PE}} q_m$ **then**
          $q_m \Leftarrow c_n$
        else if $\forall q_m \in Q(t), (c_n \nsucc_{\mathrm{PE}} p_k) \wedge (q_k \nsucc_{\mathrm{PE}} c_n) \wedge (\forall j \in \{1, 2, \cdots, M\}, (q_k, c_n \geq \underline{f}_j) \wedge (q_k, c_n < \bar{f}_j))$ **then**
          if $c_n \succ q_m$ **then**
            $q_m \Leftarrow c_n$
          else if $(c_n \nsucc q_m) \wedge (q_m \nsucc c_n)$ **then**
            if $\sum_{j=1}^{M} \sqrt{(c_{n,j} - \mathrm{box}_j)^2} > \sum_{j=1}^{M} \sqrt{(q_{m,j} - \mathrm{box}_j)^2}$ **then** /* box_j denotes the coordinates of the origin of the j^{th} grid */
              $q_m \Leftarrow c_n$
            end if
          end if
        else if $\forall q_m \in Q(t), (c_n \nsucc_{\mathrm{PE}} p_k) \wedge (q_k \nsucc_{\mathrm{PE}} c_n) \wedge (\forall j \in \{1, 2, \cdots, M\}, (q_k, c_n < \underline{f}_j) \wedge (q_k, c_n \geq \bar{f}_j))$ **then**
          $Q(t) \Leftarrow c_n$
        end if
      end for
    end for
  end for
  $t = t + 1$
end while

Method (Cont'd)

Algorithm 2 DIP-MOEA

Input: $U_j, \mu_j = \varphi(f_j), N_{\mathrm{d}}, \varepsilon_j, N_P, N_Q, \eta, T$
Output: preference Pareto set $S(t)$

for $j = 1, 2, \cdots, M$ **do**
  $\mathrm{grid}_j^{\mathrm{pre}}, \mathrm{grid}_j^{\mathrm{err}} \Leftarrow \mathrm{grid}(U_j, \mu_j, N_{\mathrm{d}})$ /* determine two grid spaces */
end for
$P(t), Q(t) \Leftarrow \mathrm{update}(N_P, N_Q, \eta, T)$
if $U_j = U'_j, \mu_j = \mu'_j$ **then**
  return $\mathrm{grid}^{\mathrm{pre}}(U_j, \mu_j)$ and $\mathrm{grid}^{\mathrm{err}}(U_j, \mu_j)$
else if U_j and μ_j remain unchanged **then**
  $R(t) \Leftarrow P(t) \cup Q(t)$
end if
for $o = 1$ to $N_P + N_Q$ **do**
  for r_o at $R(t)$ **do**
    if $(r_{o,j} \geq (\underline{f}_j + \varepsilon_j)) \wedge (r_{o,j} < (\bar{f}_j + \varepsilon_j))$ **then**
      $R(t) \Leftarrow r_o$
    end if
  end for
end for

