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A matrix-based static approach to analysis of finite state machines

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Motivation

1. The dynamic process of a finite state machine (FSM) can be represented by corresponding state transition matrix using semi-tensor product (STP) model. A drawback with STP model is the problem of "explosion of dimension". That is, the dimension of state transition matrices under STP model increases exponentially with the increase of time steps.
2. The related algorithms and criteria based on STP model for the issues of controllability and closed-loop detection are very difficult to obtain.
3. To simplify the criterion of controllability of a finite state machine with too many states.

Main idea

1. A static approach for closed-loop detection

- The definition of a single closed loop is

$$(T\delta_n^{X_s}).*\delta_n^{X_s}=\delta_n^{X_s}\wedge(T^{\mathrm{T}}\delta_n^{X_s}).*\delta_n^{X_s}=\delta_n^{X_s}.$$

  It is clear that the reachable state set of a single closed loop is itself.

- From the definition, the criterion of closed-loop detection for a finite state machine with $\|T\|=0$ is derived.

- Then the criterion is generalized to the case $\|T\|>0$.

Main idea (Cont'd)

2. A static approach for controllability

- By the fact that if two states are controllable in one direction, then the loop these two states form in a reverse direction is closed, closed-loop detection can be integrated into the issue of controllability.
- To cope with the problem of "explosion of dimension". A procedure that virtualizes a connection from the goal state to the start state is presented, and uses the idea of closed-loop by virtual state method.

Main idea (Cont’d)

3. A static approach for controllable equivalent form

- Clearly, for any deterministic FSM (DFSM), all states in a single closed loop are mutually controllable. Therefore, we can combine a single closed loop into a single “aggregate” state without changing controllability of the whole DFSM.
- For a DFSM, the controllable equivalent form is often not unique. In practice, we are more concerned with the one with the least number of states.

Major results

1. The algorithms and criteria for closed-loop detection based on the static approach are presented.

- The case $\|T\| = 1$

$$(\tilde{T} - I_{n\times n})\delta_n^{X_s} = T\delta_n^{\alpha} \vee (T - I_{n\times n})\delta_n^{X_s} = 0$$

- The case $\|T\| = 2$

$$(\tilde{T} - I_{n\times n})\delta_n^{X_s} = T\delta_n^{S} \vee (\hat{T} - I_{n\times n})\delta_n^{X_s} = T\delta_n^{S} \vee (T - I_{n\times n})\delta_n^{X_s} = 0$$

- The case $\|T\| > 2$

$$(\widehat{T} - I_{n\times n})\delta_n^{X_s} = T\delta_n^{S} \vee (T - I_{n\times n})\delta_n^{X_s} = 0$$

Major results (Cont'd)

2. The algorithms and criteria for controllability based on the static approach are presented.

Lemma 5 Consider DFSM $M = (X = \Delta_n, E, f, x_0)$. If $\boldsymbol{x}_a \in \Delta_n$ is controllable to $\boldsymbol{x}_b \in \Delta_n$, then there exists a closed loop containing x_a and x_b in $\boldsymbol{M}^\kappa$, where $\boldsymbol{M}^\kappa$ is defined as

$$M^\kappa(i,j) = \begin{cases} 1, & \text{if } i = a \wedge j = b, \\ T(i,j), & \text{otherwise.} \end{cases}$$

Algorithm 7 Two-state controllability with the virtual connection method for $M = (X = \Delta_n, E, f, x_0)$

Input: $\boldsymbol{T}$, goal state x_b, start state x_a, $S := \{i \,||\Psi(\mathrm{col}_i(\boldsymbol{T}))| > 1\}$

$\boldsymbol{T}(x_a, x_b) = 1$
Construct $\boldsymbol{T}_{V(S)}$
Construct $\widehat{\boldsymbol{T}}_{V(S)}$ where α is specified as the start state x_a
$C := \Big\{\Psi(x) \,\Big|\, (\widehat{\boldsymbol{T}}_{V(S)} - \boldsymbol{I}_{(n+|H^*|)\times(n+|H^*|)})\boldsymbol{x} = \boldsymbol{T}_{V(S)}\delta^S_{n+|H^*|} \wedge (\boldsymbol{T}\boldsymbol{x}).*\boldsymbol{x} = \boldsymbol{x} \wedge (\boldsymbol{T}^{\mathrm{T}}\boldsymbol{x}).*\boldsymbol{x} = \boldsymbol{x}\Big\}$
if $C \neq \varnothing$ **then**
    BREAK: x_a is controllable to x_b
else
    BREAK: x_a is not controllable to x_b
end if

Major results (Cont'd)

3. The algorithms for controllable equivalent form based on the static approach are presented.

Algorithm 5 Controllable equivalent form of M

Input: $\boldsymbol{T}$, W_M, $\boldsymbol{T}^* := [\,]$, $\boldsymbol{T}^\circ := [\,]$

Output: $\boldsymbol{T}^\circ$ is the TM model of the controllable equivalent form of M; the controllable equivalent function f is defined as $y_i = f(x_i) = \Psi(\mathrm{col}_i(\boldsymbol{T}^*))$

for $i \in W_M$ **do**
  $a = \min(\mathrm{find}(\delta_n^i))$
  $\mathbf{1}_n = \mathrm{ones}(n,1)$
  $\boldsymbol{b} = \mathbf{1}_n - \delta_n^i$
  $\boldsymbol{T}^* = \mathrm{diag}(\boldsymbol{b})$
  $\boldsymbol{T}^*(:,a) = \delta_n^i$
  $\boldsymbol{T}^*(:,\ \mathrm{all}(\boldsymbol{T}^* == 0)) = [\,]$
  $\boldsymbol{A} = ((\boldsymbol{T}.') * \boldsymbol{T}^*)$
  $\boldsymbol{A}(:,a) = \boldsymbol{A}(:,a) - \delta_n^i$
  $\boldsymbol{T}^\circ = ((\boldsymbol{A}.') * \boldsymbol{T}^*)$
end for

Algorithm 6 Finding all closed loops and the minimal controllable equivalent form for M

Input: $\boldsymbol{T}$, $\boldsymbol{T}_{\min} := \boldsymbol{T}$, $W^* := \varnothing$

Output: $\boldsymbol{T}_{\min}^\circ$ is the TM model of the minimal controllable equivalent form; W^* contains all closed loops in M

repeat
  $\boldsymbol{T}_{\min} := \boldsymbol{T}_{\min}^\circ$
  Apply Algorithm 4 to obtain $W_{\min}$ for $\boldsymbol{T}_{\min}$
  Apply Algorithm 5 to obtain $\boldsymbol{T}_{\min}^\circ$ for $\boldsymbol{T}_{\min}$
  $W^* := W^* \cup W_{\min}$
until $\boldsymbol{T}_{\min}^\circ = \boldsymbol{T}_{\min}$

Conclusions

1. A matrix-based static approach for detection of closed-loop has been proposed. Based on the static view, we propose the definitions of controllable equivalent form and minimal controllable equivalent form.

2. The static approach is then extended for controllability and eliminates the “explosion of complexity” inherent in the existing approaches.

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