Major results

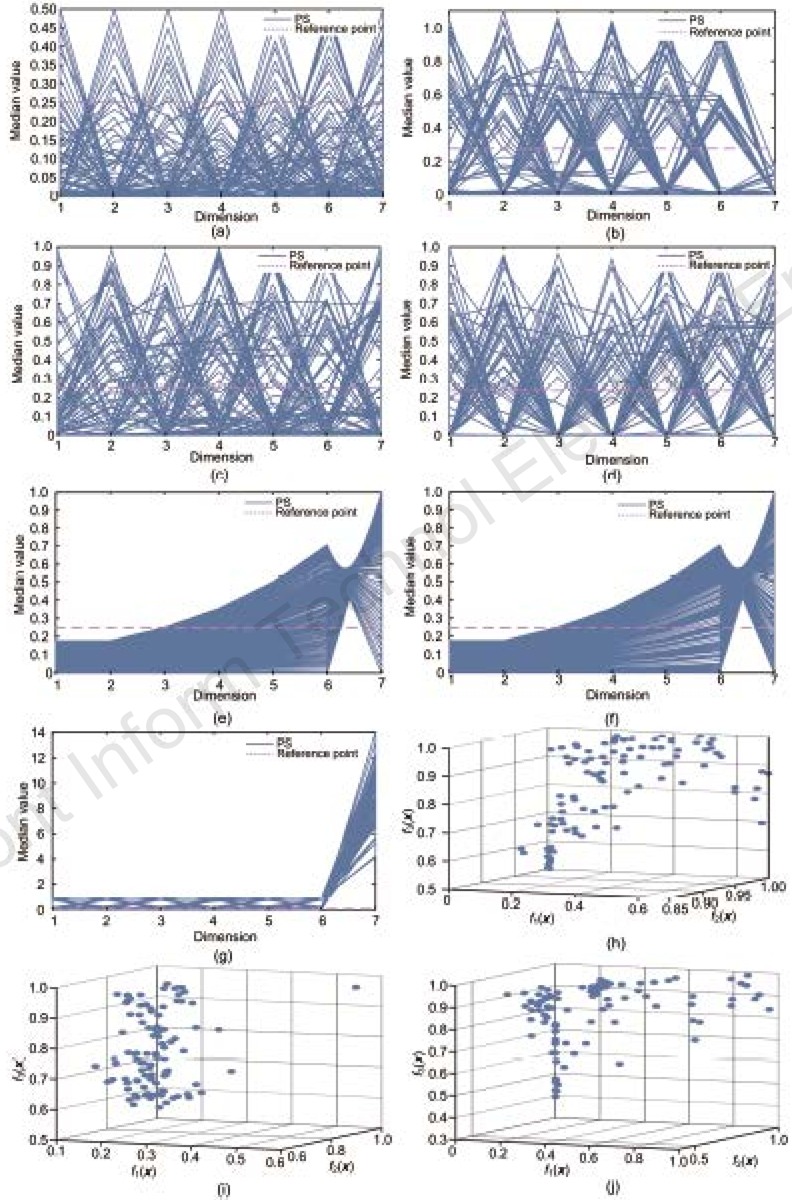

Fig. 5 An illustration of the simulation results of DIP-MOEA with DTLZ 1–7 and DDMOP 1–3 test functions: (a) DTLZ 1; (b) DTLZ 2; (c) DTLZ 3; (d) DTLZ 4; (e) DTLZ 5; (f) DTLZ 6; (g) DTLZ 7; (h) DDMOP 1; (i) DDMOP 2; (j) DDMOP 3

Major results (Cont'd)

Table 4 Mean and variance of the IGD-CF of function evaluations of 3-, 5-, and 7-dimensional DTLZ 1–7 and DDMOP 1–3 test functions with reference points for six algorithms

Test function	Mean (variance) of the IGD-CF		
	g-NSGA-II	AD-NSGA-II	AP-ε-MOEA
DTLZ 1 ($M=3$)	2.6255×10^{-1} (1.61×10^{-1})	2.4931×10^{-1} (1.61×10^{-1})	5.8680×10^{-2} (3.78×10^{-2})
DTLZ 1 ($M=5$)	1.3276×10^{-1} (9.64×10^{-2})	4.3605×10^{-2} (6.72×10^{-2})	4.3936×10^{-2} (2.63×10^{-2})
DTLZ 1 ($M=7$)	1.2549×10^{-3} (1.82×10^{-3})	1.2583×10^{-3} (1.15×10^{-3})	6.2768×10^{-4} (1.30×10^{-3})
DTLZ 2 ($M=3$)	2.6990×10^{-1} (3.68×10^{-2})	2.2418×10^{-1} (3.61×10^{-2})	2.6541×10^{-1} (5.76×10^{-2})
DTLZ 2 ($M=5$)	2.0993×10^{-1} (4.43×10^{-2})	3.9606×10^{-1} ($\mathbf{2.88 \times 10^{-2}}$)	4.2071×10^{-1} (4.67×10^{-2})
DTLZ 2 ($M=7$)	1.9323×10^{-1} (4.70×10^{-2})	4.6105×10^{-1} (3.80×10^{-2})	5.5307×10^{-1} (5.49×10^{-2})
DTLZ 3 ($M=3$)	2.5829×10^{-1} (6.73×10^{-2})	1.2965×10^{-1} (5.74×10^{-2})	1.9056×10^{-1} (9.26×10^{-2})
DTLZ 3 ($M=5$)	2.0633×10^{-1} (5.82×10^{-2})	2.7526×10^{-1} (6.30×10^{-2})	2.2535×10^{-1} (4.83×10^{-2})
DTLZ 3 ($M=7$)	2.1268×10^{-1} (4.44×10^{-2})	4.0089×10^{-1} (4.08×10^{-2})	$\mathbf{2.0262 \times 10^{-1}}$ (6.20×10^{-2})
DTLZ 4 ($M=3$)	7.1956×10^{-2} (5.09×10^{-2})	5.9021×10^{-2} (4.66×10^{-2})	1.4804×10^{-1} (6.92×10^{-2})
DTLZ 4 ($M=5$)	2.2731×10^{-2} (1.67×10^{-2})	8.8650×10^{-2} (3.19×10^{-2})	1.0215×10^{-1} (3.62×10^{-2})
DTLZ 4 ($M=7$)	1.5615×10^{-2} ($\mathbf{1.10 \times 10^{-2}}$)	2.3290×10^{-1} (8.75×10^{-2})	1.1789×10^{-1} (3.70×10^{-2})
DTLZ 5 ($M=3$)	5.7853×10^{-1} (8.60×10^{-2})	6.4711×10^{-1} (8.50×10^{-2})	5.9458×10^{-1} (1.05×10^{-1})
DTLZ 5 ($M=5$)	5.5285×10^{-1} (9.13×10^{-2})	7.4160×10^{-1} (7.11×10^{-2})	7.1218×10^{-1} (7.93×10^{-2})
DTLZ 5 ($M=7$)	6.7855×10^{-1} (7.66×10^{-2})	7.4559×10^{-1} (6.93×10^{-2})	8.2694×10^{-1} (5.27×10^{-2})
DTLZ 6 ($M=3$)	6.6938×10^{-1} (7.76×10^{-2})	6.8920×10^{-1} (6.76×10^{-2})	6.8968×10^{-1} ($\mathbf{4.29 \times 10^{-2}}$)
DTLZ 6 ($M=5$)	6.9735×10^{-1} (5.45×10^{-2})	7.2304×10^{-1} (5.16×10^{-2})	7.4542×10^{-1} (4.71×10^{-2})
DTLZ 6 ($M=7$)	6.4356×10^{-1} (7.12×10^{-2})	6.5466×10^{-1} (7.16×10^{-2})	7.1894×10^{-1} (6.85×10^{-2})
DTLZ 7 ($M=3$)	1.0137×10^{-2} (9.89×10^{-3})	2.3433×10^{-2} (2.46×10^{-2})	6.1486×10^{-3} ($\mathbf{1.01 \times 10^{-2}}$)
DTLZ 7 ($M=5$)	5.2773×10^{-2} (2.11×10^{-2})	6.4419×10^{-2} (2.39×10^{-2})	8.3321×10^{-2} (3.36×10^{-2})
DTLZ 7 ($M=7$)	3.4073×10^{-1} ($\mathbf{6.68 \times 10^{-2}}$)	$\mathbf{2.4502 \times 10^{-1}}$ (6.69×10^{-2})	4.5600×10^{-1} (9.00×10^{-2})
DDMOP 1_2021	2.8891×10^{-2} (6.55×10^{-2})	1.2381×10^{-2} (3.96×10^{-2})	2.0156×10^{-2} (3.19×10^{-2})
DDMOP 2_2021	5.8935×10^{-2} (5.90×10^{-2})	9.0942×10^{-2} (1.32×10^{-1})	7.7689×10^{-2} (8.55×10^{-2})
DDMOP 3_2021	3.3624×10^{-2} (4.45×10^{-2})	6.2386×10^{-2} (6.03×10^{-2})	7.1752×10^{-2} (1.01×10^{-1})

Test function	Mean (variance) of the IGD-CF		
	MOEA/D-PRE	RVEA-iGNG	DIP-MOEA
DTLZ 1 ($M=3$)	1.2707×10^{-2} (2.58×10^{-2})	$\mathbf{5.6749 \times 10^{-3}}$ ($\mathbf{7.78 \times 10^{-3}}$)	1.4484×10^{-2} (2.01×10^{-2})
DTLZ 1 ($M=5$)	$\mathbf{9.4270 \times 10^{-4}}$ ($\mathbf{1.23 \times 10^{-3}}$)	1.4327×10^{-3} (2.25×10^{-3})	6.6227×10^{-3} (7.94×10^{-3})
DTLZ 1 ($M=7$)	6.4750×10^{-4} (7.51×10^{-4})	1.7007×10^{-3} (2.35×10^{-3})	$\mathbf{3.5007 \times 10^{-4}}$ ($\mathbf{2.91 \times 10^{-4}}$)
DTLZ 2 ($M=3$)	3.9745×10^{-1} (7.36×10^{-2})	2.3938×10^{-1} ($\mathbf{3.50 \times 10^{-2}}$)	$\mathbf{2.0827 \times 10^{-1}}$ (5.55×10^{-2})
DTLZ 2 ($M=5$)	3.3851×10^{-1} (3.92×10^{-2})	$\mathbf{1.9406 \times 10^{-1}}$ (6.06×10^{-2})	4.7566×10^{-1} (5.99×10^{-2})
DTLZ 2 ($M=7$)	3.4325×10^{-1} (4.61×10^{-2})	$\mathbf{1.6575 \times 10^{-1}}$ ($\mathbf{3.67 \times 10^{-2}}$)	4.6784×10^{-1} (7.17×10^{-2})
DTLZ 3 ($M=3$)	1.3214×10^{-1} (8.25×10^{-2})	1.4193×10^{-1} (5.98×10^{-2})	$\mathbf{3.4416 \times 10^{-2}}$ ($\mathbf{1.06 \times 10^{-2}}$)
DTLZ 3 ($M=5$)	$\mathbf{1.6954 \times 10^{-1}}$ ($\mathbf{4.51 \times 10^{-2}}$)	2.1470×10^{-1} (5.72×10^{-2})	1.9729×10^{-1} (6.50×10^{-2})
DTLZ 3 ($M=7$)	2.1734×10^{-1} ($\mathbf{3.97 \times 10^{-2}}$)	3.7662×10^{-1} (6.93×10^{-2})	3.8277×10^{-1} (8.42×10^{-2})
DTLZ 4 ($M=3$)	2.1556×10^{-1} (9.71×10^{-2})	1.6176×10^{-1} (8.71×10^{-2})	$\mathbf{2.8781 \times 10^{-2}}$ ($\mathbf{2.26 \times 10^{-2}}$)
DTLZ 4 ($M=5$)	1.4945×10^{-1} (1.17×10^{-1})	1.0026×10^{-1} (4.74×10^{-2})	$\mathbf{2.1862 \times 10^{-2}}$ ($\mathbf{1.11 \times 10^{-2}}$)
DTLZ 4 ($M=7$)	1.6275×10^{-1} (7.24×10^{-2})	1.0274×10^{-1} (5.74×10^{-2})	$\mathbf{9.998 \times 10^{-3}}$ (6.93×10^{-2})
DTLZ 5 ($M=3$)	7.7206×10^{-1} (1.15×10^{-1})	6.9176×10^{-1} ($\mathbf{7.78 \times 10^{-2}}$)	$\mathbf{4.9604 \times 10^{-1}}$ (8.60×10^{-2})
DTLZ 5 ($M=5$)	8.1837×10^{-1} (6.04×10^{-2})	7.7451×10^{-1} ($\mathbf{4.07 \times 10^{-2}}$)	$\mathbf{5.5001 \times 10^{-1}}$ (6.32×10^{-2})
DTLZ 5 ($M=7$)	7.8470×10^{-1} (6.36×10^{-2})	8.1557×10^{-1} (6.03×10^{-2})	$\mathbf{5.9275 \times 10^{-1}}$ ($\mathbf{4.83 \times 10^{-2}}$)
DTLZ 6 ($M=3$)	7.1370×10^{-1} (6.77×10^{-2})	6.7280×10^{-1} (6.21×10^{-2})	$\mathbf{5.7414 \times 10^{-1}}$ (6.34×10^{-2})
DTLZ 6 ($M=5$)	6.6666×10^{-1} ($\mathbf{3.90 \times 10^{-2}}$)	7.5065×10^{-1} (6.50×10^{-2})	$\mathbf{5.3934 \times 10^{-1}}$ (5.79×10^{-2})
DTLZ 6 ($M=7$)	5.9220×10^{-1} ($\mathbf{4.07 \times 10^{-2}}$)	6.7846×10^{-1} (5.14×10^{-2})	$\mathbf{4.4023 \times 10^{-1}}$ (7.15×10^{-2})
DTLZ 7 ($M=3$)	$\mathbf{4.6170 \times 10^{-3}}$ (1.07×10^{-2})	1.4878×10^{-2} (2.55×10^{-2})	2.7153×10^{-2} (5.17×10^{-2})
DTLZ 7 ($M=5$)	1.4211×10^{-1} (8.65×10^{-2})	$\mathbf{4.4412 \times 10^{-2}}$ ($\mathbf{2.08 \times 10^{-2}}$)	9.8335×10^{-2} (6.11×10^{-2})
DTLZ 7 ($M=7$)	3.1091×10^{-1} (7.16×10^{-2})	3.8073×10^{-1} (7.79×10^{-2})	3.2533×10^{-1} (1.12×10^{-1})
DDMOP 1_2021	3.8989×10^{-2} (6.90×10^{-2})	6.7612×10^{-3} (1.63×10^{-2})	$\mathbf{1.4955 \times 10^{-3}}$ ($\mathbf{1.13 \times 10^{-2}}$)
DDMOP 2_2021	6.9621×10^{-2} (5.52×10^{-2})	5.2734×10^{-2} (5.42×10^{-2})	$\mathbf{2.0811 \times 10^{-2}}$ ($\mathbf{1.68 \times 10^{-2}}$)
DDMOP 3_2021	7.2575×10^{-2} (9.48×10^{-2})	4.6514×10^{-2} (4.02×10^{-2})	$\mathbf{2.0344 \times 10^{-2}}$ ($\mathbf{1.83 \times 10^{-2}}$)

Best results are in bold

Major results (Cont'd)

Table 5 Mean and variance of the IGD and HV metrics of the 3-, 5-, and 7-dimensional DTLZ 1–7 and DDMOP 1–3 test functions of the three test algorithms

Test function	Mean (variance) of the IGD		
	PICEA-g	iPICEA-g	DIP-MOEA
DTLZ 1 ($M=3$)	1.8519×10^{-1} (2.05×10^{-1})	**1.3426×10^{-1} (1.21×10^{-1})**	5.8616 (2.53)
DTLZ 1 ($M=5$)	8.4811 (7.69)	**2.7903×10^{-1} (2.45×10^{-1})**	9.2005 (3.49)
DTLZ 1 ($M=7$)	1.5385×10 (4.22)	**1.2306×10** (4.28)	7.1718 (**3.12**)
DTLZ 2 ($M=3$)	1.7296×10^{-1} (1.91×10^{-2})	1.5563×10^{-1} (1.84×10^{-2})	**1.3392×10^{-1} (1.17×10^{-2})**
DTLZ 2 ($M=5$)	3.9389×10^{-1} (2.60×10^{-2})	3.5520×10^{-1} (2.07×10^{-2})	**3.2925×10^{-1} (1.89×10^{-2})**
DTLZ 2 ($M=7$)	5.5892×10^{-1} (3.59×10^{-2})	5.1515×10^{-1} (**2.29×10^{-2}**)	**4.9718×10^{-1}** (2.53×10^{-2})
DTLZ 3 ($M=3$)	1.8070×10^{2} (3.77×10)	1.8947×10^{2} (3.62×10)	**9.3716×10 (2.20×10)**
DTLZ 3 ($M=5$)	2.1748×10^{2} (4.95×10)	2.1046×10^{2} (5.41×10)	**1.4563×10^{2} (3.20×10)**
DTLZ 3 ($M=7$)	2.6285×10^{2} (4.73×10)	2.2471×10^{2} (4.67×10)	**1.4404×10^{2} (2.46×10)**
DTLZ 4 ($M=3$)	4.6685×10^{-1} (1.78×10^{-1})	**2.7990×10^{-1} (1.52×10^{-1})**	4.5781×10^{-1} (2.93×10^{-1})
DTLZ 4 ($M=5$)	5.5357×10^{-1} (**6.96×10^{-2}**)	**4.4337×10^{-1}** (8.03×10^{-2})	4.8289×10^{-1} (1.16×10^{-1})
DTLZ 4 ($M=7$)	6.7195×10^{-1} (4.90×10^{-2})	5.8134×10^{-1} (7.61×10^{-2})	**5.6333×10^{-1} (4.70×10^{-2})**
DTLZ 5 ($M=3$)	1.0826×10^{-1} (**1.61×10^{-2}**)	1.0051×10^{-1} (2.20×10^{-2})	**7.7690×10^{-2}** (1.73×10^{-2})
DTLZ 5 ($M=5$)	2.4964×10^{-1} (3.30×10^{-2})	2.0411×10^{-1} (**3.08×10^{-2}**)	**1.8006×10^{-1}** (3.75×10^{-2})
DTLZ 5 ($M=7$)	2.8807×10^{-1} (3.60×10^{-2})	**2.5830×10^{-1}** (3.43×10^{-2})	4.1907 (**2.56×10^{-2}**)
DTLZ 6 ($M=3$)	5.9943 (**4.68×10^{-1}**)	5.8519 (5.05×10^{-1})	**5.1665** (6.58×10^{-1})
DTLZ 6 ($M=5$)	7.3646 (**3.90×10^{-1}**)	6.8479 (4.63×10^{-1})	**6.3735** (4.60×10^{-1})
DTLZ 6 ($M=7$)	7.6944 (**4.66×10^{-1}**)	6.9277 (5.09×10^{-1})	**6.6311** (5.14×10^{-1})
DTLZ 7 ($M=3$)	4.2392 (**7.74×10^{-1}**)	4.3811 (8.24×10^{-1})	**2.8257** (9.16×10^{-1})
DTLZ 7 ($M=5$)	**6.7129** (8.95×10^{-1})	9.4522 (**1.05**)	7.5228 (1.43)
DTLZ 7 ($M=7$)	**9.9699 (1.46)**	1.4307×10 (1.64)	1.2156×10 (1.76)
DDMOP 1_2021	2.0801×10 (7.20)	2.3210×10 (5.73)	**9.0423 (4.25)**
DDMOP 2_2021	3.0203×10 (9.43)	2.7787×10 (**9.41**)	**1.8086×10** (1.09×10)
DDMOP 3_2021	3.0252×10 (1.20×10)	3.6388×10 (1.47×10)	**2.4773×10 (1.18×10)**

Test function	Mean (variance) of the HV		
	PICEA-g	iPICEA-g	DIP-MOEA
DTLZ 1 ($M=3$)	3.4334×10^{-1} (3.55×10^{-1})	2.3117×10^{-1} (2.76×10^{-2})	**4.0571×10^{-1} (2.49×10^{-2})**
DTLZ 1 ($M=5$)	2.7726×10^{-1} (4.16×10^{-2})	1.6326×10^{-1} (**3.50×10^{-2}**)	**3.9156×10^{-1}** (4.10×10^{-2})
DTLZ 1 ($M=7$)	2.4125×10^{-1} (4.86×10^{-2})	1.0126×10^{-1} (**3.57×10^{-2}**)	**3.5914×10^{-1}** (3.67×10^{-2})
DTLZ 2 ($M=3$)	2.5922×10^{-1} (**2.72×10^{-1}**)	5.1173×10^{-1} (3.42×10^{-1})	**5.5925×10^{-1}** (2.95×10^{-1})
DTLZ 2 ($M=5$)	1.7515×10^{-1} (**2.35×10^{-1}**)	1.6814×10^{-1} (3.29×10^{-1})	**5.5482×10^{-1}** (3.64×10^{-1})
DTLZ 2 ($M=7$)	0.0000 (0.00)	0.0000 (0.00)	0.0000 (0.00)
DTLZ 3 ($M=3$)	1.5815×10^{-1} (6.58×10^{-2})	1.7634×10^{-1} (7.90×10^{-2})	**3.1287×10^{-1} (5.86×10^{-2})**
DTLZ 3 ($M=5$)	1.2097×10^{-1} (7.44×10^{-2})	2.0397×10^{-1} (7.77×10^{-2})	**4.7002×10^{-1} (5.97×10^{-2})**
DTLZ 3 ($M=7$)	1.4592×10^{-1} (8.85×10^{-2})	1.4855×10^{-1} (7.39×10^{-2})	**4.9381×10^{-1} (6.18×10^{-2})**
DTLZ 4 ($M=3$)	9.2503×10^{-2} (1.94×10^{-2})	6.3336×10^{-2} (1.78×10^{-2})	**1.3063×10^{-1} (1.62×10^{-2})**
DTLZ 4 ($M=5$)	3.0108×10^{-3} (4.15×10^{-3})	7.3813×10^{-3} (**1.01×10^{-2}**)	**1.7863×10^{-2}** (1.81×10^{-2})
DTLZ 4 ($M=7$)	1.0089×10^{-3} (2.95×10^{-3})	1.2846×10^{-3} (4.56×10^{-3})	**1.1347×10^{-2} (1.41×10^{-2})**
DTLZ 5 ($M=3$)	0.0000 (0.00)	0.0000 (**0.00**)	**2.9057×10^{-4}** (1.19×10^{-3})
DTLZ 5 ($M=5$)	0.0000 (0.00)	0.0000 (0.00)	**0.0000 (0.00)**
DTLZ 5 ($M=7$)	6.6314×10^{-11} (3.63×10^{-10})	0.0000 (0.00)	**0.0000 (0.00)**
DTLZ 6 ($M=3$)	2.3731×10^{-1} (4.03×10^{-2})	3.4334×10^{-1} (3.55×10^{-2})	**3.7465×10^{-1} (3.44×10^{-2})**
DTLZ 6 ($M=5$)	3.1946×10^{-1} (**3.93×10^{-2}**)	2.7726×10^{-1} (4.16×10^{-2})	**3.6681×10^{-1}** (4.61×10^{-2})
DTLZ 6 ($M=7$)	3.0356×10^{-1} (**4.23×10^{-2}**)	2.4125×10^{-1} (4.86×10^{-2})	**3.2726×10^{-1}** (4.94×10^{-2})
DTLZ 7 ($M=3$)	2.6318×10^{-1} (1.22×10^{-1})	2.5273×10^{-1} (8.16×10^{-2})	**8.1368×10^{-2} (5.40×10^{-2})**
DTLZ 7 ($M=5$)	**4.2776×10^{-1}** (9.47×10^{-2})	2.9851×10^{-1} (9.33×10^{-2})	5.1528×10^{-2} (**5.73×10^{-2}**)
DTLZ 7 ($M=7$)	**4.6807×10^{-1}** (8.14×10^{-2})	2.5790×10^{-1} (8.04×10^{-2})	2.3490×10^{-2} (**2.42×10^{-2}**)
DDMOP 1_2021	5.4154×10^{-2} (2.40×10^{-2})	**1.0945×10^{-1} (1.88×10^{-2})**	9.6948×10^{-2} (1.97×10^{-2})
DDMOP 2_2021	4.5959×10^{-3} (4.26×10^{-3})	**9.8423×10^{-3} (1.19×10^{-2})**	5.1734×10^{-3} (4.53×10^{-3})
DDMOP 3_2021	8.5418×10^{-4} (**1.68×10^{-3}**)	1.1760×10^{-3} (1.78×10^{-3})	**1.6264×10^{-3}** (3.67×10^{-3})

Best results are in bold

Major results (Cont'd)

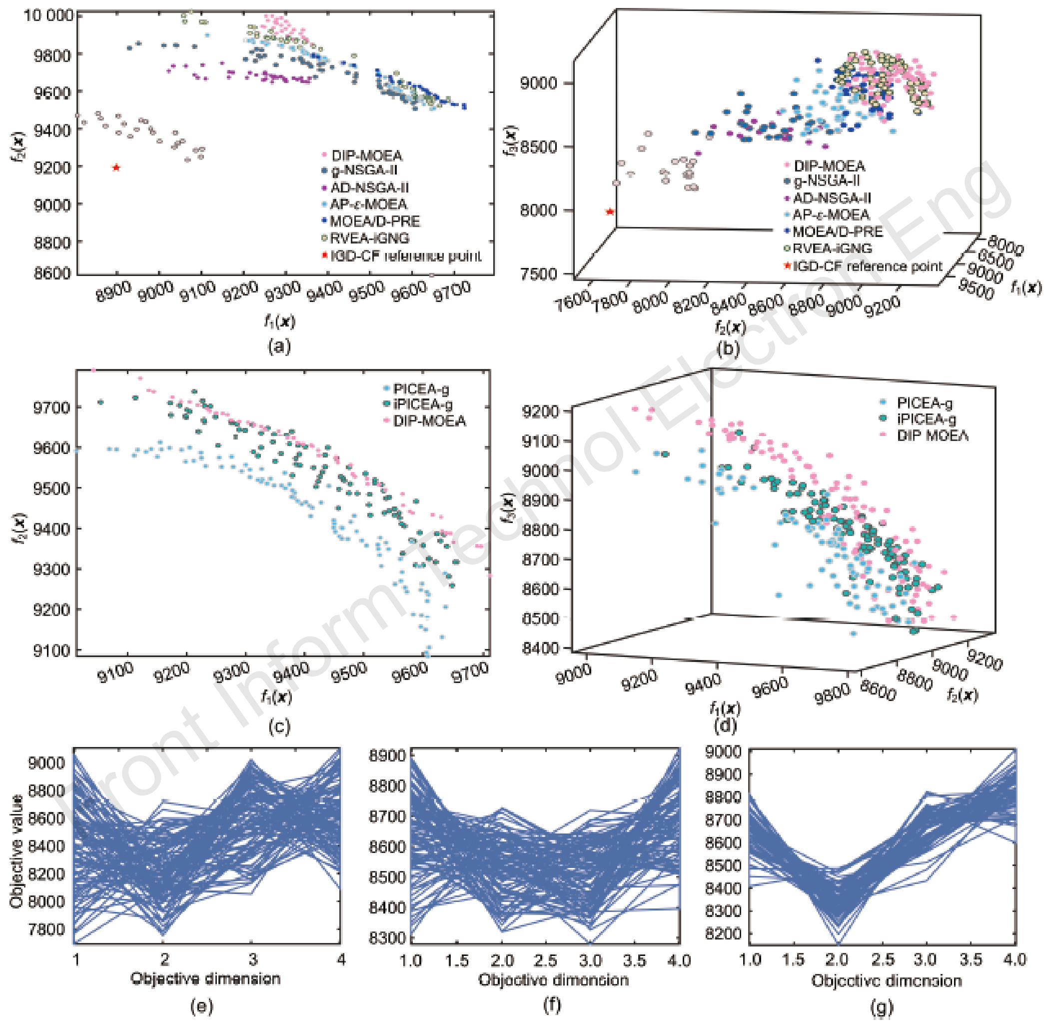

Fig. 6 An illustration of the simulation results of the preference-based MOEAs (particle preference and all preferences) on MOKP: (a) 2D MOKP with six kinds of preference-based MOEAs (partial preference); (b) 3D MOKP with six kinds of preference-based MOEAs (partial preference); (c) 2D MOKP with three kinds of preference-based MOEAs (all preferences); (d) 3D MOKP with three kinds of preference-based MOEAs (all preferences); (e) 4D MOKP with MOEA/D-PRE (all preferences); (f) 4D MOKP with RVEA-iGNG (all preferences); (g) 4D MOKP with DIP-MOEA (all preferences)

Major results (Cont'd)

Table 6 Values of the HV metric and runtime

Method	HV 2D	HV 3D	HV 4D	Runtime (s) 2D	Runtime (s) 3D	Runtime (s) 4D
g-NSGA-II	9.65×10^{7}	8.12×10^{12}	9.11×10^{16}	33.2	51.6	80.1
AD-NSGA-II	8.01×10^{8}	3.23×10^{12}	6.00×10^{17}	**20.1**	**44.4**	**71.6**
AP-ε-MOEA	8.30×10^{8}	2.46×10^{13}	8.77×10^{16}	26.3	47.2	74.4
MOEA/D-PRE	8.45×10^{8}	3.77×10^{12}	6.23×10^{17}	23.1	46.2	75.2
RVEA-iGNG	8.57×10^{8}	2.12×10^{13}	6.11×10^{17}	22.9	47.8	77.6
PICEA-g	8.37×10^{8}	2.63×10^{13}	6.65×10^{17}	25.6	47.1	74.3
iPICEA-g	8.28×10^{8}	2.61×10^{13}	6.03×10^{17}	25.4	47.7	73.3
DIP-MOEA	$\mathbf{8.84 \times 10^{8}}$	$\mathbf{2.75 \times 10^{13}}$	$\mathbf{6.82 \times 10^{17}}$	21.2	46.6	72.1

Best results are in bold

Conclusions

1. To solve preference-based MOPs, the corresponding relationship between formal preference and model preference of the DM is established by fuzzifying the preference, and the individual dominance relationship and updating strategy are reset in the objective space.

2. We consider the preference interaction problem when solving MOPs, and adjust the DM preferences in real time by adjusting the grid, retaining the accuracy of population distribution and preference area in the final solution set.

3. The experimental results show that DIP-MOEA can quickly solve the test problems and has good performance concerning the distribution of the Pareto front and the uniformity in the final solution set.

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